Real-Life Mathematics Tasks

A Singapore Experience

Paper Recycling  Water Water Water  Money Money Money  Which Mobile Plan?

Red or Black?  Up Down Up Down  3 Rockstars On the Wall  Outing to the Zoo

Malacca Trip  Singapore Got Talent  When to Retire?

Singapore Mathematics Assessment and Pedagogy Project
Real-Life Mathematics Tasks:  
A Singapore Experience

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Introduction

The ability to apply mathematics to real-life contexts has been a major aim of mathematics instruction in Singapore and many countries. This aim has been recently emphasised in the 2007 and 2013 versions of the Singapore Mathematics Curriculum, and many steps have been taken to familiarise teachers with everyday applications of mathematics including mathematical modelling. One key factor in the implementation of this policy is that teachers should have access to rich mathematics tasks that embed real-life contexts. The Singapore Mathematics Assessment and Pedagogy Project (SMAPP) was implemented to build an assessment system that includes real-life mathematics tasks, and this book has included two types of SMAPP tasks that teachers can use in their lessons.

The first type covers extended tasks that begin with a real-life context, followed by questions and activities that require students to use the given data, to solve problems, and to explain their reasoning. The competencies to explain, to solve, and to reason are important ones that define the disciplinarity of mathematics that we want our students to develop through mathematics learning. Eleven extended tasks are included in this book, and they deal with contexts such as paper recycling, mobile plan, water consumption, Singapore population, and so on. These tasks cover topics in the Secondary 1 Express syllabus, such as arithmetic, statistics, and geometry. These tasks were designed by a team of mathematicians (Zhao, Cheang, Teo & Lee, 2011). They stress that the quality of a good disciplinary task should include (1) links to real life, (2) use of real and relevant data, (3) connect to the curriculum, (4) assess multiple competencies and content knowledge, (5) enrich student experiences, and (6) scaled levels of difficulty. At a teacher workshop, some teachers applied these principles to design the three tasks on retirement, mobile plan, and zoo outing, which were later refined by the SMAPP design team. We believe that these principles will help teachers develop the skills to design similar extended tasks of their own.

Some of these extended tasks were converted into online format and piloted in several schools using Mozilla Firefox and Internet Explorer. The IT platform comprises six major components: (1) delivery of the extended tasks, (2) capture of student answers to the tasks, (3) automatic marking of closed questions by the system and semi-automatic marking of open questions by the teachers using a given marking scheme, (4) a customisable feedback component to be used by the teachers, (5) capture of students’ responses to teacher’s feedback, and (6) a report system that summarises different types of results by students and by class. This IT platform in relation to assessment for learning is illustrated by Figure 1 below:
This system was developed based on pedagogical principles and objectives rather than sophisticated technological features. These principles are explained in the two papers by Wong, Oh, Ng and Cheong (2012a, 2012b). However, with the completion of the SMAPP project, this online system will no longer be supported. During the SMAPP study, some students mentioned that they preferred working on the print version to the online one. Hence, the print version provided here may still fulfil the purpose of exposing students to these tasks.

Even though these extended tasks were designed with Secondary 1 Express syllabus in mind, some teachers feel that they can use the tasks for students at upper levels, for example, when they teach Everyday Arithmetic in Secondary 3/4. The marking schemes and teaching notes for most of these extended tasks are given to provide teaching ideas for the teachers. The teachers and students who took part in the study generally found these tasks to be relevant but also challenging, in particular questions that require careful reading. Cheang, Teo, and Zhao (2012) reported some results of students on the Paper Recycling task.

The second type of real-life tasks looks like standard problems but with simulated everyday situations. Ten such questions are included in the Everyday Mathematics Test, and they are arranged in
increasing order of difficulty based on the performance of about 900 Secondary 1 Express students in 2011. Even though some of these questions are consistent with the use of real-life contexts in PISA (Programme for International Student Assessment) items, they were not designed to follow the PISA assessment framework. These questions can be readily modified and used in standard school tests. Students’ answers will provide teachers with insights about the ability of their students to solve problems with real-life contexts. By working through these tasks, the students will hopefully gain some knowledge about the real contexts, for example, that sound intensity is measured in decibels. This assessment as learning is also aligned with the curriculum aim mentioned above.

Students should develop positive attitudes towards their learning of mathematics. The 24-item Attitudes toward Learning Mathematics Questionnaire can be used to measure students’ attitudes in six dimensions: (1) Checking solutions, (2) Confidence, (3) Enjoyment, (4) Use of IT in mathematics learning, (5) Multiple solutions, and (6) Usefulness of mathematics. These six scales have acceptable internal consistency, and some of their psychometric properties are described in Wong and Chen (2012). Teachers can modify the questionnaire for their own research and practical use.

The SMAPP project was funded by the Centre for Research in Pedagogy and Practice (CRPP) at the National Institute of Education (NIE), Nanyang Technological University (NTU), Singapore (project code: OER 1/10 FLH) from 2008 to 2012. The overall framework of the project is explained in the paper by Fan, Zhao, Cheang, Teo, and Ling (2010); see Figure 2.
We are grateful to the students and teachers who voluntarily participated in this project and gave their invaluable feedback to many aspects of the project. We have learnt much from the students and teachers about how mathematics with real-life contexts could be embedded in an assessment system.

Singapore teachers are encouraged to use the materials in this book, without further request for permission, for their own teaching so that their students can see the relevance of mathematics in daily life, in ways that can enrich the students’ learning experiences. These materials can be used by the teachers alone with their students or collaboratively with colleagues in action research to investigate techniques that can best incorporate such tasks into their teaching. The final outcome should be strong competence of using mathematics in everyday situations for all our students.

Wong Khoon Yoong
September 2012

References


SMAPP Team and Schools

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SMAPP Extended Tasks

Brief Notes about Implementation

1. These tasks were developed for topics in the Secondary 1 Express syllabus. They are of different difficulty levels. Teachers should modify the questions to suit the ability levels of their students.

2. Students can work on the tasks individually or in pairs. They can discuss the questions with their partners when in doubt. This will be especially useful for students who have difficulty following the instructions.

3. Some questions may contain substantial amount of information. Teachers may explain some of the key words or concepts to the students before they work on the tasks. Some key terms are explained in simple language in the Glossary of selected tasks.

4. A few tasks have several parts, which are generally related but can be completed separately. Teachers need to judge how much time to let their students work on these parts. Most of these parts can be completed in about 40 minutes. The teacher may wish to spend about 20 minutes going through the parts after the students have completed them.

5. Teaching notes have been included for a few tasks. They cover pre-requisites, objectives and rationale for the questions, instructional strategies, and follow-up actions. These are only suggestions rather than prescriptions for the use of the tasks as a form of formative assessment.

6. The feedback systems for the first two tasks (Paper Recycling and Red or Black?) were designed to investigate the types of feedback that teachers could provide to their students as part of assessment for learning. Teachers may use these two cases as prototypes to design their own feedback system to promote effective learning.

7. The marking schemes serve as rough guides to the difficulty levels of the questions. Teachers should amend the marking schemes to match the abilities of their students and their assessment objectives.

8. Alternative solutions and marking schemes are included, whenever appropriate, to alert teachers to these possibilities. These solutions are obviously not exhaustive. It is often a pleasant surprise for teachers to discover that their students have tackled the questions in quite unexpected ways. These alternatives should be welcome and discussed in lessons to promote mathematical thinking among the students.
Paper Recycling:

Arithmetic:

Paper is the most common type of waste in Singapore. About 1.26 million tonnes of paper waste were generated in 2008. Recycling paper conserves forest resources and produces fewer pollutants than conventional pulping and bleaching processes.

Shamila is a member of her school's Green Club. Her school is using paper made from virgin pulp for printing teaching materials. She wants to find out how many trees can be saved each year if the school uses only recycled paper.

She also wants to present some facts and figures to her teachers and schoolmates on the benefits to the environment if the school recycles paper and uses recycled paper.

In this task, you will make calculations to help Shamila complete the above tasks.
Read the text below carefully. Then fill in the blanks in the summary box with numerical values.

There are 1200 students in Shamila’s school. She estimates that each student uses about 3 reams of paper per year (for notes, test and exam papers, etc.).

Looking at the packaging of the printing paper, she sees “80 g/m²” (80 grams per square metre) printed on it. Each ream of paper contains 500 sheets of printing paper.

She learns from a National Environment Agency (NEA) poster that it takes about 17 mature trees to make one tonne (1000 kg) of printing paper.

If she can calculate the mass of the printing paper, excluding the packaging, she will be able to know the number of trees that can be saved by converting to recycled paper.

She does not have a weighing machine. You are approached to help her solve the problems.

SUMMARY

- There are _________ students in Shamila’s school.
- Each student uses about ____ reams of paper per year.
- Each ream of paper contains 500 sheets of paper.
- Mass of paper = 80 grams per square metre.
- 17 mature trees are required to produce 1 tonne (_______ kg) of printing paper.
Number of Trees to be saved

1. (a) The length and breadth of an A0 paper are 0.841 m and 1.189 m respectively. Which of the following expressions is correct to obtain the area of an A0 paper?

(1) \((0.841 + 1.189) \times 2\)
(2) \(0.841 \times 1.189\)
(3) \(1.189 \div 0.841\)
(4) \(1.189 – 0.841\)

(b) An A1 paper is obtained by folding an A0 paper into two equal halves lengthwise.

Similarly, an A2 paper is obtained by folding an A1 paper into two equal halves lengthwise.

Table 1 shows the estimated area of the “A” series paper size and the number of sheets of paper that can be obtained from an A0 paper.

Fill in the blanks with the correct answers.

Table 1

<table>
<thead>
<tr>
<th>“A” series paper size</th>
<th>Paper Size</th>
<th>Estimated Area (m²)</th>
<th>Number of sheets of paper that can be obtained from an A0 paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>0.5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>0.25</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>0.0625</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td></td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>0.015625</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A7</td>
<td>0.0078125</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>A8</td>
<td>0.00390625</td>
<td>256</td>
<td></td>
</tr>
</tbody>
</table>

(c) i. Based on your answer to the area of an A5 paper, find its estimated area in square centimetres. 
\((1 \text{ m}^2 = 10 000 \text{ cm}^2)\).

ii. Find the number of sheets of A5 paper that can be obtained from a sheet of A2 paper.

(d) Looking at the packaging of the printing paper, Shamila sees “80 g/m²” (80 grams per square metre) printed on it.

i. What is the mass of one sheet of A4 paper?

ii. One ream of paper contains 500 sheets of printing paper. What is the total mass of one ream of A4 paper (excluding the mass of the packaging)? Give your answer in kg.
There are 1200 students in Shamila's school. It is estimated that each student uses about 3 reams of A4 paper per year (for notes, test and exam papers, etc.).

If the school uses only recycled paper, how many trees can be saved each year? (Recall that 1 tonne of paper ≈ 17 trees.)

If the school uses a lighter type of paper (70 g/m² instead of 80 g/m²), how many more trees can be saved each year? Give your answer as a whole number.

Benefits of using Recycled paper

Shamila wants to make a poster to present some of the facts and figures on the benefits of recycling paper and using recycled paper. Besides the number of trees that can be saved, she plans to include other information on the benefits to the environment if the school converts to recycled paper. You are approached to help her solve the rest of the problems.

2.

(a) By searching for “advantages of recycling paper” on a search engine, find and write down 2 benefits of recycling paper. [2]

(b) It is estimated that each mature tree can absorb 16 kg of carbon dioxide and produce 10 kg of oxygen each year. If the school saves 150 trees per year by using only recycled paper,

i. how much carbon dioxide could have been absorbed by those 150 trees per year? [1]

ii. how much oxygen could have been produced by those 150 trees per year? [1]

(c) On average, a car emits about 0.16 kg of carbon dioxide for every kilometre travelled (http://www.carpages.co.uk/co2). If it emits the same amount of carbon dioxide as found in (b)(i), how far has it travelled in km? [2]

Save Water!

3.

(a) Fill in the blanks based on the information from the poster below.

A wall poster at Changi Airport states that every tonne of paper recycled can save __________ litres of water and __________ litres of oil. [2]
(b) Shamila calculated that the school uses about 9 tonnes of paper per year and will therefore save 238 455 ℓ of water. Complete the factor tree below. [3]

(c) Given that 757 is a prime number, what are the factors of 757? [1]

(d) Write down the prime factorisation of 238 455. [1]

(e) Shamila wants to draw a pictogram to show the amount of water saved (238 455 ℓ).

Which pictogram below should she use? [1]

One bottle represents 50 000 ℓ of water.

(1)  

(2)  

(3)  

(4)
**Glossary**

**Bleaching**
The process of removing the colour from something or making it lighter in colour using chemicals.

**Conserve**
To keep and protect something from damage, change or waste.

**Emit**
To send out a beam, noise, smell or gas.

**Pollutant**
A substance that makes air, water or oil dirty or harmful to people, animals and plants.

**Pulp**
A soft wet mass of fibres derived from rags or wood, used in papermaking.

**Ream**
Quantity of 500 sheets of paper.

**Recycle**
To collect and treat rubbish to produce useful materials which can be used again.

**Virgin pulp**
Pulp manufactured and used for the first time.

**General Description of Task**

This task involves topics in arithmetic and pictogram.

Question 1 requires students to perform the four operations on numbers, interpret tables and convert units.

Question 2 requires students to use the Internet to search for information.

Question 3 involves expressing a composite number as a product of prime factors and represent information on a pictogram.

**Mathematics Content Covered**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Content covered</th>
</tr>
</thead>
</table>
| **Numbers and the four operations** | - Primes and prime factorisation.  
                                  | - Calculations with the use of online calculator.  
                                  | - Approximation and estimation (including rounding off numbers in a required number of decimal places or significant figures). |
| **Ratio, Rate, and Proportion**   | - Problems involving rates.                                                   |
| **Mensuration**            | - Conversion of units.                                                         |
| **Data handling**         | - Pictogram.                                                                  |
**Solutions**

**Summary**

- There are 1200 students in Shamila’s school.
- Each student uses about 3 reams of paper per year.
- Each ream of paper contains 500 sheets of paper.
- Mass of paper = 80 grams per square metre.
- 17 mature trees are required to produce 1 tonne (1000 kg) of printing paper.

1. (a) Option (2)  
(b) 

<table>
<thead>
<tr>
<th>Paper Size</th>
<th>Estimated Area (m²)</th>
<th>Number of sheets of paper that can be obtained from an A0 paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A1</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>A2</td>
<td>0.25</td>
<td>4</td>
</tr>
<tr>
<td>A3</td>
<td>0.125</td>
<td>8</td>
</tr>
<tr>
<td>A4</td>
<td>0.0625</td>
<td>16</td>
</tr>
<tr>
<td>A5</td>
<td>0.03125</td>
<td>32</td>
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<tr>
<td>A6</td>
<td>0.015625</td>
<td>64</td>
</tr>
<tr>
<td>A7</td>
<td>0.0078125</td>
<td>128</td>
</tr>
<tr>
<td>A8</td>
<td>0.00390625</td>
<td>256</td>
</tr>
</tbody>
</table>

(c) i. 312.5 cm²  
(c) ii. 8 sheets  

(d) i. Mass of a sheet of A4 paper  
= 0.0625 m² x 80 g/m²  
= 5 g  

(d) ii. Mass of a ream of A4 paper  
= 5 g x 500  
= 2500 g  
= 2.5 kg  

(e) Amount of paper used  
= 3 x 1200 x 2.5 kg  
= 9000 kg  
Number of trees saved  
= $\frac{9000}{1000}$ x 17  
= 153 trees  

(f) 153 x $\frac{10}{80}$ ≈ 19 trees  
Alternative answer,  
Mass of a ream of paper (70 g/m²)  
= 31.25 x 70 g  
= 2187.5 g  
= 2.1875 kg  
Reduction in mass of a ream of paper  
= 2.5 - 2.1875 kg  
= 0.3125 kg  
Reduction in total mass of paper  
= 1200 x 3 x 0.3125 kg  
= 1125 kg  
Number of trees saved  
= $\frac{1125}{1000}$ x 17  
= 19.125 trees  
≈ 19 trees
2. (a) Some possible reasons are:
- reduces greenhouse gas emissions that can contribute to climate changes;
- reduces energy and water consumption;
- extends the fiber supply and contributes to carbon sequestration (the uptake and storage of atmospheric carbon; for example, in soil and vegetation);
- saves considerable landfill space;
- decreases the need for disposal (i.e., landfill or incineration which decreases the amount of \( \text{CO}_2 \) produced). [A2]

(b) i. Amount of carbon dioxide absorbed
\[ = 16 \times 150 \text{ kg} \]
\[ = 2400 \text{ kg or 2.4 tonnes} \] [A1]

(b) ii. Amount of oxygen produced
\[ = 150 \times 10 \text{ kg} = 1500 \text{ kg} \] [A1]

(c) Kilometres travelled
\[ = 2400 \div 0.16 \text{ km} \]
\[ = 15000 \text{ km} \] [A1]

3. (a) 26 495 , 1438 [A2]

(b) Complete the factor tree below. [A3]

(c) 1 and 757 [A1]

(d) \( 238 \ 455 = 3^2 \times 5 \times 7 \times 757 \) [A1]

(e) Option (2) [A1]
# Marking Scheme and Feedback

<table>
<thead>
<tr>
<th>Task Name</th>
<th>Task Category</th>
<th>Number of Questions</th>
<th>Max Score</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>Open</td>
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<tr>
<td>Paper Recycling</td>
<td>Arithmetic</td>
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<table>
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<tr>
<th>Question Number</th>
<th>Solutions</th>
<th>Scoring</th>
<th>Question Type</th>
<th>Initial Feedback</th>
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<tbody>
<tr>
<td>Summary</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3[3]: All correct answers &lt;F1&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2[2]: Two correct answers &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Closed</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1[1]: One correct answer &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[0]: All wrong answers &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Q1a             | 2         |         |               |                  |
|                 |           | 1[1]: Correct answer <F1> |               |                  |
|                 |           | 0[0]: Wrong answer <F2> |               |                  |
|                 |           | 0[-1]: Not attempted <F0> |               |                  |
|                 |           |         | Closed        |                  |

F0: You seem to have skipped this question. Next time, try to answer all the questions.
F1: Well done! You have successfully identified all the correct items.
F2: You had not identified all the correct items. Read the text carefully and try to identify the items.
F0: You seem to have skipped this question. Next time, try to answer all the questions.
F1: Well done! You know how to find the area of a rectangle!
F2: The A0 paper is a rectangle. To find the area of a rectangle, multiply its length by ____.
| Q1b | | | | |
|---|---|---|---|
| i | 0.125 | 4[4]: All correct answers <F1> | F0: You seem to have skipped this question. Next time, try to answer all the questions. |
| ii | 16 | 3[3]: Three correct answers <F4> | F1: Good work! You have noticed a correct pattern! |
| iii | 0.03125 | 2[2]: Two correct answers <F4> | F2: To find the estimated area of an A3 paper, look for a pattern under the column “estimated area”. A0 paper is 1 m$^2$, A1 paper is 0.5 m$^2$, A2 paper is 0.25 m$^2$, … The sequence is 1, 0.5, 0.25…..so what is the next number? |
| iv | 64 | 1[1]: One correct answer <F4> | F3: To find the number of sheets of paper that can be obtained from an A0 paper, look for a pattern. Number of sheets for A0 is 1, A1 is 2, A2 is 4, A3 is 8, ….. The sequence is 1, 2, 4, 8…..So how many sheets of A4 paper can be obtained from an A0 paper? |
| | | 0[0]: All wrong answers <F4> | F4: Look at the sequences {1, 0.5, 0.25, …} and {1, 2, 4, …}. |
| | | 0[-1]: Not attempted <F0> | |

<p>| Q1ci | 312.5 cm$^2$ | 1[1]: Correct answer &lt;F1&gt; | F0: You seem to have skipped this question. Next time, try to answer all the questions. |
| | | 0[0]: Wrong answer &lt;F2&gt; | F1: Great work! You have successfully used the correct answer from Q1b and converted it from m$^2$ to cm$^2$. |
| | | 0[-1]: Not attempted &lt;F0&gt; | F2: The answer from Q1b is 0.03125 m$^2$. To convert from m$^2$ to cm$^2$, think of multiplication. |</p>
<table>
<thead>
<tr>
<th>Q1cii</th>
<th>8</th>
<th>1[1]: Correct answer &lt;F1&gt;</th>
<th>Closed</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>0[0]: Wrong answer &lt;F2&gt;</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td></td>
</tr>
</tbody>
</table>

F0: You seem to have skipped this question. Next time, try to answer all the questions.

