The Mathematics Educator
1998, Vol. 3, No. 2, 54-69

The Knowledge Of Percent Of Pre-Service Teachers

Koay Phong Lee

Abstract

Percent is an important concept in mathematics and yet both children and adults find it difficult to conceptualise the concept and apply it to solve problems. To expand pupils' approaches to percent problems, the teachers themselves must have a deep understanding of the concept and be more reflective in their choice of approaches to the percent problems. This article reports on the findings of a study on the knowledge of percent that pre-service teachers have. It also provides suggestions for the use of IT to improve the teaching of percent in the classroom.

Introduction

Percent is prevalent in our daily life. It is present in the food we eat (e.g., 8% fat), the clothes we wear (e.g., 100% silk), the financial transactions we carry out (e.g., 2.5% interest), the things we use (e.g., 80% graphite), the games we play (e.g., 75% hits) and the survey reports that we read (e.g., 30% of the population). It is one of the most useful and important concepts in the mathematics curriculum. However, pupils often find it difficult to conceptualize percent and apply it to solve problems in context (Kouba et als., 1988). This inability to understand percent is also widespread among the adults (Cockcroft, 1985; Parker, 1994; Ginsburg & Gal, 1995) including the in-service and pre-service teachers (Eisenberg, 1976; Parker, 1994).

Percent is a complex concept. Teachers and textbook writers often use it interchangeably with percentage. For example, in the following questions found in a primary mathematics textbook.

- Express 0.35 as a percentage.
- 15 out of 100 oranges in a box are rotten. What percentage of the oranges are rotten?

Percent indicates rate or ratio while percentage refers to the quantity or amount. The correct term to use in the questions above is percent rather than percentage. It is no wonder that many pupils are confused and consequently, some pupils consider percent as a label that can be dropped and inserted at will anywhere in the solution to a problem.

E.g., pupils often write incorrectly $^1/_2 \times 100 = 50\%$.

Percent is used intensively to describe a relationship or comparison. The comparison can be either a fractional comparison (fractional part-whole sense) or a ratio comparison (involving disjoint sets). Formal instruction on percent often begins with the introduction of percent in the fractional part-whole sense as a certain quantity out of 100, followed by the procedures for finding the unknowns (the percent, the percentage and the base/whole) in computation problems and solving problems in context. The critical difference between the two models of percent is often not highlighted in the classroom instruction. Unless pupils have a deep understanding of the concept, they would not be able to explain the inconsistencies in the following applications of the concept.

- If 40% of P6 pupils are girls and 50% of P5 pupils are girls, we cannot say 90% of P5 and P6 pupils are girls.
- If 40% of the pupils come to school by bus and 50% by MRT, we can say that 90% of the pupils come to school by public transport.

Research (Risacher, 1992; Parker, 1994) has found that pupils usually have intuitive knowledge of percent and can solve proportion problems (base 100) if the problems are expressed in simple language. However, formal instruction on percent tends to restrict pupils' creativity and flexibility in their approaches to percent problems. Formal instruction tends to ignore the relationships underlying the referents and makes pupils to be more reliant on the procedural knowledge and less on the proportional relationships. Probably, the teachers themselves do not see the relationships.

This study examined the knowledge base of the pre-service teachers, their approaches to computation problems and their performance on percent problems in context. The findings of this exploratory study would help mathematics educators identify the areas of concern and design instructional approaches to improve the understanding of percent of the future teachers who will be teaching percent in the upper primary schools.

Method

The subjects of this study were 96 Postgraduate Diploma in Education (PGDE) students and 128 Diploma in Education (Dip Ed) students. A test on percent was administered to these 224 pre-service teachers before the teaching of percent was discussed in their elementary mathematics methods courses at the National Institute of Education. The PGDE pre-service teachers were university graduates studying for their diploma in elementary education. The Dip Ed pre-service teachers were students studying for their diploma in education. Most of them were young adults with A-level certificates or polytechnic diplomas.

The test items required these pre-service teachers to:

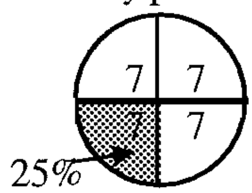
1. explain the percent concept and identify examples of its uses;
2. use different approaches to solve computation problems; and
3. solve percent problems in context.

There were three types of computation problems on percent - to find the percentage, the percent and the whole. The pre-service teachers were required to solve each problem in two different ways. The categories as shown in Table 1 (modified from Lembke & Reys, 1994) were used to code the responses of each pre-service teacher to each computation problem by approach type as well as accuracy.

