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Abstract
SINGAPORE STUDENTS' PERFORMANCE IN TWO "REAL-LIFE" MATHEMATICS WORD PROBLEMS
About 900 upper Primary students from 20 Singapore schools were given two written word problems. One word problem asked for the number of taxis needed for 9 adult passengers and the other was a nonsensical one, in that the information given (the number of passengers in a bus) had nothing to do with the question asked (the age of the bus driver). Data collection (which includes answer scripts of the students and interview data on about 60 students) will be completed by mid-1996 and the resulting analysis should be able to indicate whether students actually take into consideration all the information given and relate them to sensible solutions or whether they are prone to indiscriminate and mechanical application of procedures. The results will be reported at this conference and should prove interesting and instructive, given that problem solving is the focus of the Singapore Primary school mathematics curriculum.

Programme descriptionKey words
Assessment, learning and cognitive processesMathematics word problems, problem solving

Biodata
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SINGAPORE STUDENTS' PERFORMANCE IN TWO "REAL-LIFE" MATHEMATICS WORD PROBLEMS

Introduction

Of late, policy makers and employers have expressed concern that our schools seem to be turning out students who cannot think. For example, the Prime Minister of Singapore, Mr. Goh Chok Tong recently stated that we need to encourage children to think and be creative, and to this end, the curriculum would have to be trimmed. Prior to this, the Deputy Prime Minister, BG Lee Hsien Loong, had said that he found it difficult to believe that students of his time were "dimmer" than students nowadays just because they managed only 50 As, whereas it is commonplace nowadays to get 1000 As at 'A' levels (Straits Times, Monday, June 3, 1996). Also, the President of the Nanyang Technological University was concerned that the inordinate number of As at the GCE 'A' level could be a sign of students just being more examination-smart rather than being good/able students. Employers, too, have expressed dissatisfaction with many job interviewees who were unable to answer questions that required thinking rather than mere regurgitating and questioned how well education was meeting the needs of society. Clements & Ellerton (1995) have documented that

pencil-and-paper tests do not effectively measure mathematical understanding. Zhang (1996) found that students in China generally did very well in computational exercises and routine problems, yet gave an answer to a word problem which actually had no answer because sufficient data had not been provided.

The Singapore Primary Mathematics Syllabus has problem solving as its focus. But there seems to be a gap between the expressed aims of the curriculum and the implemented curriculum, as evidenced by statements from the policy makers, etc., above. So, I wanted to study whether upper Primary school students in Singapore could use number sense and reasoning to solve two "real-life" word problems, and see whether there is or how widespread the perceived gap is between the planned and implemented curriculum, as revealed by performance in two "real-life" word problems.

The study

A total of 987 students (462 boys, 525 girls) from 29 Singapore schools

(23 P6, 3 P5 and 3 P4) were given the following two "real life" word problems to solve:

Question 1.

If a taxi is only allowed to take 4 adult passengers, how many taxis would be needed for 9 adult passengers? Show your working. Use diagrams or pictures if you need to.

Question 2.

There are 60 adults and 10 children as passengers in a bus. What is the age of the bus driver? Show your working. Use diagrams or pictures if you need to.

These questions were written on a paper with enough space for working to be shown. The mathematics teachers of the classes concerned (who were attending an inservice class I was conducting) administered the tasks according to the directions given in Appendix 1. Prior to this, the teachers themselves were briefed by me as to the purpose of the tasks and how to administer the tasks/test. Three students, one high ability (Hi), one medium ability (Me), and one low ability (Lo)--based on the teacher's familiarity with students' ability--were then chosen from each class, and interviewed, while doing the two tasks again. Results of the pencil and paper tasks were analysed and are summarised in Tables 1 to 4. Results from the interviews are also discussed.

Results and Discussion

P6 (23 classes)	Boys	Girls	Total
# of Students	364	388	752
Question 1	74%	84%	79%
Question 2	5%	14%	10%

Table 1

From Table 1, Question 1, (on the number of taxis), was answered correctly by 74% of the boys and 84% of the girls in Primary 6.

Overall, 79% of the Primary 6 students could correctly answer Question 1. The corresponding results for Question 2 are 5%, 14% and 10% respectively, showing a marked drop in the performance.

P5 (3 classes)	Boys	Girls	Total
# of Students	37	76	113
Question 1	76%	79%	78%
Question 2	0%	5%	4%

Table 2

From Table 2, it can be seen that the performance of Primary 5 students do not differ too much from that of Primary 6 students (although the sample is appreciably smaller) for Question 1. However, the Primary 5 did worse in Question 2, compared to the Primary 6.

P4 (3 classes)	Boys	Girls	Total
# of Students	61	61	122
Question 1	80%	74%	77%
Question 2	2%	8%	5%

Table 3

Table 3 shows that, once again, there was not much difference in the performance in Question 1, and that Primary 4 students performed just as well as their counterparts in Primary 5 and 6. However, the Primary 4 boys did slightly better than girls in Question 1 which was the reverse in Primary 5 and 6. In question 2, however, the Primary 4 girls outperformed the boys, even though the overall performance was still low.