F1: Good work! Can you think of a different method to do this?

F2: One A0 paper can be folded into 4 sheets of A2 paper and 32 sheets of A5 paper. Can you use this information?

<table>
<thead>
<tr>
<th>Q1di</th>
<th>Solution (1): Mass of a sheet of A4 paper = (0.0625 \text{ m}^2 \times 80 \text{ g/m}^2) = 5 g</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solution (2): Mass of a sheet of A4 paper = (80 \times \frac{1}{16}) = 5 g Or</td>
</tr>
<tr>
<td></td>
<td>Mass of a sheet of A4 paper = (80 \div 16) = 5 g</td>
</tr>
<tr>
<td></td>
<td>2[2]: Correct method and answer (if no unit, assume unit is g) &lt;F1&gt;</td>
</tr>
<tr>
<td></td>
<td>1[1.5]: Correct answer (5) with no working &lt;F99&gt;</td>
</tr>
<tr>
<td></td>
<td>1[1]: Correct method but wrong answer &lt;F3&gt;</td>
</tr>
<tr>
<td></td>
<td>0[0]: Wrong method and answer &lt;F2&gt;</td>
</tr>
<tr>
<td></td>
<td>0[-2]: Correct answer but wrong method &lt;F98&gt;</td>
</tr>
<tr>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
</tr>
</tbody>
</table>

F0: You seem to have skipped this question. Next time, try to answer all the questions.

F1: Excellent! Can you think of something that weighs 5 g? Discuss your answer with your friends.

F2: 80 g/m\(^2\) means that 1 square metre of the paper weighs 80 grams. The area of the A4 paper is \(\_\_\_\_\_\_\) Therefore it weighs \(\_\_\_\_\_\). |

F3: You have the correct method but there is calculation error.

F4: Your method for finding the answer is correct. Have you chosen the correct paper size?

F5: Your answer is correct. Specify the correct unit.

F98: You have the correct answer but your method is wrong or not clear.

F99: Next time, show your working.
<table>
<thead>
<tr>
<th>Q1dii</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q1dii</strong></td>
<td><strong>Mass of a ream of A4 paper</strong></td>
<td><strong>2[2]</strong>: Correct method and answer (if no unit, assume unit is kg) &lt;F1&gt;</td>
<td><strong>Open</strong></td>
<td><strong>F0</strong>: You seem to have skipped this question. Next time, try to answer all the questions.</td>
</tr>
<tr>
<td></td>
<td>(\text{Mass of a ream of A4 paper} = 5 \text{g} \times 500)</td>
<td><strong>1[1.5]</strong>: Correct answer (2.5) with no working &lt;F99&gt;</td>
<td></td>
<td><strong>F1</strong>: Excellent! Can you think of something that weighs 2.5 kg? Discuss your answer with your friends.</td>
</tr>
<tr>
<td></td>
<td>(= 2500 \text{g})</td>
<td><strong>1[1]</strong>: Correct method but wrong answer in kilograms &lt;F2&gt;</td>
<td></td>
<td><strong>F2</strong>: You have the correct method but there is calculation error.</td>
</tr>
<tr>
<td></td>
<td>(= 2.5 \text{kg})</td>
<td><strong>0[0]</strong>: Wrong method and answer &lt;F99&gt;</td>
<td></td>
<td><strong>F3</strong>: Use multiplication.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>0[-2]</strong>: Correct answer but wrong method &lt;F98&gt;</td>
<td></td>
<td><strong>F4</strong>: Your method is correct. However you did not get the correct mass of one sheet of A4 paper.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>0[-1]</strong>: Not attempted &lt;F0&gt;</td>
<td></td>
<td><strong>F5</strong>: Your answer is correct. Specify the unit.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>F6</strong>: You have the correct answer in grams. Convert it to kilograms.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>F98</strong>: You have the correct answer but your method is wrong or not clear.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>F99</strong>: Next time, show your working.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q1e</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q1e</strong></td>
<td><strong>Amount of paper used</strong></td>
<td><strong>3[3]</strong>: Two correct methods and answers &lt;F1&gt;</td>
<td><strong>Open</strong></td>
<td><strong>F0</strong>: You seem to have skipped this question. Next time, try to answer all the questions.</td>
</tr>
<tr>
<td></td>
<td>(= 3 \times 1200 \times 2.5)</td>
<td><strong>2[2]</strong>: Two correct steps &lt;F2&gt;</td>
<td></td>
<td><strong>F1</strong>: Great work! It is surprising to know that we can save so many trees just by using recycled paper!</td>
</tr>
<tr>
<td></td>
<td>(= 9000 \text{kg})</td>
<td><strong>1[1]</strong>: One correct step &lt;F2&gt;</td>
<td></td>
<td><strong>F2</strong>: You have the correct method/s but there is/are calculation error/s.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>1[1.5]</strong>: Correct answer (153) with no working &lt;F99&gt;</td>
<td></td>
<td><strong>F3</strong>: You have 2 correct steps. Check the units.</td>
</tr>
<tr>
<td></td>
<td><strong>Number of trees that can be saved each year</strong></td>
<td><strong>0[0]</strong>: Wrong method and wrong answer &lt;F5&gt;</td>
<td></td>
<td><strong>F4</strong>: Your method is correct. However you did not get the correct mass of one sheet of A4 paper.</td>
</tr>
<tr>
<td></td>
<td>(= \frac{9000}{1000} \times 17 = 153 \text{trees})</td>
<td></td>
<td></td>
<td><strong>F5</strong>: Your answer is correct. Specify the unit.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>F6</strong>: You have the correct answer in grams. Convert it to kilograms.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>F98</strong>: You have the correct answer but your method is wrong or not clear.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>F99</strong>: Next time, show your working.</td>
</tr>
<tr>
<td>Question</td>
<td>Solution (1)</td>
<td>Solution (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>--------------</td>
<td>--------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Solution (1)</strong></td>
<td>Mass of a ream of paper (70 g/m²) = 31.25 \times 70 = 2187.5 g = 2.1875 kg</td>
<td>Difference between 80 g/m² and 70 g/m² = 80 - 70 = 10 (0 mark if no follow-up steps)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduction in mass of a ream of paper = 2.5 - 2.1875 = 0.3125 kg</td>
<td>Number of trees saved = 153 \times \frac{10}{80} \approx 19 trees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduction in total mass of paper = 1200 students \times 3 reams \times 0.3125 kg = 1125 kg</td>
<td>Number of trees saved = \frac{1125}{1000} \times 17 = 19.125 trees \approx 19 trees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Solution (2)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Open</td>
<td>0[-2]: Correct answer but wrong method &lt;F98&gt;</td>
<td>0[-2]: Correct answer but wrong method &lt;F98&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0[1]: Not attempted &lt;F0&gt;</td>
<td>1[1]: Correct method and wrong answer &lt;F2&gt;</td>
<td>0[0]: Wrong method and answer &lt;F4&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F4: You have 1 correct step. Check the other step.</td>
<td>F2: You have the correct method/s but there is/are calculation error/s.</td>
<td>F4: Try to use ratio or proportion or model drawing to solve this problem.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F5: First, find the total mass of paper that is used by the students. Second, convert the mass to tonnes. Third, use ratio or proportion to find the answer.</td>
<td>F3: Your methods are correct. Check your calculation. The answer is 19 trees. Can you use ratio or proportion or model drawing to solve this problem?</td>
<td>F98: You have the correct answer but your method is wrong or not clear.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F99: Next time, show your working.</td>
<td>F99: Next time, show your working.</td>
<td>F99: Next time, show your working.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Solution (3)

Mass of one A4 paper

\[
\text{(70 g/m}^2) = 0.0625 \times 70 = 4.375 \text{ (grams)}
\]

Mass of one ream of paper

\[
(70 \text{ g/m}^2) = 4.375 \times 500 = 2187.5 \text{ g}
\]

Mass of 3 reams of paper

\[
(70 \text{ g/m}^2) = 2187.5 \times 3 \times 1200 = 7875 \text{ 000 (g)} = 7875 \text{ (kg)}
\]

1 tonne = 1000 kg

Mass of 3 reams of paper

\[
(70 \text{ g/m}^2) = 7875 + 1000 = 7.875
\]

Number of trees that can be saved each year by using 70 g/m² paper

\[
= 7.875 \times 17 = 133.875
\]

Number of trees saved

\[
= 153 - 133.875 = 19 \text{ trees}
\]

### Q2a

Some possible reasons are:
- reduces greenhouse gas emissions that can contribute to climate;
- reduces energy and water consumption;
- extends the fiber supply and contributes to carbon sequestration (the uptake and storage of atmospheric carbon; for example, in soil and vegetation);
- saves considerable landfill space;
- decreases the need for disposal (i.e., landfill or incineration which decreases the amount of CO₂ produced).

2[2]: Any 2 possible reasons <F1>

1[1]: Only one reason given <F2>

0[0]: Nonsensical answer <F2>

0[-1]: Not attempted <F0>

F0: You seem to have skipped this question. Next time, try to answer all the questions.

F1: Great work! You have used the internet to find two pieces of relevant information!

| Q2bi | Amount of carbon dioxide absorbed  
\[16 \times 150 = 2400 \text{ kg}\] | 1[1]: Correct answer \(<F1>\)  
0[0]: Wrong answer \(<F2>\)  
0[-1]: Not attempted \(<F0>\) | Closed | F0: You seem to have skipped this question. Next time, try to answer all the questions.  
F1: Excellent! How many “you” would weigh 2400 kg?  
F2: 1 tree can absorb 16 kg of carbon dioxide. Use proportion: 150 trees can absorb ____ kg of carbon dioxide. |
| Q2bii | Amount of oxygen produced  
\[150 \times 10 = 1500 \text{ kg}\] | 1[1]: Correct answer \(<F1>\)  
0[0]: Wrong answer \(<F2>\)  
0[-1]: Not attempted \(<F0>\) | Closed | F0: You seem to have skipped this question. Next time, try to answer all the questions.  
F1: Very good! Four people on the moon need about 1500 kg of oxygen per year to survive!  
F2: One tree can produce 10 kg of oxygen. Therefore 150 trees can produce ____ kg of oxygen. |
| Q2c | Kilometres travelled  
\[2400 \div 0.16 = 15000 \text{ km}\] | 2[2]: Correct method and answer (no unit, assume km) \(<F1>\)  
1[1.5]: Correct answer (15 000) with no working \(<F99>\)  
1[1]: Correct method and wrong answer \(<F2>\)  
1[0.5]: Partially correct (e.g. 16 + 0.16 or 100 km) \(<F4>\) | Open | F0: You seem to have skipped this question. Next time, try to answer all the questions.  
F1: Great work! With this distance, how many times do you think you can travel from one end of Singapore to the other?  
F2: You have the correct method but there is calculation error.  
F3: You have used the correct method but your value from Q2b(ii) is not correct. |
### Q3a

<table>
<thead>
<tr>
<th>Score</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/2</td>
<td>Two correct answers &lt;F1&gt;</td>
</tr>
<tr>
<td>1/1</td>
<td>One correct answer &lt;F2&gt;</td>
</tr>
<tr>
<td>0/0</td>
<td>Two wrong answers &lt;F2&gt;</td>
</tr>
<tr>
<td>-1</td>
<td>Not attempted &lt;F0&gt;</td>
</tr>
</tbody>
</table>

F4: Try to use ratio or proportion or model drawing to solve this problem.

F98: You have the correct answer but your method is wrong or not clear.

F99: Next time, show your working.

### Q3b

<table>
<thead>
<tr>
<th>Score</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/3</td>
<td>All correct answers &lt;F1&gt;</td>
</tr>
<tr>
<td>2/2</td>
<td>Two correct answers &lt;F2&gt;</td>
</tr>
<tr>
<td>1/1</td>
<td>One correct answer &lt;F2&gt;</td>
</tr>
<tr>
<td>0/0</td>
<td>Wrong answers &lt;F2&gt;</td>
</tr>
<tr>
<td>-1</td>
<td>Not attempted &lt;F0&gt;</td>
</tr>
</tbody>
</table>

F0: You seem to have skipped this question. Next time, try to answer all the questions.

F1: Excellent work! You know about factors!

F2: You have made one or more errors. Check your calculation.

### Q3c

<table>
<thead>
<tr>
<th>Score</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1</td>
<td>All correct answers &lt;F1&gt;</td>
</tr>
<tr>
<td>0/0</td>
<td>One or two wrong answers &lt;F2&gt;</td>
</tr>
</tbody>
</table>

F0: You seem to have skipped this question. Next time, try to answer all the questions.

F1: Good! You know about prime numbers!
<table>
<thead>
<tr>
<th>Q3d</th>
<th>238 455 = 3^2 \times 5 \times 7 \times 757</th>
<th>0[-1]: Not attempted &lt;F0&gt;</th>
<th>F2: For example, 7 is a prime number and its factors are 1 and 7. Now think about 757.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Or 3 \times 3 \times 5 \times 7 \times 757</td>
<td>1[1]: Correct answer &lt;F1&gt;</td>
<td>F0: You seem to have skipped this question. Next time, try to answer all the questions.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[0]: Wrong answer &lt;F2&gt;</td>
<td>F1: Excellent! You know about prime factorization!</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td>F2: Some of your factors are wrong or not prime. Use the prime numbers that you have found from Q3(b).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>F3: You should not include 1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>F4: Give your answer in the correct factorization form.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q3e</th>
<th>2</th>
<th>1[1]: Correct answer &lt;F1&gt;</th>
<th>F0: You seem to have skipped this question. Next time, try to answer all the questions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0[0]: Wrong answer &lt;F2&gt;</td>
<td>F1: Well done! You can interpret the pictogram. Based on your daily intake of water, how long does it take for you to consume 238 455 litres of water?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td>F2: Check your calculation.</td>
</tr>
</tbody>
</table>

**Note:**
Under the scoring column, numbers in brackets, [ ], are codes.

- Code -2: Correct answer but wrong method
- Code -1: Did not attempt the question
- Code 1.5: Correct answer with no working
Student Performance in Paper Recycling Task

1. The mean scores were based on the results collected from about 400 Secondary 1 Express students in 2011. The students completed the task online in the school’s computer laboratory. The student answers were marked by the teachers using the marking scheme above.

2. The overall performance was 63%, and there were no significant differences between performance of girls and boys on this task.

<table>
<thead>
<tr>
<th>Items</th>
<th>Maximum Score</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>3</td>
<td>2.35</td>
<td>1.20</td>
</tr>
<tr>
<td>1a</td>
<td>1</td>
<td>0.94</td>
<td>0.23</td>
</tr>
<tr>
<td>1b</td>
<td>4</td>
<td>3.67</td>
<td>0.85</td>
</tr>
<tr>
<td>1ci</td>
<td>1</td>
<td>0.58</td>
<td>0.49</td>
</tr>
<tr>
<td>1cii</td>
<td>1</td>
<td>0.75</td>
<td>0.43</td>
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<tr>
<td>1di</td>
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</tr>
<tr>
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<td>0.78</td>
<td>0.42</td>
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<tr>
<td>2bii</td>
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<tr>
<td>2c</td>
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<td>3</td>
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<tr>
<td>3c</td>
<td>1</td>
<td>0.68</td>
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<td>3d</td>
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<td>0.48</td>
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<tr>
<td>3e</td>
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<tr>
<td>Marks of Separate Parts</td>
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<td></td>
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</tr>
<tr>
<td>Summary</td>
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<td>1.20</td>
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<td>9.38</td>
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<td>2</td>
<td>6</td>
<td>3.95</td>
<td>1.93</td>
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<tr>
<td>3</td>
<td>8</td>
<td>5.26</td>
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</tr>
<tr>
<td>Total Score</td>
<td>33</td>
<td>20.95</td>
<td>6.74</td>
</tr>
</tbody>
</table>
Teaching Notes

1. (a), (b) and (c)

Pre-requisites
- Interpretation of tables.
- Conversion of units.
- Rates.