The test also included two problems in context: the juice problem and the mixed nuts problem. The first problem involved the comparison of two disjoint referent sets. The second problem involved the combination of two sets and the comparison of a subset with the combined set.

Table 1 : Approaches to Solving Percent Problems (Computation)

Approach	Example
Benchmarks: use common reference points (often % that can be replaced by unit fractions) to establish initial values so that exact value can be found.	*Find 75% of 160.* Since 25% of 160 is 40, 75% of 160 is 120.
Fraction: transform the given percent/sets as a fraction and then solve the problem by carrying out the appropriate operation on the fraction.	*Find 75% of 160.* $75\% = \frac{75}{100}$ $\frac{75}{100} \times 160 = 120$ *7 is how many percent of 28?* $\frac{7}{28} = \frac{1}{4} = \frac{25}{100} = 25\%$
Decimal: express the given percent as a decimal and carry out the necessary operation on the decimal.	*Find 75% of 160.* 0.75 x 160 = 120
Unitary: find 1 unit or 1% of the quantity first and then proceed to find the proportion.	*7 is how many percent of 28?* 28→ 100% $1 \rightarrow \frac{100}{28}\%$ $7 \rightarrow \frac{100}{28} \times 7 = 25\%$
Ratio: The % is renamed as fraction and set equal to another fraction with 100 as base.	7 is how many percent of 28? 28 → 100% 7 → ?% 28 x ?% = 7 x 100% $?\% = \frac{100 \times 7}{28}\% = 25\%$

Equation/rule : Use a letter to represent the unknown and factor - factor - product type of equation or a rule such as "number over number total times 100%"	80% of what number is 40? $\frac{80}{100}\times n = 40$ 7 is how many percent of 28? $\frac{7}{28}\times 100\% = 25\%$
Draw a picture: use graphical representations to find the solution.	7 is how many percent of 28? 

Modified from Lembke & Reys (1994)

Results & Discussion

The pre-service teachers' responses are discussed under three sections:

- Explanation of percent concept and its uses
- Performance on computation problems
- Solving percent problems in context

Explanation of percent concept and its uses

Table 2 shows the distribution of the responses. Postgraduate (PGDE) pre-service teachers were more likely to mention the use of 100 as the base in percent compared to the diploma (Dip Ed) pre-service teachers. About a quarter of the diploma pre-service teachers regarded percent as a fraction or a ratio without referring to the base of 100. There are also 5% of postgraduate pre-service teachers and 12% of diploma pre-service teachers who regarded percent as a label or symbol in mathematics. 13% of postgraduate pre-service teachers and 9% of diploma pre-service teachers considered percent as a number to be operated on. They wrote,

'Percent is a number we time a hundred to it.'
'Percent is a number divided by 100.'
'Percent is $^{number\ x}/_{number\ y}$ x 100%.'

Table 2: Explanation of the concept

Responses	PGDE (n=96)	Dip Ed (n=128)
Part-whole mention the base as 100	57%	37%
Part-whole without mention the base	13%	24%
Symbol %	5%	12%
Procedure/formula	13%	9%
Ratio / Measurement of proportions	7%	9%
Others	5%	
No response		9%

There were also some diploma pre-service teachers (9%) who found it difficult to explain the concept. They were observed to ponder over the item for some time before they moved on to the next question. However, all respondents were able to list some examples of the uses of percent. The three examples cited by over 90% of the pre-service teachers were interest, discount and marks/scores.

Performance on computation problems

a) To find the percentage of a quantity: *What is 75% of 160?*

As shown in Table 3, the most popular approach among the pre-service teachers in the study was the fraction approach taught in school where the given percent 75% was first transformed to fraction $\frac{75}{100}$ and then multiplied with the 'whole'. Although only 37% of the diploma pre-service teachers mentioned 100 as the base for percent in their explanation of the concept, nearly 94% of the diploma pre-service teachers were able to use 100 as the denominator when converting 75% to fraction. This shows that many of these pre-service teachers can remember the 'right rule' for finding percentage but cannot correctly explain what 'percent' is.