Overall (29 classes)	Boys	Girls	Total
# of Students	462	525	987
Question 1	75%	82%	79%

Question 2 5% 12% 8%

Table 4
Table 4 shows that overall, girls outperformed boys in both questions. Even so, about a quarter of the boys, and a fifth of the girls did not respond correctly to Question 1. Given that the numbers involved were very small, it could not have been the computation that was a difficulty. Neither was reading and understanding of the question a drawback, as the interviews clearly showed that students could both read and understand the question asked. Also, a look at the answer scripts of these students revealed that they had mechanically done the question, generally giving the answer
"2 1/4 (two and a quarter)." These students clearly did not use number sense.
Another, perhaps greater, concern was the generally weak performance in Question 2, which required students to recognise the inappropriateness of the question.
From the answer scripts and interview responses to Question 2, (see samples below), the following categories seem to emerge:

Category 1--Operate on the numbers and relate to some topic learned in class:

a).Av. age of adult = 40, 40 x 60 = 2400
Av. age of child = 12, 10 x 12 = 120, 2400/120 = 20, age of bus driver.

b) 60 + 10 = 70, 70 /2 = 30

Category 2--Operate on the numbers, and adjust to given context

a) 60 + 10 = 70, too old to drive, so either 60 or 65.

b) 60 - 10 = 50, as 60 too old, 10 too young.

Category 3--Personal experience, ignoring numbers given

a) Most drivers I see are quite old, so the bus driver must be 40 to 50 years old, say 45.

b) The age you become an adult is 20. He probably would have had high ambitions when he was young so he couldn't be a bus driver when he was in his twenties. He probably had a high rank in an office and he decided to retire at 55 because the work was too stressful. When he retired, he felt that he was wasting time and decided to take a relaxing job and become a bus driver. Ans. 57 years old.

For Question 2, then, it looks as if students in Category 1 have been trained to just write down the answers without thinking about the reasonableness of answers (and questions). Or, where they thought about the reasonableness of answers, they tended to operate on the numbers given, to give an answer that accorded with their everyday experiences (Category 2). And, in instances where they took cognizance of the context, students based their answers on everyday experience, ignoring the numbers given in the question (Category 3).
So, to go back to the two questions I set out to study:
1. Do upper Primary school students in Singapore use number sense and reasoning to solve two "real-life" word problems? and
2. Is there a gap and/or how widespread the perceived gap is between the planned and implemented curriculum, as revealed by performance in two "real-life" word problems?
The answer to both questions is a (qualified) "no," because, although it would be tempting to say that the results of the study indicate that Singapore students do not think, and just tend to regurgitate what they have learnt, such a conclusion must be tempered with the fact that
1. The students were not told beforehand that some of the questions may not have had sufficient information. (Zhang (1996) reported that students who were given hints such as "if the answer cannot be found, explain why" performed much better than those who had not been given any hints). Moreover, they have been accustomed to giving answers to questions set by teachers, and hardly have had any experience in questions with insufficient or extraneous information. Perhaps the implication here is that the teaching emphasis must change, from a preponderance of procedural exercises, to more open-ended questions as well as questions with insufficient or extraneous information.
2. Some of the answers show originality of thought and show attempts at accommodating cognitive dissonance (between what is given and what is in students' everyday experience), even going to the extent of

ignoring given information that seems in contradiction to "common sense." The implication here seems to be to allow students more flexibility in answering questions, which open ended questions would generally do.

However, I agree that the results on the performance in these two tasks, especially on the latter task, are cause for concern, especially if we want students to reason and have number sense. But how can these results be reconciled with the fact that 8 500 Singapore students beat about half a million students all over the world (in 41 countries), and

came out first in both Mathematics and Science, in the recent Third International Mathematics and Science Study (TIMSS) by the International Association for the Evaluation of Educational Achievement, the results of which were just released a few days ago? Although the latter cohort comprised 13 year olds, it is not too erroneous to imply that my present cohort would tend to give similar results, being a product of the same school system. However, rather than trying to reconcile these seemingly disparate results (which can only be undertaken if a more detailed analysis of the TIMSS results were possible) , we should study whether the present system of learning and teaching mathematics would be appropriate for a rapidly changing, technologically-driven society, which might need more critical, generative and productive skills, rather than uncritical, procedural and re-productive skills. Such a study would allow us to evaluate the appropriateness of our curriculum, assessment, teaching and learning practices.

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Appendix 1

Directions

Distribute the questions to the pupils and ask them to work out the answer individually. Tell them that this is NOT a test, and no marks will be given, but you would like to know what they understand from the questions, and to work them out to the best of their ability.

After looking through the solutions by the pupils, select 1 high ability (Hi), 1 average ability (Me) and 1 low ability (Lo) pupil (include boys and girls as far as possible). Interview them, using the following questions to guide you:

a) Read the question to me.
b) Explain what the question is asking you to do.
c) Show me how you will work it out.
d) Explain how you got your answer and why you think your answer is correct.

During the interview, try to be neutral and not to influence the answers of the pupil. Write a report of what transpired during the interview, and hand it to me on May 6, 1996. (You may want to tape record your session if you cannot remember what transpired during the interview.) Also hand in to me the (pre-interview) answer sheets of all the pupils as well as the (post-interview) answer sheets of the pupils who were interviewed.