Objectives/Rationale
- To assess ability to look for patterns.
- To assess ability to solve rate problems.

Instructional notes/Follow-up actions
- Students may face difficulty in doing part (b). Teacher can ask students questions like:
  - What did you notice about the estimated area for each paper size?
  - What did you notice about the number of sheets of paper that can be obtained from an A0 paper for different paper sizes?
  - Do you see a pattern? Can you write the answer for the next row?
- Part (c) of the question requires students to do some simple calculations as well as conversion of units. Some students may have difficulty understanding that $1 \text{ m}^2 = 10000 \text{ cm}^2$. Teacher can draw on the board to illustrate this conversion of units. Draw a square and label the sides as 1 m (as shown below).

```
+---------------------+
|                     |
|                     |
+---------------------+
     1 m
     1 m
```
- Ask scaffolding questions like:
  - How do you find the area of this 1 m by 1 m square?
  - What is the area of this square?
  - How many centimeters are there in 1 m?

- The teacher can then add in $1 \text{ m} = 100 \text{ cm}$ on the same drawing as shown below:

```
1 m = 100 cm
```

- Then ask students:
  - What is the area of this square in square centimeters?

1. (d), (e) and (f)

Pre-requisites
- Conversion of units.
- Perform four operations on numbers.
- Rates.

Objectives/Rationale
- To assess procedural skills for numerical calculation, measurement and estimation.
- To assess ability to solve rate problems.

Instructional notes/Follow-up actions
- Students may write 5 g/m$^2$ as the answer for question 1(d)i. Teacher should stress that this is mass per square metre and not the mass of a sheet of A4 paper.

```
1 m = 100 cm
```
2. (a), (b) and (c)

Pre-requisites
- Use of Internet.
- Perform four operations on numbers.
- Rates.

Objectives/Rationale
- To assess procedural skills for numerical calculation and measurement.
- To assess ability to solve rate problems.

Instructional notes/Follow-up actions
- Some students may not be proficient at searching for information on the Internet and might spend too much time reading many different articles. Teacher can remind students to extract only essential information from the Internet to answer part (a).

3. (a), (b), (c), (d) and (e)

Pre-requisites
- Recognise prime numbers.
- Express a composite numbers as a product of prime factors.
- Represent prime factorisation of a number in index notation.
- Draw pictogram.

Objectives/Rationale
- To assess skills in prime factorisation.
- To assess interpretation of pictogram.

Instructional notes/Follow-up actions
- Most students will not see the link between part (b) and (d). Tell students that by completing the factor tree, they can write the answer in prime factorised form quickly.
Anne’s parents are planning to buy a new car. They can either purchase a normal car or an off-peak car. An off-peak car will cost less but has restrictions on usage, compared to a normal car.

In this task, you will apply your mathematics knowledge to help Anne’s parents decide whether they should purchase a normal car or an off-peak car.
The Off-Peak Car (OPC) Scheme

The OPC scheme offers car owners the option to pay less for their cars in return for reduced usage of the cars. Under the current OPC scheme, newly registered OPCs owners will enjoy unrestricted usage on Saturdays, Sundays, Public Holidays and the eves of 5 major public holidays (New Year, Lunar New Year, Hari Raya Puasa, Deepavali and Christmas).

The restricted hours for OPCs are from 7:00 AM to 7:00 PM during other normal working days. To use an OPC during restricted hours, the owner needs to pay $20 for a day license.

The annual road tax discount for an OPC is $500, subject to a minimum annual road tax of $70. For example, if the annual road tax for a normal car is $800, the annual road tax for the same car registered as an OPC is $300. If the annual road tax for a normal car is $400, the annual road tax for the same car registered as an OPC is $70.

Owners registering a new car as an OPC are also given a vehicle tax rebate of up to $17 000.

Source: http://www.onemotoring.com.sg

Summary

- Off-peak cars can be used without restrictions on Saturdays, Sundays, Public Holidays and the eves of 5 major public holidays.
- The license for using an OPC during restricted hours costs $20 per day.
- If the annual road tax for a normal car is $750, the annual road tax for the same car registered as an OPC is $450.
- If the annual road tax for a normal car is $450, the annual road tax for the same car registered as an OPC is $70.
- If an owner registers a new car as an off-peak car, the owner can get a maximum rebate of $17 000.
1. The price of the car (normal usage) that Anne’s parents are interested in is $62 800.

(a) If they pay $15 000 as down payment (the initial payment) and take a loan (borrow money) from a bank for the remaining amount, how much loan do they need to take? Fill in the blank to complete the answer statement.
They need to take a loan of $ ____________ from the bank. [1]

(b) For a car loan, the bank charges an interest at a fixed rate. The total interest payable on the loan can be calculated using the following formula:

\[ \text{Interest} = P \times r \times t \]

where \( P \) = amount of loan taken,
\( r \) = interest rate per year, \( t \) = number of years for the loan.

Using your answer in part (a) for \( P \), calculate the interest for a 7-year loan at an interest rate of 2.5% per annum (i.e., per year). [2]
(Note: \( r \) is expressed as a decimal or fraction, so a percent must be divided by 100. For example, if the rate is 3%, then use \( \frac{3}{100} \) or 0.03 in the formula.)

(c) The monthly instalment (monthly payment) for the car can be calculated using the formula:

\[ \text{Monthly instalment} = \frac{(P + \text{Interest})}{12 \times t} \]

where \( t \) = number of years for the loan and \( P \) = amount of loan taken.

Using your answers for \( P \) in part (a) and the interest in part (b), calculate the monthly instalment for the car. Give your answer in dollars, correct to 2 decimal places. [2]

(d) The same car registered under the off-peak car scheme costs $17 000 less. Given that \( s \) = selling price of a normal car, \( d \) = down payment and \( P \) = loan amount, for the same car under the off-peak car scheme, write down the formula for \( P \) in terms of \( s \) and \( d \). Fill in the blanks to complete the formula.

\[ P = _____ - _____ - _____ \] [1]

(e) If the car that Anne’s parents are interested in is registered under the OPC scheme, calculate the monthly instalment for the car. (Recall that the car costs $62 800, down payment is $15 000, interest rate is 2.5% and the loan is for 7 years.) [4]

2. Table 1.

<table>
<thead>
<tr>
<th>Engine Capacity (EC) in cc</th>
<th>Annual Road Tax Formulae</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 600</td>
<td>$200 \times 0.782 \times 2</td>
</tr>
<tr>
<td>600 to 1000</td>
<td>([$200 + $0.125(\text{EC} - 600)] \times 0.782 \times 2)</td>
</tr>
<tr>
<td>1000 to 1600</td>
<td>([$250 + $0.375(\text{EC} - 1000)] \times 0.782 \times 2)</td>
</tr>
<tr>
<td>1600 to 3000</td>
<td>([$475 + $0.75(\text{EC} - 1600)] \times 0.782 \times 2)</td>
</tr>
<tr>
<td>More than 3000</td>
<td>([$1525 + $1(\text{EC} - 3000)] \times 0.782 \times 2)</td>
</tr>
</tbody>
</table>
Red or Black?: Arithmetic and Algebra

(a) Table 1 shows the calculation for the annual road tax for a normal car. If Anne’s mother is interested in a car with an engine capacity (EC) of 1400 cc, calculate the annual road tax for:

i. a normal car; [2]

ii. an off-peak car. [2]

(Note: The annual road tax discount for an OPC is $500, subject to a minimum annual road tax of $70.)

(b) If Anne’s father is willing to pay a maximum tax per annum of $2000 for a normal car, can he buy a car with an engine capacity of more than 3000 cc? Show your working and explain your answer. [2]

3. (For this question, use the values given in Table 2 instead of the values you’ve found in earlier questions.) On top of the monthly instalments, there are other costs involved in owning a car, as shown in Table 2.

### Table 2.

<table>
<thead>
<tr>
<th>Expenses involved in owning cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of Car = $62 800</td>
</tr>
<tr>
<td><strong>Normal Car</strong></td>
</tr>
<tr>
<td>Monthly instalment (estimated)</td>
</tr>
<tr>
<td>Road Tax (per year)</td>
</tr>
<tr>
<td>Insurance (per year)</td>
</tr>
<tr>
<td>Additional cost for usage during restricted hours on weekdays (per day)</td>
</tr>
<tr>
<td>Fuel, maintenance, etc.</td>
</tr>
<tr>
<td><strong>Off-peak Car</strong></td>
</tr>
<tr>
<td>Monthly instalment (estimated)</td>
</tr>
<tr>
<td>Road Tax (per year)</td>
</tr>
<tr>
<td>Insurance (per year)</td>
</tr>
<tr>
<td>Additional cost for usage during restricted hours on weekdays (per day)</td>
</tr>
<tr>
<td>Fuel, maintenance, etc.</td>
</tr>
</tbody>
</table>

Anne’s parents plan to use the car during the weekends, public holidays and on 2 weekdays per week during restricted hours.

(a) Calculate the expenses incurred per year for using a normal car. (Ignore fuel and maintenance costs.) [2]

(b) Study the two workings below produced by Anne’s parents on the annual cost of owning an off-peak car.

**First working**

Expenses incurred for 12 months (52 weeks) for an OPC

= Instalments + road tax + insurance + additional cost incurred

= ($430 x 12) + $250 + $1650 + 2 x 52 x $20

= $9140

**Second working**

Expenses incurred for 1 month for an OPC

= Instalments + road tax + insurance + additional cost incurred

= $430 + $250 + $1650 + (2 x 4) x $20

= $748.33

Expenses incurred for 12 months for an OPC

= $748.33 x 12

= $8979.96 or $8980

i. Why is the annual OPC cost calculated in the first working different from the second working? [1]

ii. Which estimate is more accurate?

(1) First working

(2) Second working [1]
(c) Table 3 below shows the estimated annual cost of using a normal car, an OPC car, and the difference in their costs.

i. Complete the table by finding the values for (x), (y) and (z). [3]

Table 3.

<table>
<thead>
<tr>
<th>Number of weekdays per week the car is used (n)</th>
<th>Estimated annual cost for a normal car (A)</th>
<th>Estimated annual cost for an off-peak car</th>
<th>The difference in costs between a normal car and an off-peak car (A - B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instalments + Road tax + Insurance</td>
<td>Instalments + Road tax + Insurance</td>
<td>Additional costs = n x 52 x 20</td>
<td>Total (B)</td>
</tr>
<tr>
<td>1</td>
<td>$10 590</td>
<td>$7060</td>
<td>$1040</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$8100 (x)</td>
</tr>
<tr>
<td>2</td>
<td>$10 590</td>
<td>$7060</td>
<td>$2080</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$9140 $1450</td>
</tr>
<tr>
<td>3</td>
<td>$10 590</td>
<td>$7060</td>
<td>$3120</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$10 180 $410</td>
</tr>
<tr>
<td>4</td>
<td>$10 590</td>
<td>$7060</td>
<td>$4160</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$11 220 $630</td>
</tr>
<tr>
<td>5</td>
<td>$10 590</td>
<td>$7060</td>
<td>$ (y)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$12 260 $ (z)</td>
</tr>
</tbody>
</table>

ii. There are two ways to find the value in cell (y). Show your working(s) for both methods. [2]

(d) Summarize the information by filling in the following blanks with the correct values.

i. To save money, Anne’s parents should buy an off-peak car if they plan to use the car for fewer than _______ weekdays per week. [1]

ii. To save money, Anne’s parents should buy a normal car if they plan to use the car for at least _______ weekdays per week. [1]
### Glossary

**Annual**
Occurring once every year.

**cc**
Abbreviation for cubic centimetre.

**Down-payment**
Amount of money that you pay at the time that you buy something, but it is only a part of the total cost of that thing. You usually pay the rest of the cost over a period of time.

**Expenses**
Costs required or incurred.

**Income**
Money that is earned from doing work or received from investments.

**Initial**
Existing or occurring at the beginning.

**Instalment**
A part of a total sum of payment, to be paid at regular intervals until the total amount is fully paid.

**Insurance**
An agreement in which you pay a company money and they pay your costs if you have an accident, injury, etc.

**License**
An official document which gives you permission to own, do or use something, usually after you have paid money and/or taken a test.

**Loan**
A sum of money which is borrowed, often from a bank, and has to be paid back, usually together with an additional amount of money that you have to pay as a charge for borrowing.

**Per annum**
For each year.

**Rebate**
(1) An amount of money which is returned to you, especially by the government, for example when you have paid too much tax: a tax rebate.
(2) A concession given to some groups of people meeting some criteria.

**Restricted**
Limited, especially by official rules, laws.

**Restriction**
A limiting condition or measure.

**Interest**
Money that is charged bases on the amount of money that has been borrowed or invested.

**Tax**
(An amount of) Money paid to the government which is based on your income or the cost of the goods or service you have bought.
General Description of Task

This task involves topics in arithmetic and algebra.

The first question is designed to assess students’ ability at extracting and summarising the information from a given text. The different parts in question 1 involve interpretation and evaluation of algebraic expressions and formulae.

Question 2 requires students to have a clear understanding of simple inequality in order to work with numbers, four operations and brackets in a contextualised problem. In particular, they need to select the right expression and substitute the correct value accordingly.

Question 3 further assesses students’ mathematical reasoning and application.

Mathematics Content Covered

<table>
<thead>
<tr>
<th>Topic</th>
<th>Content covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers and the four operations</td>
<td>• Approximation and estimation, including rounding off numbers to a required number of decimal places or significant figures.</td>
</tr>
<tr>
<td>Algebraic representation and formulae</td>
<td>• Use letters to represent numbers or variables.</td>
</tr>
<tr>
<td></td>
<td>• Interpret algebraic notations.</td>
</tr>
<tr>
<td></td>
<td>• Translate real-life situations into algebraic expressions.</td>
</tr>
<tr>
<td>Percentage</td>
<td>• Problems involving percentages.</td>
</tr>
<tr>
<td>Data handling</td>
<td>• Interpretation of information given in tables.</td>
</tr>
</tbody>
</table>

Solutions

Summary

• Off-peak cars can be used without restrictions on Saturdays, Sundays, Public Holidays and the eves of 5 major public holidays.
• The license for using an OPC during restricted hours cost $20 per day.
• If the annual road tax for a normal car is $750, the annual road tax for the same car registered as an OPC is $250.
• If the annual road tax for a normal car is $450, the annual road tax for the same car registered as an OPC is $70.
• If an owner registers a new car as an off peak car, the owner can get a maximum rebate of $17 000.

1. (a) Loan amount
   = $62 800 - $15 000
   = $47 800       [A1]

   (b) Interest payable = \( P \times r \times t \)
   = $47 800 \times \frac{2.5}{100} \times 7
   = $8365.00       [A1]

   (c) Monthly instalment = \( \frac{(P + \text{interest})}{12 \times t} \)
   = \( \frac{(8365 + 47 800)}{12 \times 7} \)
   ≈ $668.63 (2 d.p.)       [A1]
(d) \[ P = s - 17000 - d \]
OR
\[ P = s - (17000 + d) \]  \[\text{[A1]}\]

(e) Loan amount
\[= 62800 - (15000 + 17000)\]
\[= 30800 \]  \[\text{[M1]}\]

Interest payable
\[= Prt\]
\[= 30800 \times \frac{2.5}{100} \times 7\]
\[= 5390.00 \]  \[\text{[M1]}\]

Monthly installment
\[= \frac{(P + \text{Interest})}{12 \times t}\]
\[= \frac{(30800 + 5390)}{12 \times 7}\]
\[\approx 430.83 \text{ (2 d.p.)} \]  \[\text{[A1]}\]

2. (a) i. Road tax per annum for normal car
\[= [250 + 0.375(1400 - 1000)]\]
\[\times 0.782 \times 2\]
\[= 625.60 \]  \[\text{[A1]}\]

(a) ii. Road tax per annum for an off-peak car
\[= 625.60 - 500\]
\[= 125.60 \]  \[\text{[A1]}\]

(b) Assume EC is 3000 cc.
\[= 1525 + 1(EC - 3000)] \times 0.782 \times 2\]
\[= 2385.10\]
No, the minimum tax is $2385.10.  \[\text{[A2]}\]

3. (a) Cost for a normal car per year
\[= \text{Instalments + Road tax + Insurance}\]
\[= (670 \times 12) + 750 + 1800\]
\[= 10590 \]  \[\text{[A1]}\]

(b) i. The second working uses 4 weeks per month, hence the total number of weeks per year is only 48 weeks.  \[\text{[A1]}\]

(b) ii. The first working is more accurate.  \[\text{[A1]}\]

(c) i. \[x = 10590 - 8100 = 2490\]  \[\text{[A1]}\]
\[y = 5 \times 52 \times 20 = 5200\]
\[z = 10590 - 12260 = -1670\]  \[\text{[A1]}\]

(c) ii. Method 1: \[5 \times 52 \times 20 = 5200\]  \[\text{[A1]}\]
Method 2: \[12260 - 7060 = 5200\]  \[\text{[A1]}\]

(d) i. 4  \[\text{[A1]}\]

(d) ii. 4  \[\text{[A1]}\]
## Marking Scheme and Feedback

<table>
<thead>
<tr>
<th>Task Name</th>
<th>Task Category</th>
<th>Number of Questions</th>
<th>Max Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>Open</td>
</tr>
<tr>
<td>Red or Black</td>
<td>Arithmetic and Algebra</td>
<td>16</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Solutions</th>
<th>Scoring</th>
<th>Question Type</th>
<th>Initial Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>1 5</td>
<td>5[5]: All correct answers &lt;F1&gt;</td>
<td>Closed</td>
<td>F0: You seem to have skipped this question. Next time, try to answer all the questions. F1: Well done! You have successfully identified all the correct items. F2: You had not identified all the correct items. Read the text carefully and try to correct the wrong items.</td>
</tr>
<tr>
<td></td>
<td>2 20</td>
<td>4[4]: Four correct answers &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 250</td>
<td>3[3]: Three correct answers &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 70</td>
<td>2[2]: Two correct answers &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1[1]: One correct answer &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[0]: All wrong answers &lt;F2&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q1a</td>
<td>Loan amount $ = $62 800 − $15 000 $ = $47 800 [A1]</td>
<td>1[1]: Correct answer &lt;F1&gt;</td>
<td>Closed</td>
<td>F0: You seem to have skipped this question. Next time, try to answer all the questions. F1: Well done! Did you check your answer by adding $15 000 + $47 800 = $62 800? F2: The price of the car is $62 800. They have already paid $15 000 as down-payment. They need to borrow $_____ from the bank.</td>
</tr>
</tbody>
</table>
Interest payable

\[ I = P \times r \times t \]

\[ = 47,800 \times \frac{2.5}{100} \times 7 \]

\[ = 8,365.00 \]

2[2]: Correct method and answer (if no unit, assume unit is $)  <F1>

1[1.5]: Correct answer ($8,365) with no working <F1>

1[1]: Correct method but wrong answer (e.g., wrong value for P) <F2>

0[0]: Wrong method and answer <F3>

0[-2]: Correct answer but wrong method <F98>

0[-1]: Not attempted <F0>

Monthly instalment

\[ M = \frac{(P + I)}{12 \times t} \]

\[ = \frac{(47,800 + 8,365)}{12 \times 7} \]

\[ = 668.63 \text{ (2 d.p.)} \]

2[2]: Correct method and answer (if no unit, assume unit is $)  <F1>

1[1.5]: Correct answer ($668.63) with no working <F1>

1[1]: Correct method but wrong answer (e.g., wrong value for interest or wrong conversion to two decimal places) <F2>

0[0]: Wrong method and answer <F3>

0[-2]: Correct answer but wrong method <F98>

0[-1]: Not attempted <F0>

F0: You seem to have skipped this question. Next time, try to answer all the questions.