Table 3: Distribution of Approaches Used to Find Percentage

	PGDE (n= 96) Attempt		Dip Ed (n = 128) Attempt	
Approach	1st	2nd	1st	2nd
Benchmark	2%	5%		5%
Fraction	85%	7%	94%	2%
Decimal	3%	24%	1%	23%
Unitary	6%	9%	2%	10%
Ratio / proportions	3%	13%	1%	9%
Equation / rule				
Diagram		5%		8%
Incorrect	1%	1%	1%	
No response		36%	1%	43%

When asked to solve the problem in another way, more than a third of the pre-service teachers could not do so. For those who did, they tended to use the decimal approach. Diagram approach was not popular among the pre-service teachers neither was the use of benchmark percents.

b) To express a quantity as a percent of the other: *Express 7 as a percent of 28.*

A great majority of the pre-service teachers used the rule taught in school to find the percent (see Table 4). That is, they first expressed 7 as a fraction of 28 and then multiplied by either 100% or 100 with the % symbol placed in the product. Many of these teachers (about 45% postgraduate and 52% diploma pre-service teachers) were unable to find the percent by an alternative approach. Among those who were able to do so, they tended to use the ratio approach.

Table 4: Distribution of Approaches Used to Find Percent

	PGDE (n= 96) Attempt		Dip Ed (n = 128) Attempt	
Approach	1st	2nd	1st	2nd
Benchmark				1%
Fraction	4%	6%	6%	8%
Decimal	2%	4%	1%	5%
Unitary		9%	1%	8%
Ratio / proportions	3%	19%	5%	9%
Equation / rule	85%	3%	67%	4%
Diagram		8%	1%	8%
Incorrect	6%	6%	13%	5%
No response		45%	6%	52%

The common incorrect response was $^{7}/_{100}$ x 28. These pre-service teachers seem to interpret 'percent' as 'division by 100'.

c) To find the whole: *80% of a number is 40. What is the number?*

Table 5 shows that the three popular approaches among the pre-service teachers were

- the equation approach where they literally translated the problem statement into an algebraic equation by replacing 'a number' by either a letter or □, and 'of' by 'x' and then solved the equation (49% postgraduate, 40% diploma pre-service teachers).
- the unitary approach
- the ratio approach

More pre-service teachers used the benchmark percent to solve this problem compared to the previous two types of problem. This is also the most difficult of the three computation problems.

Table 5: Distribution of Approaches Used to Find the Whole

	PGDE (n= 96) Attempt		Dip Ed (n = 128) Attempt	
Approach	1st	2nd	1st	2nd
Benchmark	5%	4%	6%	5%
Fraction				1%
Decimal				
Unitary	17%	15%	13%	9%
Ratio / proportions	17%	12%	24%	16%
Equation / rule	49%	10%	40%	7%
Diagram		8%		6%
Incorrect	11%	6%	13%	9%
No response	1%	45%	4%	47%

Nearly half of the pre-service teachers were unable to find the whole using an alternative strategy. There were also some pre-service teachers who had the misconception that percent computation involved division by hundred followed by multiplication, so

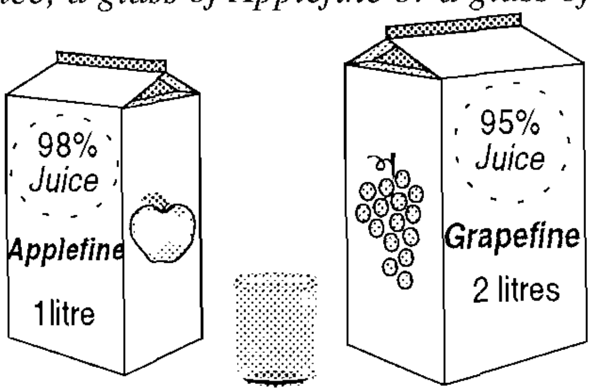
$$80\% \text{ is } \frac{80}{100} \qquad \frac{80}{100} \times 40 = 32$$

This was the most common incorrect response for this item.

Solving percent problems in context

a) The juice problem

Which has more juice, a glass of Applefine or a glass of Grapefine? Explain.



As shown in Table 6, only about half of the respondents (54% of postgraduate, 48% diploma pre-service teachers) were able to explain that a glass of Applefine would contain more juice than a glass of Grapefine. About 30% of the postgraduate pre-service teachers and 29% of the diploma pre-service teachers thought that a glass of Grapefine would contain more juice since $\frac{95}{100} \times 2 \rangle \frac{98}{100} \times 1$.

There were also some pre-service teachers (3% postgraduate, 4% diploma pre-service teachers) who claimed that a glass of Applefine would contain the same amount of juice as a glass of Grapefine since the same glass was used.