F1: Excellent! Do you think this is a reasonable amount of interest to pay over 7 years? Discuss with your friends.

F2: You have the correct method but there may be calculation errors or you have used the wrong value of P from part (a).

F3: Your method is incorrect. Use the formula \( I = P \times r \times t \) to find the interest.

F98: You have the correct answer but your method is wrong or not clear.

F99: Next time, show your working.
### Q1d

**Original Formula:**

\[ P = s - 17,000 - d \]

Or

\[ P = s - d - 17,000 \]

\[[A1]\]

**Feedback:**

0[0]: Wrong method and answer <F3>

0[-2]: Correct answer but wrong method <F98>

0[-1]: Not attempted <F0>

### Q1e

**Loan amount:**

\[ P = 62,800 - (15,000 + 17,000) = 30,800 \]

\[[M1]\]

**Interest payable:**

\[ = \frac{30,800 \times \frac{2.5}{100} \times 7}{12} \]

\[ = 5,390.00 \]

\[[M1]\]

**Monthly installment:**

\[ = \frac{P + \text{Interest}}{12 \times t} \]

\[ = \frac{30,800 + 5,390}{12 \times 7} \]

\[ \approx 430.83 \text{ (2 d.p.)} \]

\[[A1]\]

**Feedback:**

4[4]: Three correct steps and answers <F1>

3[3]: Three correct steps and wrong final answer <F2>

2[2]: Two correct steps and wrong final answer <F3>

1[1.5]: Correct answer ($430.83) with no working <F99>
### Q2

#### (i) Road tax per annum for normal car (1400 cc)

\[
\text{Road tax per annum} = \left[\$250 + \$0.375(1400 - 1000)\right] \times 0.782 \times 2
\]

\[
= \$625.60
\]

**M1**

**A1**

1[1]: One correct step and wrong final answer. <F3>

0[0]: Wrong methods and wrong answers <F4>

0[-2]: Correct answer but wrong method/s <F98>

0[-1]: Not attempted <F0>

F99: Next time, show your working.

#### (ii) Road tax per annum for an off-peak car (1400 cc)

\[
\text{Road tax per annum} = \$625.60 - \$500
\]

\[
= \$125.60
\]

**M1**

**A1**

2[2]: Correct method and answer (if no unit, assume unit is $) <F1>

1[1.5]: Correct answer ($625.60) with no working <F99>

1[1]: Correct method but wrong answer <F2>

0[0]: Wrong methods and answer <F3>

0[-2]: Correct answer but wrong method <F98>

0[-1]: Not attempted <F0>

F0: You seem to have skipped this question. Next time, try to answer all the questions.

F1: Brilliant! You have chosen the correct formula to find the answer!

F2: You have the correct method but there may be calculation errors.

F3: The engine capacity of the car is 1400 cc, EC = 1400. Which row should you look at in Table 1? Apply the formula to find the answer.

F98: You have the correct answer but your method is wrong or not clear.

F99: Next time, show your working.

Open
1[1.5]: Correct answer ($125.60) with no working <F99>
1[1]: Correct method but wrong answer <F2>
0[0]: Wrong method and answer <F3>
0[-2]: Correct answer but wrong method <F98>
0[-1]: Not attempted <F0>

F3: The road tax per annum for a normal car (engine capacity of 1400 cc) is $625.60. The road tax discount for an OPC is $500. Therefore the road tax per annum for an off-peak car is __________.
F98: You have the correct answer but your method is wrong or not clear.
F99: Next time, show your working.

b

Assume EC is 3000 cc.

\[
[1525 + \$1(EC - 3000)] \times 0.782 \times 2 = 2385.10
\]
No, the minimum tax is $2385.10.

[2][2]: Reasonable answer with working <F1>
1[1]: Partially reasonable answer <F2>
0[0]: Wrong reason or unclear working <F3>
0[-1]: Not attempted <F0>

F0: You seem to have skipped this question. Next time, try to answer all the questions.
F1: Your explanation is very good!
F2: You have a good reason. Can you convince others with some calculations?
F3: First, work out the road tax per annum for a 3000 cc car. Then look at your answer and give a reason.

Q3a

Cost for a normal car per year

\[
= \text{Instalments} + \text{road tax} + \text{insurance}
= ($670 \times 12) + 750 + 1800
= $10 590
\]

2[2]: Correct method and answer (if no unit, assume unit is $) <F1>
1[1.5]: Correct answer ($10 590) with no working <F99>

F0: You seem to have skipped this question. Next time, try to answer all the questions.
F1: Excellent!
F2: You have the correct method but there may be calculation errors.
| Q3b |  | 1[1]: Correct method but wrong answer <F2> | F3: First, find the total amount of instalments for 12 months (1 year). Then add the road tax and insurance to get the answer. |
|-----|  | 0[0]: Wrong method and answer <F3> | F98: You have the correct answer but your method is wrong or not clear. |
|     |   | 0[-2]: Correct answer but wrong method <F98> | F99: Next time, show your working. |
|     |   | 0[-1]: Not attempted <F0> | |
| Q3b | i | The second working uses 4 weeks per month, hence the total number of weeks per year is only 48 weeks. [A1] | |
|     |   | 1[1]: Reasonable answer <F1> | F0: You seem to have skipped this question. Next time, try to answer all the questions. |
|     |   | 0[0]: Wrong or unclear reason/s <F2> | F1: Good! You have noticed the difference between the number of weeks and months in a year. |
|     |   | 0[-1]: Not attempted <F0> | F2: Look at the second working. How many weeks were used in the calculation? Do you think the number of weeks in a year is correct? |
| Q3b | ii | Working 1 [A1] | F0: You seem to have skipped this question. Next time, try to answer all the questions. |
|     |   | 1[1]: Correct answer <F1> | F1: Good! Is this the most accurate method? |
|     |   | 0[0]: Wrong answer <F2> | F2: First working uses ________ weeks for calculation. Second working uses ________ weeks for calculation. Therefore ________ working is more accurate. |
|     |   | 0[-1]: Not attempted <F0> | |
| Q3ci | x | 2490 | 3[3]: Three correct answers <F1> | F0: You seem to have skipped this question. Next time, try to answer all the questions. |
|     | y | 5200 | 2[2]: Two correct answers <F2> | F1: Well done! You understand the meanings of the items in the table. |
|     | z | - 1670 | Closed |
### Q3c

#### ii

<table>
<thead>
<tr>
<th></th>
<th>1[1]: One correct answer &lt;F2&gt;</th>
<th>2[2]: Two correct methods &lt;F1&gt;</th>
<th>F2: You have one or more errors. Try again.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0[0]: All wrong answers &lt;F2&gt;</td>
<td>1[1]: One correct method &lt;F2&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td>0[0]: Wrong methods &lt;F2&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td></td>
</tr>
</tbody>
</table>

**Method 1:**
\[5 \times 52 \times 20 = 5200\]  
\([A1]\)

**Method 2:**
\[12260 - 7060 = 5200\]  
\([A1]\)

---

### Q3d

#### i

<table>
<thead>
<tr>
<th></th>
<th>1[1]: Correct answer &lt;F1&gt;</th>
<th>F0: You seem to have skipped this question. Next time, try to answer all the questions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0[0]: Wrong answer &lt;F2&gt;</td>
<td>F1: Well done! You can now help Anne’s parents decide what kind of car to buy!</td>
</tr>
<tr>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td>F2: Look at the last column. What is the meaning of the negative value?</td>
</tr>
</tbody>
</table>

**4**

---

#### ii

<table>
<thead>
<tr>
<th></th>
<th>1[1]: Correct answer &lt;F1&gt;</th>
<th>F0: You seem to have skipped this question. Next time, try to answer all the questions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0[0]: Wrong answer &lt;F2&gt;</td>
<td>F1: Well done! You can now help Anne’s parents decide what kind of car to buy!</td>
</tr>
<tr>
<td></td>
<td>0[-1]: Not attempted &lt;F0&gt;</td>
<td>F2: Look at the last column. What is the meaning of the negative value?</td>
</tr>
</tbody>
</table>

**4**

---

**Note:**
Under the scoring column, numbers in brackets, [ ], are codes.

- **Code -2:** Correct answer but wrong method
- **Code -1:** Did not attempt the question
- **Code 1.5:** Correct answer with no working
Student Performance in Red or Black? Task

1. The mean scores were based on the results collected from about 400 Secondary 1 Express students in 2011. The students completed the task online in the school’s computer laboratory. The student answers were marked by the teachers using the marking scheme above.

2. The overall performance was 49%, and there were no significant differences between performance of girls and boys on this task.

<table>
<thead>
<tr>
<th>Items</th>
<th>Maximum Score</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>5</td>
<td>3.61</td>
<td>1.53</td>
</tr>
<tr>
<td>1a</td>
<td>1</td>
<td>0.86</td>
<td>0.35</td>
</tr>
<tr>
<td>1b</td>
<td>2</td>
<td>1.42</td>
<td>0.83</td>
</tr>
<tr>
<td>1c</td>
<td>2</td>
<td>1.19</td>
<td>0.87</td>
</tr>
<tr>
<td>1d</td>
<td>1</td>
<td>0.63</td>
<td>0.48</td>
</tr>
<tr>
<td>1e</td>
<td>4</td>
<td>0.97</td>
<td>1.49</td>
</tr>
<tr>
<td>2ai</td>
<td>2</td>
<td>1.25</td>
<td>0.87</td>
</tr>
<tr>
<td>2a(ii)</td>
<td>1</td>
<td>0.74</td>
<td>0.94</td>
</tr>
<tr>
<td>2b</td>
<td>2</td>
<td>0.79</td>
<td>0.92</td>
</tr>
<tr>
<td>3a</td>
<td>2</td>
<td>0.48</td>
<td>0.85</td>
</tr>
<tr>
<td>3b(i)</td>
<td>1</td>
<td>0.12</td>
<td>0.33</td>
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<tr>
<td>3b(ii)</td>
<td>1</td>
<td>0.33</td>
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<tr>
<td>3c(i)</td>
<td>3</td>
<td>1.74</td>
<td>1.34</td>
</tr>
<tr>
<td>3c(ii)</td>
<td>2</td>
<td>0.86</td>
<td>0.88</td>
</tr>
<tr>
<td>3d(i)</td>
<td>1</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>3d(ii)</td>
<td>1</td>
<td>0.22</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Marks of Separate Parts

<table>
<thead>
<tr>
<th>Items</th>
<th>Maximum Score</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>5</td>
<td>3.61</td>
<td>1.53</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>5.07</td>
<td>2.93</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2.78</td>
<td>2.13</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>3.85</td>
<td>3.01</td>
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<tr>
<td>Total Score</td>
<td>31</td>
<td>15.31</td>
<td>7.64</td>
</tr>
</tbody>
</table>
Teaching Notes

Summary
Pre-requisite
- Careful reading and interpretation of terms such as “subject to a minimum” and “up to xxx value”.

Objectives/Rationale
- To assess ability to gather and extract essential information from text.

1. (a)
Pre-requisite
- Down payment and loan.

Objectives/Rationale
- To check pre-requisite knowledge.

Instructional notes/Follow-up actions
- Explain the terms down payment and loan to those who cannot recall the meanings.

1. (b) to (e)
Pre-requisites
- Interpret algebraic notations in formula.
- Algebraic substitution.
- Approximation in terms of number of decimal places.

Objectives/Rationale
- To assess ability to evaluate algebraic expressions and formula by substitution.
- To assess ability to compute interest, instalment, loan based on given formula and information.

Instructional notes/Follow-up actions
- Introduce the simple interest formula to students, if new to them.
- Useful websites about simple interest:
  - http://www.vertex42.com/Calculators/simple-interest.html
  - http://www.learningwave.com/lwonline/percent/interest.html
- Check skill of algebraic substitution.
- Remind students to give the answer to 2 decimal places as required by the question.
- Give more examples to students who have difficulty writing algebraic expressions from statements as required in 1(d).

2. (a) and (b)
Pre-requisite
- Read and interpret values given in tables.

Objectives/Rationale
- To assess ability to extract relevant information from a table and complete the required computation.

Instructional notes/Follow-up actions
- The different formulae given in the table can be confusing to some students. Take the first row and ask students what do they know about the road tax for car with engine capacity less than 600 cc. Then look at the second row; ask students to substitute different values of EC and interpret the answer. Continue with other rows, if necessary.
- Remind students to refer to the Summary if they do not see the difference between part (a)i and (a)ii.

Useful websites about simple interest:
- http://www.mathexpression.com/simple-interest-formula.html
3. (a) and (b)
Pre-requisite
- Interpret tabulated information

Objectives/Rationale
- To assess students’ ability in interpreting information and comparing different methods of computation.
- To assess students’ mathematical reasoning skills.

Instructional notes/Follow-up actions
- Students may face difficulty answering part (b) as it requires them to think critically about the two methods of computation. To scaffold the students, teachers can ask questions like:
  - What is the difference between these two workings?
  - Why do you think these workings use different numbers for calculation? Where do these numbers are from?
- Take note that both workings are only estimates because 52 weeks multiplied 7 days a week is only 364 days and a “normal” year has 365.25 days. Discuss the properties of calendar.

3(c) and (d).
Pre-requisite
- Interpret tabulated diagram.

Objectives/Rationale
- To assess ability in interpreting information for evaluation.
- To assess ability to find alternative methods.

Instructional notes/Follow-up actions
- For a large table, explain to students to focus on one row or one column at a time.
Rate, Speed, Algebra, and Inequalities:

Malacca Trip

Aziz and Bryan are planning to drive to Malacca during the June holidays. Aziz lives in Ang Mo Kio Avenue 1 and Bryan in Bedok South Avenue 1. On the way from their home to Malacca, they plan to meet at the Woodlands Checkpoint and again at the rest point in Yong Peng.

You are to help them to plan the trip by working through Tasks A to D.
Task A – From Home to Woodland Checkpoint

From the Google Maps website, we find that the distance to the Woodlands Checkpoint from Ang Mo Kio Avenue 1 is 20 km, and from Bedok South Avenue 1 is 30 km.

A1. Aziz wants to meet Bryan at 07 50 at the Woodlands Checkpoint. If he travels at an average speed of 60 km/h, find his departure time from Ang Mo Kio Avenue 1. [2]

A2. Suppose Bryan plans to leave his home at Bedok South Avenue 1 at 07 30 and arrives at the Woodlands Checkpoint at 07 50. What would be his average speed? [2]

A3. The speed limit on Singapore roads is less than or equal to 90 km/h. If Bryan adheres to the speed regulations, would the average speed you obtained in question A2 be realistic? Give one reason for your answer. [2]

A4. Suppose Bryan leaves his home x minutes before 07 30 and reaches the Woodlands Checkpoint at 07 50.

(a) Express the total time he would take in terms of x (in hours). [1]

(b) Bryan plans to drive at an average speed of 50 km/h. How long would he take to drive from his home to the Woodlands Checkpoint? Give your answer in hours. [1]

(c) Equating your answers in (a) and (b), form an equation in x in order to find how many minutes before 07 30 he needs to depart from his home. [1]

(d) Solve the equation in (c) for x. What would be Bryan’s actual departure time? [2]
Aziz and Bryan reach the Woodlands Checkpoint as planned. After clearing both the Singapore and Malaysia Customs, Aziz continues his journey from Johor Bahru to Yong Peng while Bryan picks up a friend in Johor Bahru before driving to Yong Peng. The next place they plan to meet is at the rest point in Yong Peng.

**B1.** From Johor Bahru, Aziz travels at an average speed of 80 km/h to Yong Peng, which is 100 km away.

(a) If he leaves Johor Bahru at 08 10, what time will he reach Yong Peng? [2]

(b) If he sets off from Yong Peng at 10 00, how long can he stop there? [1]

**B2.** Let $y$ be the time taken (in minutes) for Aziz to clear both Customs, and $u$ be the average speed of Aziz (in km/h) from Johor Bahru to Yong Peng.

Write down an algebraic expression in terms of $u$ and $y$ for the time taken (in minutes) from the Woodlands Checkpoint to Yong Peng. [2]

**B3.** Recall that $y$ is the time taken (in minutes) for Aziz to clear both Customs, and $u$ is the average speed of Aziz (in km/h) from Johor Bahru to Yong Peng.

Let $z$ be the time (in minutes) that Aziz stops at Yong Peng.

(a) Find the duration from the time Aziz reaches the Woodlands Checkpoint to the time he leaves Yong Peng (in minutes). Give your answer as an algebraic expression in terms of $u$, $y$, and $z$. [1]

(b) Given that $y = 20$ and $u = 80$. If Aziz plans to leave Yong Peng at 10 00, how long can he stop at Yong Peng? Using the expression in (a), form an equation and find the value of $z$. (Recall that Aziz reached the Woodlands Checkpoint at 07 50) [2]
B4. Bryan also takes $y$ minutes to clear both Customs and $w$ minutes to pick up his friend. Bryan then drives for 100 km to Yong Peng at an average speed of $v$ km/h.

(a) Find the time taken for Bryan to reach Yong Peng (in minutes after 07 50). Give your answer as an algebraic expression in terms of $v$, $w$ and $y$. [1]

(b) If Bryan reaches Yong Peng at exactly 10 00, use the expressions obtained in B4(a), to form an equation in $v$, given that $y = 20$ and $w = 10$. [1]

(c) If Bryan reaches Yong Peng before 10 00, use the expressions obtained in B4(a), to form an inequality in $v$, given that $y = 20$ and $w = 10$. [1]

B5. Given that $y = 20$ and $w = 10$ in B4.

(a) If Bryan reaches Yong Peng at 09 55, what is his average speed $v$? [3]

(b) If Bryan wants to reach Yong Peng before 09 55, what should his average speed be? Write down an inequality in $v$ using your answer from part (a). [1]

---

Task C – From Yong Peng to Malacca

Aziz leaves Yong Peng at 10 00. Bryan leaves Yong Peng at 10 15 because he needs more time to purchase some local products at the rest point. He plans to catch up with Aziz before reaching Malacca.

C1. Aziz travels at an average speed of 75 km/h and Bryan at 100 km/h. Bryan catches up with Aziz $x$ minutes after 10 00.

(a) Based on Aziz's speed and time, write down an algebraic expression in $x$ for the distance between Yong Peng and the point where Bryan catches up with Aziz. [1]

(b) Next, based on Bryan's speed and time, write down an algebraic expression for the distance between Yong Peng and the point where Bryan catches up with Aziz. (Recall that Bryan leaves Yong Peng at 10 15) [1]
C2. By solving parts (a) to (c), find the time taken and distance travelled for Bryan to catch up with Aziz.

(a) Using the two expressions obtained in C1, form an equation to represent the relationship between the distances travelled by Aziz and Bryan. [1]

(b) Solve the equation obtained in (a), and hence find the time at which Bryan catches up with Aziz. [1]

(c) How far was Aziz from Yong Peng when Bryan catches up with him? Give your answer in km. [2]

C3. The graph on the next page shows the distances travelled by Aziz and Bryan from Yong Peng at a given time. Given that the distance between Yong Peng and Malacca is 115 km, how can you tell from the graph that Bryan catches up with Aziz before reaching Malacca? [2]
C4. By entering different values (up to 1 decimal place) for \( v \) and observing how the graph changes, estimate Bryan's minimum average speed for him to catch up with Aziz before reaching Malacca. 

[1]
C5. By solving parts (a) to (c) below, find the minimum average speed for Bryan to catch up with Aziz before reaching Malacca.

(a) For Bryan to catch up with Aziz before reaching Malacca, the distance travelled from Yong Peng to the meeting point has to be less than 115 km.

Form an inequality using your expression in C1, and solve the inequality for \( x \).

\[ x < \frac{15v}{v - 75} \]  

(b) Recall that Bryan’s average speed is \( v \) (km/h). Using your equation in C2(a), show that

\[ x = \frac{15v}{v - 75} \]  

(c) Using your answers in (a) and (b), find the minimum average speed for Bryan to catch up with Aziz before reaching Malacca.

Task D – Walking Tour in Malacca

Aziz and Bryan plans to stop in Malacca for a walking tour. Using a brochure for tourists, they marked some places to visit. They have to plan their trip carefully due to time constraints.

They made a list of the places of interest and the estimated time required to tour the location.

<table>
<thead>
<tr>
<th>Node</th>
<th>Places</th>
<th>Visiting Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>The Stadhuys, Christ Church, A’ Famosa</td>
<td>40 minutes</td>
</tr>
<tr>
<td>B</td>
<td>St. Peter’s Church</td>
<td>25 minutes</td>
</tr>
<tr>
<td>C</td>
<td>The Baba and Nyonya Heritage Museum.</td>
<td>30 minutes</td>
</tr>
<tr>
<td>D</td>
<td>Malacca Sultanate Palace</td>
<td>25 minutes</td>
</tr>
<tr>
<td>E</td>
<td>Malacca Taming Sari Tower</td>
<td>20 minutes</td>
</tr>
<tr>
<td>F</td>
<td>St. John’s Fort</td>
<td>15 minutes</td>
</tr>
</tbody>
</table>
Aziz and Bryan will start their walking tour from location A (The Stadhuys, Christ Church, A’ Famosa), the most famous landmark in Malacca. They have simplified the actual map into the following diagram below.

**D1.** Aziz and Bryan decide to visit A, D, and E in the given order. How much time do they need if their walking speed is 5 km/h? Give your answers in hours, minutes and seconds.  

**D2.** Let \( x \) be their average walking speed (in km/h). Write down an algebraic expression in terms of \( x \) for the total time required (in minutes) for them to visit A, D and E.  

**D3.** Suppose Aziz and Bryan have 3 hours 5 minutes to visit A, D, E and F in that order.  

(a) What is the total time they can spend on walking between these places?  

(b) Let their average walking speed be \( x \) km/h. Form an inequality in \( x \) for them to complete their visits within the time limit.  

(c) Solve the inequality to find their required average walking speed.
D4. Aziz and Bryan plans to park their vehicles at Point A. They will eventually come back to their cars at Point A. One way for completing the tour is to travel in the following order:

\[ A \rightarrow F \rightarrow B \rightarrow C \rightarrow E \rightarrow D \rightarrow A \]

Find the total time required for the tour if they are walking at an average speed of 5 km/h. [4]

D5. Can you find a route that is shorter than the one given in D4? (Note: the route must start from Point A, passing through all the places of interest, and end at Point A, without repeating any path taken.) [2]

D6. Can you find the shortest path starting from point A, visiting all the places of interest, and coming back to point A? [5]

---

**Glossary**

**Average Speed**
Total distance travelled divided by total time taken.

**Checkpoint**
A place where people and vehicles are stopped and examined, especially at a border between two countries.

**Inequality**
A mathematical statement indicating that two quantities are not equal, represented by the symbol \(<\), \(>\), \(\leq\), \(\geq\) or \(\neq\).
General Description of Task
This task involves topics on speed, algebra and inequalities. A strong mastery of algebraic equation and inequalities is required.

In Part A, students are required to work on questions related to average speed. The scenario involves two vehicles moving off from different locations and then meeting up at the destination at the same time.

Part B is an extension of Part A. The questions focus mainly on speed and algebra. The students are required to solve more challenging questions involving formulating and solving algebraic expressions, equations, and simple inequalities.

Part C is a continuation of Part B. Apart from speed and algebra, students are required to interpret graphs and solve questions involving inequalities. The scenario involves two vehicles moving from the rest point at two different times, with the second car travelling faster and planning to catch up with the first car.

Finally, Part D requires students to solve problems involving speed and shortest routes. The questions require students to make calculations based on information presented in table and diagram. The scenario is about planning the route for a walking tour, taking into consideration time constraints. Students need to be careful in listing all possibilities.

Mathematics Content Covered

<table>
<thead>
<tr>
<th>Topic</th>
<th>Content covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance, Time and Speed</td>
<td>• Speed, uniform speed and average speed</td>
</tr>
<tr>
<td></td>
<td>• Conversion of units (e.g., km/h to m/s)</td>
</tr>
<tr>
<td></td>
<td>• Problems involving speed, uniform speed and average speed.</td>
</tr>
<tr>
<td>Solutions of equations and</td>
<td>• Solving linear equations in one unknown (including fractional coefficients).</td>
</tr>
<tr>
<td>inequalities</td>
<td>• Solving simple inequality (e.g., $3x \leq 5$).</td>
</tr>
<tr>
<td></td>
<td>• Solving simple fractional equations that can be reduced to linear equations, e.g. $\frac{x}{3} + \frac{x - 2}{4} = 3$</td>
</tr>
<tr>
<td></td>
<td>• Formulating a linear equation in one unknown to solve problems.</td>
</tr>
</tbody>
</table>

In addition, students need to use heuristics such as “list possible cases” to solve problems.
Solutions

Task A

A1. Travelling time required at average speed of 60km/h
\[
\frac{20 \text{ km}}{60 \text{ km/h}} = \frac{1}{3} \text{h (or 20 minutes).} \quad [M1]
\]
Departure from Ang Mo Kio will be 20 minutes before 07 50, i.e., 07 30. \[A1\]

A2. Time taken (Number of minutes between 07 30 and 07 50) = 20 minutes \[M1\]
\[
30 \text{ km} + \frac{20}{60} \text{ h} = 90 \text{ km/h} \quad [A1]
\]

A3. It is not realistic. If Bryan adheres to the speed regulations, his maximum speed will be 90 km/h. Given that waiting time maybe caused by traffic lights, accidents, road works and other road conditions, his average speed will be less than 90 km/h. \[A2\]

A4. (a) \(\frac{x + 20}{60}\) \(\text{h}\) \[A1\]
A4. (b) \(\frac{30}{50}\) \(\text{h}\) (or other equivalent fractions) \[A1\]
A4. (c) \(\frac{x + 20}{60} = \frac{30}{50}\) \[A1\]
A4. (d) \(\frac{x + 20}{60} = \frac{30}{50}\)
\[
x = \frac{30}{50} \times 60 - 20
\]
\[
x = 16 \quad [M1]
\]
16 minutes before 07 30, i.e., 07 14 \[A1\]

Task B

B1. (a) \(\frac{100}{80} = 1 \text{ h 15 min}\) \[M1\]
Arrival time at Yong Peng is 09 25 \[A1\]

B1. (b) \((10 \ 00 - 09 \ 25)\text{min} = 35 \text{ min}\) \[A1\]

B2. Time taken to clear both Customs: \(y\)
Time taken to cover the distance from Johor Bahru to Yong Peng: \(\frac{100}{u} \times 60 \text{ min}\) \[M1\]
Time taken from Woodlands Checkpoint to Yong Peng: \((y + \frac{100}{u} \times 60) \text{ min}\) \[A1\]

[Teacher can award 1 mark for answers in hours rather than in minutes, \((\frac{y}{60} + \frac{100}{u}) \text{ h.}]\]

B3. (a) Time taken from Woodlands Checkpoint to Yong Peng:
\((y + \frac{100}{u} \times 60) \text{ min}\)
Time taken from Woodlands Checkpoint till Aziz leaves Yong Peng:
\((y + \frac{100}{u} \times 60 + z) \text{ min}\) \[A1\]

B3. (b) Substituting \(y = 20\) and \(u = 80\) into the equation \(y + \frac{100}{u} \times 60 + z\), we have
\[
20 + \frac{100}{80} \times 60 + z = 130 \quad [M1]
\]
\[
z = 35 \quad [A1]\]
B4. (a) \( y + w + \frac{100}{v} \times 60 \) [A1]

B4. (b) \( 20 + 10 + \frac{100}{v} \times 60 = 130 \) [A1]

B4. (c) \( 20 + 10 + \frac{100}{v} \times 60 < 130 \) [A1]

B5. (a) \((09 55 - 07 50) \) min = 125 min [M1]
\[ 20 + 10 + \frac{100}{v} \times 60 = 125 \] [M1]
\[ \frac{100}{v} \times 60 = 95 \]
\[ v = \frac{100}{95} \times 60 \]
\[ v = 63.2 \text{ (3 S.F.)} \] [A1]

B5. (b) According to answer in (a), the average speed of Bryan in order for him to reach Yong Peng at 09 55 is 63.2 km/h.
To reach earlier, he must drive faster, so \( v > 63.2 \) (3 S.F.) [A1]

Task C

C1. (a) \( 75 \times \frac{x}{60} \) [A1]

C1. (b) \( 100 \times \frac{x - 15}{60} \) [A1]

C2. (a) \( 100 \times \frac{x - 15}{60} = 75 \times \frac{x}{60} \) [A1]

C2. (b) \( 100(x - 15) = 75x \)
\[ 100x - 75x = 1500 \] [M1]
\[ x = \frac{1500}{25} = 60 \]

Bryan catches up with Aziz at 11 00. [A1]

C2. (c) \( \frac{60}{60} \times 75 = 75 \) [A1]

Aziz has travelled 75 km when Bryan catches up with him.

C3. Aziz and Bryan meet when the two lines intersect. [A1]
Since the point of intersection occurs at the 75 km mark before the final destination at the 115 km mark, Bryan does catch up with Aziz. [A1]

C4. From the graph, Bryan’s minimum average speed is 89.7 km/h (1 decimal place) in order to catch up with Aziz. [A1]

C5. (a) \( 75 \times \frac{x}{60} < 115 \) [M1]
\[ 75 \times x < 6900 \]
\[ x < 92 \] [A1]

C5. (b) \( v \times \frac{x - 15}{60} = 75 \times \frac{x}{60} \)
\[ v \times (x - 15) = 75x \] [M1]
\[ -15v = 75x - vx \]
\[ x = \frac{-15v}{(75-v)} \text{ or } \frac{15v}{(v-75)} \] [A1]

C5. (c) \( \frac{15v}{(v-75)} = 92 \)
\[ (\text{from } x \leq 92 \text{ and } x = \frac{15v}{(v-75)} \) \]
\[ 15v = 92 x (v - 75) \] [M1]
\[ -77v = -6900 \]
\[ v = 89.6 \] [M1]
The minimum average speed is 89.6 km/h for Bryan to catch up with Aziz before reaching Malacca. [A1]
### Task D

<table>
<thead>
<tr>
<th>D1.</th>
<th>Time taken to visit the 3 places</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 40 + 25 + 20 min</td>
<td></td>
</tr>
<tr>
<td>= 1 h 25 min [M1]</td>
<td></td>
</tr>
<tr>
<td>Total time taken to walk</td>
<td></td>
</tr>
<tr>
<td>= $\frac{0.6}{x} \times 60$ min [M1]</td>
<td></td>
</tr>
<tr>
<td>= 7 min 12 sec</td>
<td></td>
</tr>
<tr>
<td>Total time taken</td>
<td></td>
</tr>
<tr>
<td>= 1 h 32 min 12 sec [A1]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D2.</th>
<th>Time taken to visit the 3 places</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 40 + 25 + 20 min</td>
<td></td>
</tr>
<tr>
<td>= 1 h 25 min</td>
<td></td>
</tr>
<tr>
<td>= 85 min</td>
<td></td>
</tr>
<tr>
<td>Total time taken to walk</td>
<td></td>
</tr>
<tr>
<td>= $\frac{0.6}{x} \times 60$ min [M1]</td>
<td></td>
</tr>
<tr>
<td>= $\frac{36}{x}$ min [M1]</td>
<td></td>
</tr>
<tr>
<td>Total time taken</td>
<td></td>
</tr>
<tr>
<td>= 85 + $\frac{36}{x}$ min [A1]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D3. (a)</th>
<th>Time taken to visit the 3 places</th>
</tr>
</thead>
<tbody>
<tr>
<td>= (40 + 25 + 20 + 15) min</td>
<td></td>
</tr>
<tr>
<td>= 1 h 40 min [M1]</td>
<td></td>
</tr>
<tr>
<td>Time left for walking</td>
<td></td>
</tr>
<tr>
<td>= 3 h 5 min – 1 h 40 min</td>
<td></td>
</tr>
<tr>
<td>= 1 h 25 min [A1]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D3. (b)</th>
<th>Total distance to walk: 2.7 km [M1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inequality will be: $\frac{85}{60} x \geq 2.7$ [A1]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D3. (c)</th>
<th>Inequality: $\frac{85}{60} x \geq 2.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x \geq 2.7 \times \frac{12}{17}$</td>
<td></td>
</tr>
<tr>
<td>$x \geq 1.91$</td>
<td></td>
</tr>
<tr>
<td>Their required average walking speed is 1.91 km/h [A1]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D4.</th>
<th>Total time taken to visit the places</th>
</tr>
</thead>
<tbody>
<tr>
<td>= (40 + 15 + 25 + 30 + 20 + 25) min</td>
<td></td>
</tr>
<tr>
<td>= 2 h 35 min [M1]</td>
<td></td>
</tr>
<tr>
<td>Total distance</td>
<td></td>
</tr>
<tr>
<td>= (2 + 2.6 + 1.7 + 0.7 + 0.4 + 0.2) km</td>
<td></td>
</tr>
<tr>
<td>= 7.6 km</td>
<td></td>
</tr>
<tr>
<td>Total walking time</td>
<td></td>
</tr>
<tr>
<td>= $\frac{7.6}{5}$ h [M1]</td>
<td></td>
</tr>
<tr>
<td>= 1 h 31 min 12 sec [M1]</td>
<td></td>
</tr>
<tr>
<td>Total time taken</td>
<td></td>
</tr>
<tr>
<td>= 4 h 6 min 12 sec [A1]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D5.</th>
<th>Listing the possible answers [M2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ADEFBCA – 7.5 km</td>
<td></td>
</tr>
<tr>
<td>2) ADFECBA – 7.5 km</td>
<td></td>
</tr>
<tr>
<td>3) ABFDECA – 7.0 km</td>
<td></td>
</tr>
<tr>
<td>4) ACBFEDA – 7.5 km</td>
<td></td>
</tr>
<tr>
<td>5) ABCFEDA – 7.5 km</td>
<td></td>
</tr>
<tr>
<td>6) ACEDFBA – 7.0 km</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D6.</th>
<th>Either (a) ABFDECA or (b) ACEDFBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(By listing all possibilities).</td>
<td></td>
</tr>
<tr>
<td>Listing the possible answers [M2]</td>
<td></td>
</tr>
<tr>
<td>Finding the distances for the answers [M2]</td>
<td></td>
</tr>
<tr>
<td>Identifying the correct answer [A1]</td>
<td></td>
</tr>
</tbody>
</table>
Teaching Notes

Task A

A1. and A2.

Pre-requisite
- Computation and conversion involving distance, average speed and time.

Objective/Rationale
- To assess skills involving speed, distance and time in a given context.