Table 6: Distribution of Responses to Juice Problem

Responses	PGDE (n = 96)	Dip Ed (n = 128)
Applefine		
with no explanation	13%	16%
with explanation	54%	48%
Grapefine		
with no explanation	3%	3%
with explanation	27%	26%
Same	3%	4%
No response	0	3%

b) The nuts problem

Question: There are 50g of cashew nuts in each of the following packets of mixed nuts. Amy mixes a packet of Sunshine Mixed Nuts with a packet of Tong's Mixed Nuts in a bowl. What is the percent of cashew nuts in the bowl?

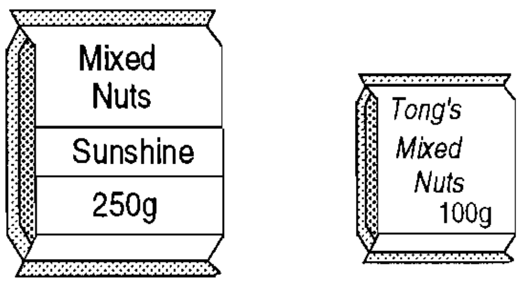


Table 7: Distribution of Responses to the Mixed Nuts Problem

Responses	PGDE (n = 96)	Dip Ed (n = 128)
$\frac{100}{350} \times 100\%$ *	76%	71%
50g → 50%	1%	1%
20% + 50%	13%	16%
20% x 50%	9%	6%
No response	1%	6%

*correct answer

76% of postgraduate pre-service teachers and 71% of diploma pre-service teachers were able to answer this question correctly (see Table 7). The most common incorrect procedure used by the pre-service teachers was to determine the percent of the cashew nuts in each packet and then either add (13% postgraduate, 16% diploma pre-service teachers) or multiply (9% postgraduate, 6% diploma pre-service teachers) the percents. These pre-service teachers did not reflect on their solutions. Their ridiculous answers (20% + 50%, 20% x 50%) indicate their misunderstanding of the percent sign and their inability to connect the school mathematics with their everyday life experiences.

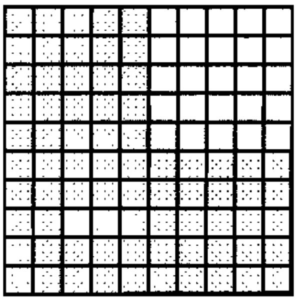
Implications & Suggestions

Like their overseas counterparts, the pre-service teachers in this study did not have a good understanding of the percent concept. They found it easier to perform computation problems than to explain the concept of percent. The pre-service teachers were still 'rule-driven' when solving percent problems years after the formal instruction on percent. Many of them were so rigid in their thinking that they were not able to think of an alternative approach to a computation problem. Percent is an important concept in mathematics that has wide applications in our daily life. Yet many of the pre-service teachers seem to have difficulties applying the concept correctly in context. Like the pre-service teachers in Parker's study (1994), the pre-service teachers in this study lack the sense of comparisons of specific referent quantities. They were not able to correctly identify the referents and their relationships to each other. Moreover, these pre-service teachers did not look back reflectively at their answers and link the problem situations with their everyday life experiences.

Hence, there is a need to enrich and extend the knowledge base of the pre-service teachers. Instruction on percent should emphasise on the proportional interpretation of percent, focusing on percent in context and the analysis of the proportional relationships of the referent quantities rather than the procedural knowledge for solving the three types of percent problem and the routine word problems.

A model for proportional interpretation of percent

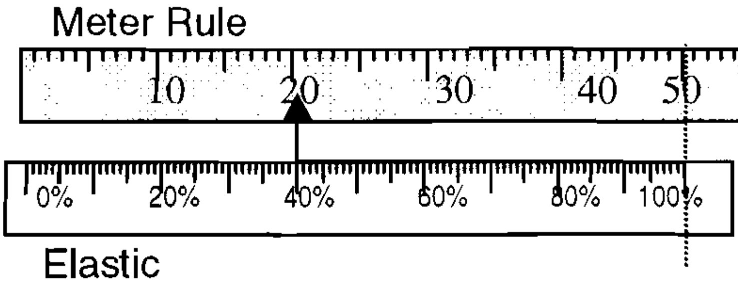
In the mathematics curriculum, percent is introduced after fractions and decimals. The same 100-square grid in the teaching of decimals (hundredths) is often used to illustrate the part-whole model of percent. For example,



$$\frac{75}{100} = 75\%$$

This part-whole model is restrictive. Unless the comparative relationship of one region to the whole is clearly explained, percent would be seen as synonymous with fractions and decimals. Moreover, it is problematic to use the grid to represent a percent greater than 100 or the comparative relationships between the referent and the other quantities in a percent problem.