Instructional notes/ Follow-up actions
- Some students use \( \text{time} = \frac{\text{speed}}{\text{distance}} \) instead of \( \text{time} = \frac{\text{distance}}{\text{speed}} \). Teachers may encourage pupils to draw the DST triangle before performing the computation.

- Some students may obtain 08 10 as the answer instead of 07 30. Ask these students to read the question carefully, and consider what is required, before deciding to add 20 minutes to 07 50 or subtract 20 minutes from 07 50.
- For A2, a common wrong answer is 1.5 km/h. Explain that the unit for speed is km/h.
- Some students may give their answers as 1.5 km/min instead of 90 km/h as required, in this case only 1 mark is awarded.

A3.

Pre-requisite
- Concept of average speed.

Objective/Rationale
- To highlight the concept that the average speed cannot exceed the maximum speed.
- To require students to explain their answers.

Instructional notes/ Follow-up actions
- This is a difficult question for many pupils. Most pupils (about 75%) in the trial group answered that the answer is realistic.
- Emphasise that Bryan cannot drive faster than 90 km/h as required by the speed regulation.
- Some students may draw conclusions from their experience rather than from the information given in the question, saying that 90 km/h is realistic as it is normal speed.
- Some students answered that 90 km/h is realistic because it is equal to the speed limit. They should take note of waiting time due to traffic conditions and time is also required for changing speeds.

A4.

Pre-requisites
- Concept of average speed.
- Algebraic representation and formulae.
- Algebraic manipulation and solving algebraic equations.

Objective/Rationale
- To assess ability to represent a situation \( (x \text{ minutes before 07 30}) \) mathematically using algebra.
- To assess ability to make calculations and solve equations involving algebra.

Instructional notes/ Follow-up actions
- Many students have difficulty formulating the equation in A4(c) directly. A4(a) and A4(b) are added to guide them to formulate the equation in A4(c).
## Task B

### B1.
**Pre-requisite**
- Computation and conversion involving distance, average speed and time.

**Objective/Rationale**
- To assess skills involving speed, distance and time.

**Instructional notes/ Follow-up actions**
- Two mistakes when adding up time.
  
  \[
  \frac{100}{80} = 1.25 \text{ h} \\
  1.25 \text{ h} + 08 \text{ 10} = 09 \text{ 35} \\
  \text{Highlight to student that} \\
  1.25 \text{ h} = 1 \text{ h} 15 \text{ min}
  \]

  \[10 \text{ 00} - 09 \text{ 25} = 75 \text{ min.}\]
  This arises because the student treats the time format as whole numbers.

### B2.
**Pre-requisite**
- Algebraic representation and formulae.

**Objective/Rationale**
- To assess ability to translate scenario-based situation into algebraic expression.
- This question prepares students for B3, which is more demanding.

**Instructional notes/ Follow-up actions**
- Some students have difficulty understanding the situation described. Encourage students to draw a diagram or timeline (example as shown below) to help them visualize the scenario.

### B3.
**Pre-requisites**
- Algebraic substitution.
- Solve algebraic equations.

**Objective/Rationale**
- To assess ability to translate scenario-based situation into algebraic expression.
- To solve algebraic expression and interpret the answer.

**Instructional notes/ Follow-up actions**
- This question is very difficult as there are several values and variable \((u, y \text{ and } z)\).
- Suggest to students to break the question down into 3 smaller sub-problems, a common skill in problem-solving, and solve each part before putting them together.
- Encourage students to extend the diagram drawn in B2 to help them visualize the scenario and understand the question.

---

### Total Time Calculation

\[
\text{Total Time} = y \text{ minutes to clear customs} + \frac{100}{u} \times 60 \text{ min}
\]

![Diagram](image)
B4.

Pre-requisite
- Knowledge of inequalities.

Objective/Rationale
- To assess ability to formulate a given situation using linear equations and linear inequalities.

Instructional notes/ Follow-up actions
- B4 is similar to B3, except for B4(c), where inequalities is introduced.
- This is a difficult problem. Suggest to students to break the question down into several smaller sub-problems and represent the parts in a diagram.

B5.

Pre-requisite
- Knowledge of inequalities.

Objective/Rationale
- To assess ability to write down linear inequality.

Instructional notes/ Follow-up actions
- Some students may not be able to do this question because they cannot do B4. Give more guidance to students about B4.

\[
\text{Total Time} = \begin{array}{c}
\text{y minutes to clear customs} \\
\text{Travelling Time} = \frac{100}{u} \times 60 \text{ min} \\
\text{Stopped for } z \text{ minutes in Yong Peng}
\end{array}
\]

Woodlands Checkpoint \hspace{1cm} Yong Peng
### Task C

**C1. and C2.**

**Pre-requisites**
- Computation using distance, average speed and time.
- Solve linear equation.

**Objective/Rationale**
- To assess ability to translate a given situation into algebraic expression.
- To solve linear equation and interpret the result.

**Instructional notes/ Follow-up actions**
- Suggest that the students break down the question down into smaller parts.
  - (1) Write down the formula for finding distance.
  - (2) Write down Aziz’s speed.
  - (3) Write down Aziz’s time.
- **C2** is more challenging for students as it has more mathematics content in terms of algebraic manipulation.
- For **C2**, highlight that the distances travelled by Aziz and Bryan are the same at the meeting point (when Bryan catches up with Aziz).

**C3.**

**Pre-requisite**
- Knowledge of line graphs.

**Objectives/Rationale**
- To assess ability to interpret a distance-time graph.
- To assess ability to explain own reasons.

**Instructional notes/ Follow-up actions**
- Distance-time graph (linear case) is normally not assessed at this level. Hence, this question only requires ability to interpret a line graph given the variables for the x- and y- axis.
- Knowledge of computing gradient or drawing the graph is not required.
- For students who have studied gradient of straight line graphs, teacher may mention that (1) the gradient of the line represents the average speed of the car, (2) the actual distance-time graph of the car will be of very irregular shape due to acceleration, retardation and stopping time.

**C4.**

**Pre-requisites**
- Knowledge of line graphs.
- IT skills.

**Objectives/Rationale**
- To assess ability to interpret a distance-time graph.
- To expose students to a graphical method using IT exploration.

**Instructional notes/ Follow-up actions**
- Explain that this is a graphical method to find solution of **C5(c)**. This trial and error method is made possible with the use of information technology.
- Ask students to observe the graph as speed increases/decreases, and relate average speed to the concept of gradient.
C5. Pre-requisite
- Algebraic skills involving equations and inequalities.

Objectives/Rationale
- To challenge students mathematically in the topics of inequality and speed.

Instructional notes/ Follow-up actions
- C5(a) and C5(b) are scaffolding questions to help students answer C5(c). To make the question more challenging, encourage students to attempt C5(c) directly.
- Ask students to check whether the answer obtained in C5(c) is equal to their answer in C4, and why.

Task D

D1. Pre-requisite
- Computation using average speed.

Objective/Rationale
- To assess ability to interpret information presented in a table.

Instructional notes/ Follow-up actions
- Students should be able to see that total time taken is equal to travelling + visiting time.
- For enrichment, mention graph theory and its use.

D2. Pre-requisite
- Algebraic expression.

Objective/Rationale
- To assess ability to represent a situation algebraically.

Instructional notes/ Follow-up actions
- Refer students to D1 and look for similarities in the two questions.

D3. Pre-requisites
- Algebraic manipulations.
- Solve algebraic equations.
- Handle inequalities.

Objective/Rationale
- To assess ability to represent the problem in inequalities and to find the range of values that a variable can take, given conditions stated in the question.

Instructional notes/ Follow-up actions
- D3(a) and D3(b) are scaffolding questions to help students answer D3(c).
D4.
Pre-requisite
• Average speed.

Objective/Rationale
• To assess ability to extract relevant information.

Instructional notes/ Follow-up actions
• This question prepares students for D5 and D6 by showing students one way that Aziz and Bryan can tour all the places of interest, and come back to the starting point. It is a relatively simple question.

D5.
Pre-requisite
• Average speed.

Objective/Rationale
• To assess ability to identify a route shorter than the one given, and compute the distance of the route.

Instructional notes/ Follow-up actions
• This question is a follow up of D4 by asking students to identify a path of their own.
• This question also aims to highlight to students that travelling in different ways will give rise to different distances, hence takes different time.

D6.
Pre-requisite
• Average speed.

Objective/Rationale
• To assess ability to identify the shortest route by systematically making a list that includes all possible combinations of routes.

Instructional notes/ Follow-up actions
• Suggest that one way to list the possible path is by writing down all possible paths. For example, from A, the next possible paths are AB, AD, AC and AF. Then from AB, the next possible paths are ABC and ABF. And the list goes on.
• Mention that every path can be taken in two directions, so half of the paths are repeated. For example, A C E D F B A and A B F D E C A are the same path travelled in different directions.
• For enrichment purpose, highlight that the task is similar to the Travelling Salesman Problem (TSP) in Graph Theory, and encourage students to read up on the topic if they are interested.
Mensuration and Statistics:

**Water Water Water**

This task involves geometrical shapes, measurements, percentages and statistics.

In Part A, students are to find the area and capacity of a swimming pool. In Part B, students are to estimate the area and capacity of the Marina Reservoir. Finally, in Part C, students are to investigate water consumption and supply issues for our nation.
Task A

The picture below shows an L-shape swimming pool. Its surface can be divided into two rectangular shapes, measuring 18 m by 16 m and 12 m by 10 m, respectively.

A1. Find the perimeter and area of the surface of the pool. (You may refer to the diagram below.)

A2. The diagram below shows a three dimensional view of the pool. The depth of water in Section A increases gradually from 0.9 m at the shallow end to 1.8 m at the deep end. The depth of water in Section B is 0.9 m throughout.

(a) Find the floor area of the pool. [2]
(b) Find the total area of the walls of the pool. [2]

A3. Find the volume of water in the pool, assuming it is completely filled. Give your answer in litres. [3]
A4. The water level is measured from the deepest end of the pool in Section A. Given that the capacity of the pool is 496.8 m$^3$, complete the following table.

<table>
<thead>
<tr>
<th>Water level</th>
<th>Volume of water in pool</th>
<th>Percentage of pool capacity filled (3 S.F.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 0.9 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) 1.2 m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A5. Recall that the capacity of the pool is 496 800 ℓ.

(a) If the pool is empty, how much time is required to completely fill the pool at a rate of 800 litres per minute? Give your answer in hours and minutes.

(b) Hence, if water is pumped into the pool at a rate of $x$ litres per minute, express the time needed to completely fill the pool in terms of $x$. Give your answer in hours.

A6. The graph below shows the change in the water level as water is pumped into the pool, at a rate of 800 litres per minute.

(a) Before the water level reaches 0.9 m, is the rise in the water level getting faster or slower? Give a reason for your answer.

(b) After the water level exceeds 0.9 m, why is the graph a straight line? Give a reason for your answer.

A7. Let $x$ denote the water level measured from the deepest end of the pool in Section A.

Find an algebraic expression in $x$ for the amount of water in the pool.
Task B

The following picture shows the Marina Reservoir (as marked below) near the Singapore River and the Kallang River.

In this task, you are to estimate the area of the Marina Reservoir and make some calculations based on your estimation.

B1. We can use geometrical shapes to approximate the area of the Marina Reservoir as shown below.

The figures below show the pieces used to approximate the area of the Marina Reservoir. Name the shapes of these figures.

B2. The dimensions of the figures (not drawn to scale) are indicated below.

(a) Find the areas of Figures C to E (the areas of Figures A, B and F are given.)

(b) Hence, find the approximate area of the reservoir (give your answer correct to 3 S.F.).

<table>
<thead>
<tr>
<th>A</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>E</td>
</tr>
<tr>
<td>C</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A</th>
<th>328 125 m²</th>
<th>D</th>
<th>m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>187 500 m²</td>
<td>E</td>
<td>m²</td>
</tr>
<tr>
<td>C</td>
<td>m²</td>
<td>F</td>
<td>312 500 m²</td>
</tr>
</tbody>
</table>
B3. The following table shows the average depth of water in the geographical area marked out by each of the 6 figures.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 m</td>
<td>15 m</td>
<td>20 m</td>
<td>15 m</td>
<td>10 m</td>
<td>10 m</td>
</tr>
</tbody>
</table>

Find the approximate volume of water in the Marina Reservoir (give your answer correct to 3 S.F.). [2]

B4. According to national statistics, water consumption in Singapore was about 277 million gallons a day in 2008. Assuming Singapore maintains this rate of water consumption, how long will the supply of water (as approximated in B3) from the Marina Reservoir last? (1 million gallon = 4546.09 m$^3$) [2]

B5. Different shapes can be used to approximate the area of the Marina Reservoir.

(a) Draw the shapes you would use (different from the diagram given in Question 1) to approximate the area of the Marina Reservoir. [2]

(b) Based on the shapes you’ve drawn, will the measure of the area of the Marina Reservoir be exactly the same as you obtained earlier? Give a reason for your answer. [1]

B6. If we approximate the area of the Marina Reservoir using more geometrical figures that are smaller in size, will the answer be more or less accurate? Justify your reasoning. [2]
Task C

Water is precious, especially in Singapore. We have enough water for our needs, thanks to a diversified, sustainable water supply system that has been established. As part of this system, Singapore has built many reservoirs and the Marina Reservoir is the latest addition.

In Task B, we have estimated the water capacity of the Marina Reservoir.

In this part, you are to gather more information about water supply and conservation in Singapore and answer some questions.

C1. The pie chart below shows the water usage pattern of Singapore households. The domestic water consumption per day among households in Singapore was 158 litres in 2008.

![Water Usage Pattern in Singapore Households]

It was found that installing water-saving devices at water fittings and adopting good water saving habits (such as taking shorter showers), can cut household water consumption by up to 5%.

Complete the table below (some answers are already given) and find the amount of water (in litres) that can be saved per day from the activities mentioned above. Assume that water consumption is reduced by 5%

<table>
<thead>
<tr>
<th>Activity</th>
<th>Water Saved (litres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laundry</td>
<td>1.501 ℓ</td>
</tr>
<tr>
<td>Washing Basin</td>
<td>0.79 ℓ</td>
</tr>
<tr>
<td>Flushing Cistern</td>
<td>1.264 ℓ</td>
</tr>
<tr>
<td>Sink</td>
<td>1.738 ℓ</td>
</tr>
<tr>
<td>Shower</td>
<td>0.316 ℓ</td>
</tr>
<tr>
<td>Others</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

Amount of water (in litres) that can be saved per day from:

- Laundry 1.501 ℓ
- Washing Basin 0.79 ℓ
- Flushing Cistern 1.264 ℓ
- Sink 1.738 ℓ
- Shower 0.316 ℓ
- Others 0.316 ℓ

Total: 5.152 ℓ
C2. The bar graph below shows the amount of water consumed per person per day among households (called “Domestic water consumption per capita”) in Singapore for four different years.

The Public Utilities Board has taken a series of water conservation initiatives (such as the Water Efficient Homes Program), targeted to reduce domestic water consumption per capita to about 155 litres per day by 2012.

(a) Based on the trend in the graph, do you think that this target is achievable? Give an explanation for your answer. [2]

(b) The actual water consumption per capita in 2012 can be higher or lower than the target of 155 litres per day. Name one factor that can cause water consumption per capita to increase. [1]

C3. There are currently 15 reservoirs in Singapore. About \( \frac{1}{2} \) of Singapore's total land area is used as water catchments. This will increase to \( \frac{2}{3} \) of the island when the new Marina Reservoir and the first phase of the Seletar-Serangoon Reservoir Schemes are completed (see picture below).

The land area of Singapore in 2008 was 710.2 km\(^2\). When the Marina Reservoir and the first phase of the Seletar-Serangoon Reservoir Schemes are completed, what will be

(a) the total area of the water catchments? [2]

(b) the percentage increase in the water catchment area? [2]

C4. The daily water consumption in Singapore in 2008 was 1 262 000 m\(^3\).

(a) Assuming that the increase in the daily water consumption was 1.5% per year for 2009 and 2010, estimate the daily water consumption for Singapore in 2010 (give your answer correct to 4 S.F.). [3]

(b) By 2010, the NEWater factories were capable of producing a total of 95 million gallons of water per day (mgd). A target was set for NEWater to meet 30% of the total water consumption in 2010. Assuming the water consumption in 2010 was as estimated in Question (a), is the NEWater produced sufficient to meet the target set? Show your working clearly. (1 million gallon = 4546.09 m\(^3\)) [3]

C5. (a) Find the four sources of water supply (called the “Four National Taps”) in Singapore? You may use different information sources including online websites. [1]

(b) By 2010, Singapore will have five NEWater factories. Write down the location of the 5 factories. You may use different information sources including online websites. [2]

(c) Give some suggestions to meet the increasing need of water consumption in Singapore in the long run. [2]
**Glossary**

**Conservation**
Protection of valued resources.

**Consumption**
The amount used or eaten.

**Domestic**
Relating to or used in the home or everyday life within a household.

**Exceed**
To go beyond the limits of something in quantity, degree, or scope.

**Gallon**
A unit for measuring volume: A US gallon is equal to 3785 cubic centimetres.

**Per capita**
By or for each person.

**Trend**
A general development or change in a situation or in the way that people are behaving.

**Water catchment area**
The land from which rainwater is drained and collected into a body of water, for example, a reservoir.

---

**General Description of Task**
This task involves geometrical shapes, measurements, percentages and statistics.

In Part A, students are to find the area and capacity of a swimming pool.

In Part B, students are to estimate the area and capacity of the Marina Reservoir.

In Part C, students are to investigate water consumption and supply issues for our nation.