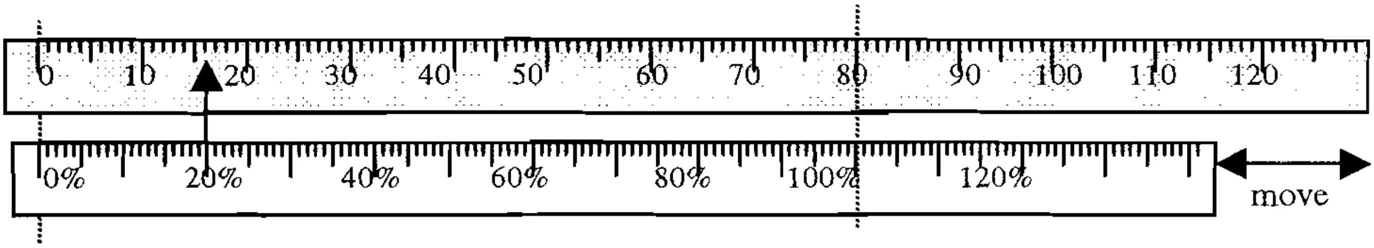
Various visual models and representations of the comparative relationships between the referent and the four quantities in a percent problem have been suggested in literature (Dewar, 1984; Weibe, 1986; Haubner, 1992). Some of them were reviewed by Parker & Leinhardt (1995). One of the models that are associated with the proportional method for solving percent problems has been suggested by Weibe (1986). His model consists of a piece of sewing elastic and a meter ruler. The sewing elastic is calibrated into 100 equal parts. To use the model, the elastic is placed next to the ruler and stretched accordingly to correspond to the given referent quantities.



The efficiency of Weibe's model can be improved with the help of computer. The model presented in this article is more flexible and gives more accurate readings. It can represent percents greater than 100 and can be easily edited or resized to deal with any given referent quantities. The procedure involves the use of the Drawing Toolbar in MS Word to construct two bars, one representing the percent and the other the referent set. First mark at equal distance, the units on the bars. Make sure that the drawings in each bar are 'grouped' as one object so that it can be 'stretched' proportionally. Keep the referent bar fixed and place the percent bar adjacent to it. The percent bar is then adjusted to correspond to the referent quantities. The following percent problems illustrate how the bars can be manipulated.

<u>Task 1</u>: 20% of 80 = ?

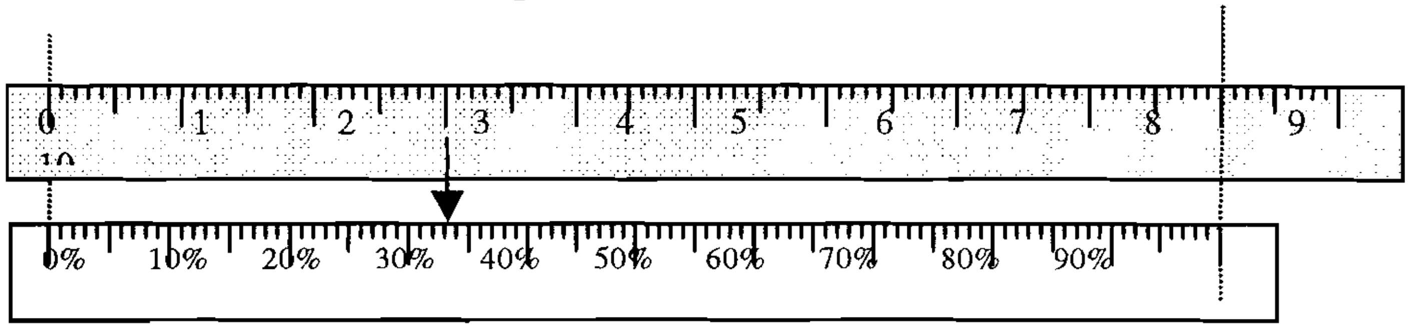
After aligning the zeros, stretch the percent bar so that the 100% mark corresponds with the 'whole' in the problem. i.e. 80.
The 20% of percent bar corresponds to 16.



Write 80 → 100%
? → 20%

Task 2: How many percent is 3 of 9?