---

**Mathematics Content Covered**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Content covered</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mensuration</strong></td>
<td>• Area of parallelogram and trapezium.</td>
</tr>
<tr>
<td></td>
<td>• Perimeter and area of composite plane figures.</td>
</tr>
<tr>
<td></td>
<td>• Volume and surface area of cube, cuboid, prism and cylinder.</td>
</tr>
<tr>
<td></td>
<td>• Conversion between cm$^2$ and m$^2$, and between cm$^3$ and m$^3$, litre and gallons.</td>
</tr>
<tr>
<td></td>
<td>• Problems involving volume and surface area of composite solids.</td>
</tr>
<tr>
<td><strong>Ratio, rate and proportion</strong></td>
<td>• Average rate.</td>
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<tr>
<td></td>
<td>• Problems involving ratio and rate.</td>
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<tr>
<td><strong>Percentages</strong></td>
<td>• Expressing one quantity as a percentage of another.</td>
</tr>
<tr>
<td></td>
<td>• Comparing two quantities by percentage.</td>
</tr>
<tr>
<td></td>
<td>• Problems involving percentages.</td>
</tr>
<tr>
<td><strong>Algebraic representation</strong></td>
<td>• Using letters to represent numbers.</td>
</tr>
<tr>
<td></td>
<td>• Translation of simple real-world situations into algebraic expressions.</td>
</tr>
<tr>
<td><strong>Data handling</strong></td>
<td>• Drawing simple inference from statistical diagrams.</td>
</tr>
</tbody>
</table>
Solutions

Task A

A1. Area
\[ \text{Area} = (18 \times 16 + 12 \times 10) \text{ m}^2 \]
\[ = 408 \text{ m}^2 \] \[\text{[A1]}\]

Perimeter
\[ = (18 + 28) \times 2 \text{ m} \]
\[ = 92 \text{ m} \] \[\text{[A1]}\]

A2.
(a) Floor Area
\[ \text{Floor Area} = (18 \times 16.03 + 12 \times 10) \text{ m}^2 \]
\[ = 288.54 + 120 \text{ m}^2 \]
\[ = 408.54 \text{ m}^2 \] \[\text{[M1]}\]

(b) Area of the walls
\[ \text{Area of the walls} = \sum \text{Perimeter} \times \text{Height} \]
\[ = 2 \times (12 \times 0.9 + 10 \times 0.9 + 8 \times 0.9 + 18 \times 1.8 + 2 \times (1.8 + 0.9) \times 16) \text{ m}^2 \]
\[ = 113.4 \text{ m}^2 \] \[\text{[M1]}\]

A3. Volume of water in the pool
\[ \text{Volume of water in pool} = \text{Volume of Section A} + \text{Volume of Section B} \]
\[ = 18 \times \left( \frac{1}{2} (1.8 + 0.9) \times 16 \right) \]
\[ + 0.9 \times 12 \times 10 \text{ m}^3 \]
\[ = 388.8 + 108 \text{ m}^3 \]
\[ = 496.8 \text{ m}^3 \] \[\text{[M1]}\]
\[ = 496 \text{ 800 000 cm}^3 \]
\[ = (496 \text{ 800 000} \div 1000) \text{ ℓ} \]
\[ = 496 \text{ 800 ℓ} \] \[\text{[A1]}\]

A4.

<table>
<thead>
<tr>
<th>Water level</th>
<th>Volume of water in pool</th>
<th>Percentage of pool capacity filled (3 S.F.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 0.9 m</td>
<td>(\frac{1}{2} \times 0.9 \times 16 \times 18) [= 129.6 \text{ m}^2 ] [\text{[M1]}]</td>
<td>26.1 % [\text{[A1]}]</td>
</tr>
<tr>
<td>(b) 1.2 m</td>
<td>129.6 + 0.3 \times 408 [= 252 \text{ m}^2 ] [\text{[M1]}]</td>
<td>50.7 % [\text{[A1]}]</td>
</tr>
</tbody>
</table>

A5. (a) To fill the pool, we need
\[ \frac{496.800}{800} \text{ min} = 621 \text{ min} \] \[\text{[M1]}\]

OR

(Some students may use the unitary method)

800 litres \(\rightarrow\) 1 min

1 litre \(\rightarrow\) \(\frac{1}{800}\) min

496 800 litres \(\rightarrow\) \(\frac{1}{800} \times 496 800 \text{ min} \]
\[ = 621 \text{ min} \] \[\text{[M1]}\]
\[ = 621 \div 60 \text{ h} \]

= 10.35 h = 10 h 21 min \[\text{[A1]}\]

(b) To fill the pool, we need
\[ \frac{496.800}{60 \times \frac{8280}{x}} \text{ h} \] \[\text{[A1]}\]

A6. (a) It is getting slower. \[\text{[A1]}\]

The horizontal cross-section area of the pool is increasing before the water level reaches 0.9 m, and more water (hence time) is required to increase the water level to cover a larger area with the same depth.

OR

For the first 60 seconds, the increase in water level is about 0.5 to 0.6 m. For the next 60 seconds, the increase in water level is about 0.2m. \[\text{[A1]}\]

OR

The gradient of graph is decreasing therefore change in the water level is getting slower. \[\text{[A1]}\]
(b) The graph is a straight line because the horizontal cross-section area of the pool is the same after the water level exceeds 0.9 m, so the water level increases at a constant rate. [A1]

OR

The graph is a straight line because the water level is increasing at a constant rate. [A1]

A7.

<table>
<thead>
<tr>
<th>Water level</th>
<th>Amount of water pumped</th>
</tr>
</thead>
</table>
| $x < 0.9$   | Show triangles are similar (AA)  
height of triangle = $x$, [M1]  
base of triangle = $\frac{16x}{0.9}$ [M1]  
Volume of water in the pool  
$= \left(\frac{1}{2} \times x \times \frac{16x}{0.9}\right) \times 18 \text{ m}^3$  
$= 160x^2 \text{ m}^3$ [A1] |
| $x \geq 0.9$ | $\frac{1}{2} \times 0.9 \times 16 \times 18$  
$+ (x - 0.9) \times 408 \text{ m}^3$ [M1]  
$= (408x - 237.6) \text{ m}^3$ [A1] |
**Task B**

**B1.**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>trapezium</th>
<th>D</th>
<th>parallelogram</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>trapezium</td>
<td>E</td>
<td>triangle</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>trapezium</td>
<td>F</td>
<td>rectangle</td>
</tr>
</tbody>
</table>

- **B2.**

(a) Area of reservoir

\[
\text{Area of reservoir} \approx \text{Area of } (A + B + C + D + E + F) \\
\approx 1671 \, 875 \, m^2 \\
\approx 1670 \, 000 \, m^2 \text{ (3 S.F.)} \\
\]

- **B3.** Approximate amount of water

\[
\approx 328 \, 125 \, x \, 10 + 187 \, 500 \, x \, 15 + 437 \, 500 \, x \, 20 + 312 \, 500 \, x \, 15 + 93 \, 750 \, x \, 10 + 312 \, 500 \, x \, 10 \, m^3 \\
\approx 23 \, 593 \, 750 \, m^3 \\
\approx 23 \, 600 \, 000 \, m^3. \\
\]

- **B4.**

\[
\frac{23 \, 600 \, 000}{4546.09 \times 277} \text{ days} \\
\approx 18.7 \, \text{days (3 S.F.)} \\
\]

- **B5.**

(a) Answers may vary (Any reasonable drawing should be accepted).

- The whole area of the reservoir is reasonably covered, and the areas of all the shapes used can be calculated using known formulae. [A2]
- The whole area of the reservoir is reasonably covered, but some or all of the shapes used are irregular, and their areas cannot be calculated using known formulae. [A1]
- No attempt, or the area of the reservoir is largely (e.g., about 50% or more) uncovered or over covered by the shapes. [0]

(b) Unlikely or No.

The measure obtained will unlikely be exactly the same because we are approximating the area with a different set of shapes. [A1]

- **B6.** Answers may vary.

- In general, we should get a more accurate answer for the area. [A1]
- This is because the area of the composite geometrical shapes will be closer to that of the actual shape of the Marina Reservoir, in other words, there will be less overestimation or underestimation. [A1]

(Accept any reasonable answer, e.g., “It depends on how you approximate the area using geometrical shapes”).
Task C

C1. Amount of water saved from shower:

\[5\% \times 29\% \times 158 \ell = 2.291 \ell \] \[\text{[A1]}\]

Total: \(5\% \times 158 \ell = 7.9 \ell\)

[Or, \(1.501 + 1.264 + 2.291 + 0.79 + 1.738 + 0.316 \ell = 7.9 \ell\)] \[\text{[A1]}\]

C2.

(a) Yes, it is achievable. \[\text{[A1]}\]

Water consumption per capita is decreasing at a rate of about 0.5 to 2 litres per year over the recent years. Based on this trend, by 2012 it should be below 155 litres per capita. \[\text{[A1]}\]

(b) Answers may vary; accept any reasonable answer. For example, a hotter weather may result in people taking more showers. \[\text{[A1]}\]

C3.

(a) \(\frac{2}{3} \times 710.2 \text{ km}^2\) \[\text{[M1]}\]

\[\approx 473 \text{ km}^2 (3 \text{ S.F.}) \] \[\text{[A1]}\]

(b) \(\frac{\frac{2}{3} \times 710.2 - \frac{1}{2} \times 710.2}{\frac{1}{2} \times 710.2} \times 100\%\)

\[= \frac{\frac{2}{3} - \frac{1}{2}}{\frac{1}{2}} \times 100\% \] \[\text{[M1]}\]

\[= 33.3\% \] \[\text{[A1]}\]

C4.

(a) The projected water consumption in 2009 will be

\[\frac{1015}{100} \times 1 \ 262 \ 000 \ m^3 = 1 \ 280 \ 930 \ m^3 \] \[\text{[M1]}\]

\[\frac{1015}{100} \times 1 \ 280 \ 930 \ m^3\]

\[= 1 \ 300 \ 143.95 \ m^3 \] \[\text{[A1]}\]

\[= 1 \ 300 \ 000 \ m^3 (4 \text{ S.F.}) \] \[\text{[A1]}\]

(b) \(30\% \text{ of } 1 \ 300 \ 143.95 \ m^3\)

\[= 390 \ 043.185 \ m^3 \] \[\text{[M1]}\]

\[390 \ 043.185 \ m^3 \div 4546.09 \ m^3 \]

\[= 85.8 \text{ mgd (3 S.F.)} \] \[\text{[M1]}\]

The 95 mgd of water produced by the 5 NEWater factories can meet the target of 30% of Singapore’s water consumption. \[\text{[A1]}\]

C5.

(a) Water from local catchment, imported water from Johor Malaysia, desalinated water and NEWater. \[\text{[A1]}\]

(b) Bedok, Kranji, Seletar, Ulu Pandan, Changi.

\[\Rightarrow 5 \text{ correct answers} \] \[\text{[A2]}\]

\[\Rightarrow 3 \text{ to 4 correct answers} \] \[\text{[A1]}\]

\[\Rightarrow \text{Less than 3 correct answers} \] \[\text{[0]}\]

(c) Increasing seawater desalination, water reclamation, increasing catchment area, having more reservoirs, water conservation initiatives. \[\text{[A2]}\]

[Or, 1 mark for any one or two reasonable suggestions, 2 for three and above]
**Teaching Notes**

**Task A**

A1. and A2.

**Pre-requisite**
- Area and perimeter of composite plane figures.

**Objectives/Rationale**
- To assess skills in computing area and perimeter of a composite plane figure.

**Instructional notes/Follow-up actions**
- Indicate two ways to find the perimeter of the shape. One way is by adding up all the sides along the shape, and the other way is given in the solution below, which makes the computation of perimeter easier. Ask students to find the area in different ways, e.g.,

\[ \text{Perimeter} = 18 + 10 + 12 + 16 = 56 \text{ m} \]

- To motivate students and make the problem more authentic, point out that finding the wall and floor area is useful if the wall and floor areas are to be tiled.
- For students with stronger mathematics background, teacher can indicate that one dimension given for the trapezium is actually extraneous, and they can find that \( 16.0 \approx \sqrt{16^2 + 0.9^2} \) after learning Pythagoras’ theorem in Secondary Two. In other words, the dimension is fixed after the other dimensions of the trapezium, as given in the diagram, are fixed.

---

A3. And A4.

**Pre-requisites**
- Volume of 3-D figures.
- Conversion of units between \( m^3 \) and \( cm^3 \) and between \( cm^3 \) and \( ℓ \).

**Objectives/Rationale**
- To assess ability to find the volume of a 3-D figure in a real-life context.
- To assess conversion of units of volume.
- To assess analytical skills, i.e., to break down a complicated problem into simpler parts.

**Instructional notes/Follow-up actions**
- Help students realize that the 3-D figure of the swimming pool is made up of 2 smaller figures. One is a cuboid (Section B) and the other is a prism (Section A).
- The base of Section A is the trapezium in Question A2.
- Students have learned the conversion \( 1 \text{ ℓ} = 1000 \text{ cm}^3 \) in primary school. The conversion of \( 1 \text{ m}^3 = 1000 \text{ ℓ} \) is not introduced in the secondary syllabus; however, students should not be penalised for using \( 1 \text{ m}^3 = 1000 \text{ ℓ} \). Point out that litre is a commonly used unit in measuring the quantity of water, gas, petrol, etc.
- In A4, the capacity of the pool is the volume of water in the pool when it is completely filled.
- Show students the vertical cross section of the swimming pool (see diagram below), to help them visualise the problem.
A5.
Pre-requisites
- Rate involving time.
- Unit conversion.
- Algebraic expression in one variable.

Objectives/Rationale
- To assess skills in solving problems involving rate and unit conversion.
- To assess ability to represent a situation using algebra.

Instructional notes/Follow-up actions
- For weak students, explain that time taken is volume divided by rate.
- Some students may not pay attention to different units used. Point out to them that the rate given is litres per minute, so conversion between minutes and hours is required.
- Remind students that the fraction in the answer for (b) should be in the simplest form.

A6.
Pre-requisite
- Gradient of a line graph. This is not covered in Secondary 1, so only an informal interpretation is required.

Objectives/Rationale
- To assess thinking skills involving analysing relationship and visualisation (A6(a)).
- To assess interpretation about a line graph (A6(b)).

Instructional notes/Follow-up actions
- A6(a) focuses on intuitive thinking and visualization of the problem, i.e., more water and hence more time is required to increase the water level to cover a larger area with the same depth.
- Suggest that students read from the graph to find the change in the water level for 0 to 60 seconds, and compare it with the change in the water level from 60 to 120 seconds.
- For A6(b), help students realise that the horizontal cross-section area of the pool is the same after the water level exceeds 0.9 m, so the water level increases at a constant rate.
- Give students a more accurate graph where the values can be read accurately. Discuss the effects of the straight-line part.

A7.
Pre-requisites
- Similar triangles. Teachers may want to omit this question.
- Algebraic manipulation.

Objectives/Rationale
- To assess skills in using similar triangles and algebraic expressions in problem solving.

Instructional notes/Follow-up actions
- Suggest the following.
**Task B**

**B1.**

**Pre-requisite**
- Names of basic geometrical shapes.

**Objectives/Rationale**
- To assess knowledge of names of basic geometrical shapes.
- To introduce a basic mathematical idea, that is, the area of an irregular shape can be approximated by dividing it into a number of shapes where areas can be computed easily.

**Instructional notes/Follow-up actions**
- Point out that it is common to approximate an irregular shape with shapes of known properties.

**B2.**

**Pre-requisites**
- Formulae for areas of triangle, rectangle, parallelogram and trapezium.
- Significant figures.

**Objectives/Rationale**
- To assess ability to compute areas of standard geometrical shapes.

**Instructional notes/Follow-up actions**
- For students who can substitute the correct values into the correct formula but get the wrong answers, ask them to check their calculations more carefully.

**B3.**

**Pre-requisite**
- Volume of prism = base area × height.

**Objectives/Rationale**
- To assess ability to calculate the volume of a prism, given the average depth of water in the geographical area marked in the question.

**Instructional notes/Follow-up actions**
- Help students visualize that the approximate volume of water above each section is the volume of water in a prism container with the base area calculated in **B2** and the height being the average depth of the water given.

**B4.**

**Pre-requisite**
- Rates.

**Objectives/Rationale**
- To assess computation involving rates.

**Instructional notes/Follow-up actions**
- Note that the number of days the water supply can last can be found using the total amount of water kept divided by daily water consumption.
- Discuss relevance of this question to water consumption and supply in Singapore.

**B5.**

**(a)**

**Pre-requisite**
- Knowledge of basic geometrical shapes.

**Objectives/Rationale**
- To assess ability to use standard shapes to approximate irregular ones in different ways.

**Instructional notes/Follow-up actions**
- Highlight that there are different ways to approximate the area of an irregular shape. In this question, students should devise their own ways to approximate the area of the Marina Reservoir.
- If students choose shapes whose areas cannot be readily calculated, remind them that the purpose of using standard shapes is to measure the area in an easier way.
### Task C

#### C1.
**Pre-requisite**
- Pie chart.

**Objectives/Rationale**
- To assess ability to read the data represented in a pie chart and perform calculation involving percentages.

**Instructional notes/Follow-up actions**
- Discuss the different methods used by the students to help them realise that only approximate values are obtained by this process.

#### C2.
**Pre-requisite**
- Bar chart.

**Objectives/Rationale**
- To assess inferences made from data presented in a bar graph.
- To assess ability to explain one's thinking.

**Instructional notes/Follow-up actions**
- Highlight that the trend offers a relatively good prediction of what might happen if there are no unexpected events.
- Explain the term “Domestic water consumption”, referring to the water used for daily activities at home. It excludes industrial water consumption.
- If the answer to (b) is “due to an increase in population”, explain that we are not comparing the total water consumption, but the water consumption per person (per capita).
### C3. and C4.  
**Pre-requisite**  
- Percentage and percentage increase.

**Objectives/Rationale**  
- To assess ability to compute percentage increase in a real-life context.

**Instructional notes/Follow-up actions**  
- A common mistake for Question C3(b) is to take \( \frac{2}{3} - \frac{1}{2} = \frac{1}{6} = 16.7\% \). Highlight that the correct answer, i.e., 33.3\%, refers to the original area of the water catchment, but the wrong answer, 16.7\%, refers to the total land area of Singapore.
- In C4, the assumption of 1.5\% increase in daily water consumption is given for easy calculation.
- The 5 NEWater plants and their water production capacities are as follows: Bedok (6 mgd), Changi (50 mgd), Kranji (9 mgd), Seletar (5 mgd), Ulu Pandan (25 mgd).
- With regard to National Education, explain that NEWater has become increasingly important in meeting the demand of water supply in Singapore. Singapore has earned international acclaims in its achievement in water research and technology.