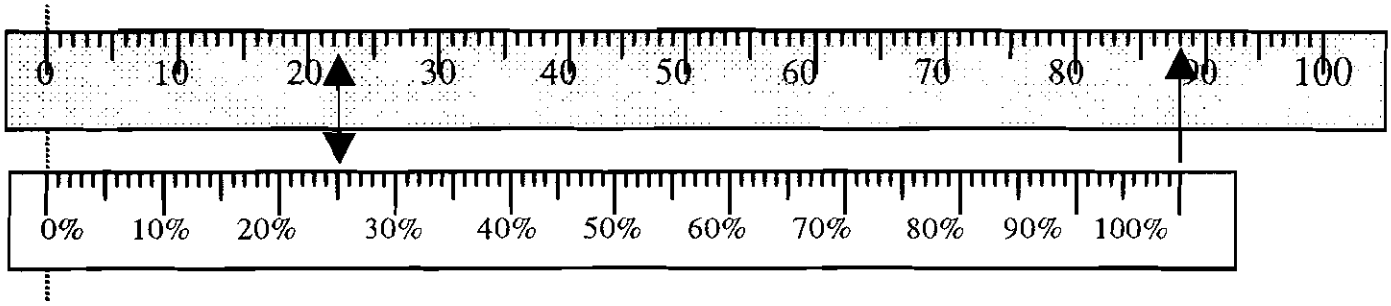
After aligning the two zeros on the bars, stretch the percent bar so that 100% is aligned with the 9 on the referent bar.
The 3 will be aligned with the percent required.



Write 9 → 100%
3 → ?%

Task 3: 25% of what number is 22?

After aligning the zeros, stretch the percent bar so that the 25% is aligned with the 22 mark. Then the 100% mark will be aligned with the 'whole'.



Write 25% → 22
100% → ?

In this model, the fact that percent always involves proportional comparison of something to 100 is emphasised. The referent set has to be identified before correct alignments can be carried out. While using the model, pupils will be expected to actively verbalise their thinking and reasoning. This approach can also be used to illustrate problems in context. An area for future research is to test the efficacy of this model in the classroom settings.

Percent in context

The concept of percent is developed gradually. The findings in this study show that ability to recite percent as per hundred and carry out the computations correctly does not lead to the ability to interpret and apply the concept in context. To improve performance on percent problems, formal instruction on percent should be designed to connect school-based knowledge with the pupils' knowledge of the world and expose pupils to percent problems that invite them to use their real-world knowledge and personal experience in the solutions to the problems. For example, the juice problem and the nuts problem are more realistic to a 10 or 11 year-old child compared to the discount and the tax problems usually found in the textbooks. Pupils must be encouraged to reflect on their solutions to percent problems and given opportunities to communicate their thinking and reasoning. Moreover, mathematics assessment should be expanded to include solving real-world problems and answering probing questions that measure depth of understanding.

References

Cockcroft, W.H. (1985). Mathematics Counts, London. Her Majesty's Stationary Office.

Dewar, J.M. (1984). Another look at the teaching of percent. *Arithmetic Teacher, 31*(7), 48-49.

Eisenberg, T.A. (1976). Computational errors made by teachers of arithmetic:1930, 1976. *Elementary School Journal, 76*, 229-237.

Ginsburg, L. & Gal, I. (1995). Linking informal knowledge and formal skills: The case of percents. *Science, Mathematics & Environmental Education*. Oct.

Haubner, M.A. (1992). Percents: Developing meaning through models. *Arithmetic Teacher, 40*, 232-234.

Kouba, V.L., Brown, C.A., Carpenter, T.P., Linquist, M.M., Silver, E.A., & Swafford, J.O. (1988). Results of the fourth NAEP assessment of mathematics: Number, operations, and word problems. *Arithmetic Teacher, 35*, 14-19.

Lembke, L.O. & Reys, B. (1994). The development of, and interaction between, intuitive and school-taught ideas about percent. *Journal of Research in Mathematics Education*, May, 237-259.

Parker, M. (1994). Instruction in percent: Moving prospective teachers under procedures and beyond conversions. *Dissertation Abstracts International*, *55*(10), 3127A.

Parker, M. & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research*, *65*(4), 421-481.

Risacher, B.F. (1992). Knowledge growth of percent during the middle school years. *Dissertation Abstracts International*, *54*(03), 853A.

Wiebe, J.H. (1986). Manipulating percentages. *Mathematics Teacher*, *79*(Jan), 23-26.