### C5.  
**Pre-requisite**  
- Ability to search for information, including using online resources.

**Objectives/Rationale**  
- To assess ability to gather, select, and organise relevant information, including search on the Internet.
- To assess ability to support one’s suggestion using relevant data.

**Instructional notes/Follow-up actions**  
- This question can be done outside the class for an extended period of time.
- The question can be used for assessment as learning. Teachers may interview a few students on how they have obtained the answers and what difficulties they encounter in the process.
“The history is this graph, from six children per woman in 1960 coming down to the mid 70s to 2.1, which is the replacement level, because you need about two children per woman to replace herself and her husband, and then continuing to go down till it’s about 1.3 today. … As the economy developed, as we educated our people, as women got jobs and they were liberated, they stopped, just having one baby after another at home, and the numbers came down. That's our history.”

(PM Lee Hsien Loong – National Day Rally 2008 Speech)

In this task, you will answer some questions related to demography, which is the statistical study of population, in Singapore. In particular, we look at issues about fertility rate, population aging and growth.
Task A - Population Growth

A1. Singapore is a harmonious multi-racial society. The pie chart below shows the ethnic composition of residents in 2008. The total number of residents was 3 642 700.

(a) Use the information presented in the pie chart to estimate the number of Chinese residents in 2008, correct to the nearest hundred thousand. [2]

(b) There were 495 100 Malay residents. Find the angle of the sector representing Malay residents in 2008. Give your answer correct to 1 decimal place. [2]

(c) If the angle of the sector representing the Indian population is 32.0°, find the number of Indian residents, correct to the nearest thousand. [2]

A2. The resident population (citizens and permanent residents) of Singapore from 1970 to 2008 are shown in the bar graph below.

(a) What was the resident population in 1990? Write down your answer in words. [1]

(b) From the graph, what is the best conclusion about the population change in Singapore from 1970 to 2008? (Select the most appropriate answer from the following options.) [1]

1. There is no trend in the resident population growth from 1970 to 2008.
2. There is a downward trend in the resident population growth from 1970 to 2008.
3. There is an upward trend in the resident population growth from 1970 to 2008.
4. The resident population remains approximately constant from 1970 to 2008.

( )
(c) Find the percentage increase in the population from 1970 to 1980. The percentage increases are given correct to 3 S.F. [2]

<table>
<thead>
<tr>
<th>Period</th>
<th>Percentage Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970 to 1980</td>
<td>19.9%</td>
</tr>
<tr>
<td>1980 to 1990</td>
<td>19.6%</td>
</tr>
<tr>
<td>1990 to 2000</td>
<td>11.3%</td>
</tr>
<tr>
<td>2000 to 2008</td>
<td></td>
</tr>
</tbody>
</table>

(d) The percentage increases in population for the two periods shown below are approximately the same.

<table>
<thead>
<tr>
<th>Period</th>
<th>Percentage Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980 to 1990</td>
<td>19.9%</td>
</tr>
<tr>
<td>1990 to 2000</td>
<td>19.6%</td>
</tr>
</tbody>
</table>

Do you think that the actual population increases for these two periods are approximately the same? Show your working and explain your answer. [2]


Do you think the percentage of residents (Singapore citizens and permanent residents) in 2008 had increased, decreased or remained unchanged compared to 2007? Show your working and explain your answer. [3]
Task B – Fertility and Aging

Total fertility rate (TFR) refers to the mean (average) number of children that would be born per female throughout her childbearing years. In Singapore, a TFR of 2.1 is considered the replacement level to sustain the current population.

B1. The bar graph below represents the TFR of Singapore residents from 2003 to 2008.

(a) Which of the following best describes the TFR trend, if any, indicated by this graph? [1]

1. Increasing trend.
2. Decreasing trend.
3. Approximately constant.
4. Others. Please specify: __________

(b) Based on this graph, do you think that the TFR has been declining over the last few decades? Give a reason for your answer. [1]

(c) The following is another bar graph based on the same data for constructing the bar graph in B1.

The two graphs in B1(a) and B1(c) appear different even though they are constructed using the same data. Which graph would you recommend to use? Give a reason for your answer.
B2. The line graph below shows the TFR from 1970 to 2008. (Based on data from www.singstat.gov.sg)

(a) Does the graph reflect that the TFR has been declining as mentioned in PM Lee’s speech? Explain your answer. [1]

(b) There is a sharp decline in the TFR from 1972 to 1975. Give possible reason(s) to explain the decline (search the web or other resources if necessary.) [2]

(c) In 1976, 1988, 2000, there are "spikes" in the TFR. Give possible reason(s) to explain these spikes (search the web or other resources if necessary). Predict the year for the next possible spike. [2]

B3. The old-age support ratio (see glossary) of residents from 1970 to 2008 is indicated in the graph below.

(a) Describe the trend in the old-age support ratio of resident population from 1980 to 2008? [1]

(b) In 1980, the old-age support ratio was 13.8. Consider 1000 residents aged 65 years and above. How many residents aged 15 to 64 years old were there for 1000 elderly residents? [1]

(c) Do you think it is better to have a high or low old-age support ratio for Singapore? Give a reason for your answer. [1]

(d) Suggest a possible reason for the trend described in B3(a). [1]

(e) In the graph above, the old-age support ratio of resident population for the year 1975 is missing. Suggest a possible value for the ratio. [1]
To predict the old-age support ratio in 2010, **student X** suggests taking the average of the ratios in 2000, 2005 and 2008.

**Student X's solution**

\[
\frac{9.9 + 8.9 + 8.4}{3} = \frac{27.2}{3} = 9.07 \text{ (3 S.F.)}
\]

Old-age support ratio in 2010 \(\approx 9.1\)

**Student Y** suggests drawing a trend line to predict the old-age support ratio in 2010.

**Student Y's solution**

Old-age support ratio in 2010 \(\approx 8\)

Which method is more reasonable? Explain your answer. [2]

B4. Based on the graph in B3(a), circle the following statements with True or False with regard to the population of Singapore.

1. There are fewer residents of working age supporting each resident above 65 years old now compared to 10 years ago. [1]
   (True / False)

2. The percentage of elderly (65 and above) in the population (15 and above) has decreased over the years. [1]
   (True / False)

3. There will likely be more elderly residents than working-age residents in 10 years' time. [1]
   (True / False)
Glossary

**Constant**
Staying the same, or not becoming less or more.

**Demography**
The study of changes in the number of births, marriages, deaths, etc. in a particular area during a period of time.

**Ethnic**
Of a national or racial group of people.

**Old-age support ratio**
The ratio of working-age residents to elderly residents. For example, if the old-age support ratio is 13.8, it means that there are 13.8 working-age residents for every elderly resident, or 138 working-age residents for every 10 elderly residents. (The ratio is computed as the number of residents aged 15 – 64 years per resident aged 65 years and above.)

**Spike**
A sudden increase in something.

**Total fertility rate (TFR)**
The mean (average) number of children that would be born per female throughout her childbearing years.

**Trend**
A general change in a situation or in the way that people are behaving.

---

**General Description of Task**
This task involves topics in statistics. The questions cover interpreting and comparing data presented in table, bar graph, line graph and pie chart. Some questions are designed to guide students in studying trends, identifying misrepresentations (B1(c)), making deductions using extrapolation and interpolation (B3(e) and B3(f)).

In Part A, the questions are related to demography of the population in Singapore, e.g., ethnic composition, percentage change in resident population, proportion of non-residents, etc.

In Part B, the questions are related to declining birth rates and aging population, e.g., total fertility rates and old-age support ratio.

**Mathematics Content Covered**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Content covered</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Percentages</strong></td>
<td>• Problems involving expressing one quantity as a percentage of another, comparing two quantities by percentage, reverse percentages, etc.</td>
</tr>
<tr>
<td><strong>Data handling</strong></td>
<td>• Interpretation of table, bar graph, line graph and pie chart.</td>
</tr>
<tr>
<td></td>
<td>• Advantages and disadvantages of the different forms of statistical charts.</td>
</tr>
<tr>
<td></td>
<td>• Draw simple inference from statistical charts.</td>
</tr>
</tbody>
</table>

Students learn about trends and make simple deductions using extrapolation and interpolation.
## Solutions

### Task A

**A1.(a)** \( \frac{270}{360} \times 3.64 \text{ million} \) \[ \approx 2.73 \text{ million} \approx 2 700 000 \] [M1] [A1]

**A1.(b)** \( \frac{495}{3 642 700} \times 360^\circ \) \[ \approx 48.9296^\circ \approx 48.9^\circ \] [M1] [A1]

**A1.(c)** \( \frac{32}{360} \times 3 642 700 \) \[ \approx 323 795.556 \] \[ \approx 324 000 \text{ (nearest thousand)} \] [M1] [A1]

**A2.(a)** Two million seven hundred thirty-five thousand and nine hundred. [A1]

**A2.(b)** Option (3). [A1]

**A2.(c)** \( \frac{2282.1 - 2013.6}{2013.6} \times 100\% \) \[ = 13.3\% \text{ (3 S.F.)} \] [M1] [A1]

**A2.(d)** No. [A1]

\[ 2735.9 - 2282.1 = 453.8 \] \[ 3273.4 - 2735.9 = 537.5 \] [A1]

If students give a valid example to explain "No", they could be given full credit.

### Task B

**B1.(a)** Option (3). [A1]

**B1.(b)** No. The timeline is too short. [A1]

**B1.(c)** The graph in B1(a) gives a better representation of the data. [A1]

The first graph starts from 0, whereas the graph in B1(c) starts from 1.25.

**B2.(a)** Yes, the TFR has declined, from above 3 to around 1.3. [A1]

**B2.(b)** Accept any reason that could cause a decrease in TFR. For example, economic recession in 1973; family planning policies introduced etc. [A2]

### A2.(e)

<table>
<thead>
<tr>
<th>Period</th>
<th>Percentage Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970 to 1980</td>
<td>13.3%</td>
</tr>
<tr>
<td>1980 to 1990</td>
<td>19.9%</td>
</tr>
<tr>
<td>1970 to 1990</td>
<td>( \frac{2735.9 - 2013.6}{2013.6} \times 100% ) [ = 35.9% \text{ (3 S.F.)} ] [M1]</td>
</tr>
</tbody>
</table>

13.3\% + 19.9\% = 33.2\% ≠ 35.9\% [M1] [A1]

No. [A1]

**A3.** The percentage of total population who were residents in 2007 was

\[ \frac{3.13 + 0.45}{3.13 + 0.45 + 1.01} \times 100\% \approx 78.0\% \] [A1]

The total population who were residents in 2008 was

\[ \frac{3.16 + 0.48}{3.16 + 0.48 + 1.2} \times 100\% \approx 75.2\% \] [A1]

The percentage of residents in 2008 had decreased. [A1]
B2.(c) Accept any reason that could cause an increase in TFR. For example, these “spikes” are dragon years, and Chinese people may think that babies born in the year of dragon are healthy and energetic. 

The next possible spike could be in 2012.

B3.(a) The ratio decreases from 1980 to 2008.

B3.(b) $13.8 \times 1000 = 13800$
There were 13800 residents aged 15 to 64 years old for 1000 elderly residents.

B3.(c) For Singapore, it is better to have a high old-age support ratio. This means that there will be more working age residents to help/support each elderly citizen.

B3.(d) Accept any reason that could cause an aging population. For example, lower birth rate caused by people choosing to be single, deciding not to have children, or having fewer children. On the other hand, medical advances and higher standard of living result in longevity.

B3.(e) A possible ratio can be 14.9 to 15.9. One possible way to make an estimate is to take the average of 17 and 13.8. 

B3.(f) Student Y’s solution is more reasonable (where the data has a downward trend).

Student X’s answer doesn’t reflect/conform to the short-term downward trend. In fact, it reverses the trend, which is not likely unless some significant events occur and dramatically affect the old-age support ratio.

B4.(1) True.
The old-age support ratio of residents ten years ago was above 10, meaning that more than 10 residents of working age support each elderly resident. In contrast, the ratio now is below 9, meaning that fewer than 9 residents of working age support each elderly resident. Therefore, this statement is correct.

B4.(2) False.
According to the graph, the old-age support ratio of residents has decreased over the years, implying that the percentage of elderly has increased. Thus, this statement is wrong.

B4.(3) False.
To have more elderly residents than working population, the old-age support ratio needs to be less than 1. From the graph, it is not likely to happen in ten years' time.
Teaching Notes

Task A

A1.
Pre-requisites
- Angle of sectors and pie charts.
- Round off numbers to specified criteria.

Objectives/Rationale
- To assess computation involving pie chart.

Instructional notes/Follow-up actions
- Help students recall angle of sector in a circle.
- Since no angle is given in the pie chart, students need to make a guess that the sector for Chinese has an angle of $270^\circ$.
- More accurate values should be used in follow-up calculations.
- Encourage students to find the latest demographic data published by the Singapore Statistical Board:

A2.
Pre-requisites
- Bar graph.
- Percentages.

Objectives/Rationale
- To assess ability to interpret information presented in a bar graph.
- To assess ability to express big numbers in words.
- To assess ability to draw conclusions about trend of population growth based on the information given in a bar graph.

Instructional notes/Follow-up actions
- The term "trend" may be new to students. Explain that trend refers to an observable pattern in the data, which can be increasing, decreasing or remaining constant.
- Citizens refer to Singaporeans, who may be a born Singaporean or someone who has been granted Singapore citizenship. They have pink colour identification card.
- Permanent residents (PR) hold foreign citizenship, but are granted permission to reside in Singapore. Foreigners can be granted PR status if they are family members of a citizen, are studying in Singapore schools, are employed or run businesses in Singapore (conditions apply). They have blue colour identification card.
- Non-residents work in Singapore but hold employment passes or work permits.
- For part (d), a common misconception is to treat the same percentage increase as the same as increase in actual quantities. Use numbers to highlight that in general, \( \frac{b-a}{a} \neq \frac{c-b}{b} \) does not imply \( (b - a) = (c - b) \), i.e., equal percentage increase does not imply equal absolute increase.
- For part (e), the misconception is to believe that percentage increase of a whole is the sum of percentage increases of the parts. Use numbers to explain that in general, \( \frac{b-a}{a} + \frac{c-b}{b} \neq \frac{c-a}{a} \), i.e., sum of percentage increase over the two periods is not equal to the actual percentage increase over the whole period.

A3.
Pre-requisites
- Compare percentages.
- Stacked bar graph.

Objectives/Rationale
- To assess ability to compare percentages.
- To assess ability to draw conclusion based on information provided by a stacked bar graph.

Instructional notes/Follow-up actions
- Graphs with multiple series of data in a column (stacked bar graphs) may need to be taught.
- Some students may just compare the heights of the bars representing citizens and PR, without paying attention to the total heights of the bars. Ask them to compute the relevant percentages.
Task B

B1.  
Pre-requisite  
- Bar graph.

Objectives/Rationale  
- To assess ability to make simple inference about trend based on the information provided by a bar graph.  
- To assess whether students can recognise misrepresentation of data in a bar graph.

Instructional notes/Follow-up actions  
- Explain the definition of “TFR” and “replacement level”. A more precise definition of TFR would be the mean (average) number of children who would be born per female, assuming all females live through their child bearing years of 15 – 49. If there were no mortality in the female population until the end of the childbearing years (generally taken as 44 or 49) then the replacement level of TFR will be very close to 2.0. That is to say, the women have just enough babies to replace themselves and their husband.  
- The replacement level is also affected by mortality, especially childhood mortality. The replacement fertility rate is roughly 2.1 births per woman for most industrialised countries (e.g., 2.075 in the UK), but ranges from 2.5 to 3.3 in developing countries because of higher mortality rates. Taken globally, the TFR at replacement is 2.33 children per woman. At this rate, global population growth would tend towards zero.  
- Some of the information above is taken from Wikipedia. More information can be found at:  
  http://en.wikipedia.org/wiki/Total_fertility_rate  
- B1(c) is an example of misuse of a bar graph, and more examples from the mass media should be discussed.

B2.  
Pre-requisites  
- Rates.  
- Line graph.

Objectives/Rationale  
- To assess whether students can understand the meaning of “spike”.  
- To assess ability to use IT tools to search for relevant information.  
- To assess ability to make simple prediction based on the information provided by the graph.

Instructional notes/Follow-up actions  
- Discuss cultural influences that may explain the increase and decrease of TFR.  
- Let students brainstorm ideas that help them to make prediction for the next spike in population.

B3.  
Pre-requisites  
- Ratio.  
- Line graph.

Objectives/Rationale  
- To assess ability to work with an unfamiliar ratio (old-age ratio).  
- To assess ability to make reasonable judgement about different solutions.

Instructional notes/Follow-up actions  
- Compare the solutions for (e) and (f). Average of values is appropriate for (e) because it involves interpolation. However, change of previous values is not appropriate for predicting trend.  
- Caution students that extrapolation may only be good for estimating values of immediate and short term data points.
B4.

Pre-requisite
- Compare ratios and percentages.

Objectives/Rationale
- To assess ability to make inferences based on ratios and percentages.

Instructional notes/Follow-up actions
- The key to judge the three statements is to have a clear understanding of the meaning of the old-age support ratio of residents. Teacher should help students differentiate ratio from percentage and absolute number.
- For the last statement, caution students that even though it seems possible from the graph that the old-age support ratio may reach zero or become negative in the long run, they should not over interpret from the graph. The old-age support ratio may stabilise or increase over the long run due to unforeseen circumstances.
Geometry:

Singapore Got Talent

The organisers of Singapore Got Talent had decided to hold the quarterfinals in the Singapore Indoor Stadium (SIS). You are to help the organisers by working from Task A to Task D, involving computation of areas and volumes of different geometric shapes as well as the conversion of units.
Task A

Base Area of Singapore Indoor Stadium

As it is a prestigious event, the organisers would like the map of this location to be as accurate as possible. A map showing the SIS was taken from Google map. The base of the SIS was found to resemble a parallelogram. Some pictures of a model of the Indoor Stadium are attached below.

Simplified Map

Pictures of a model of the Singapore Indoor Stadium
A1. Figure ABCD is a parallelogram. Complete the following:

(a) \( \angle ABC = \angle \underline{______} \) [1]
    \( \angle BAD = \angle \underline{______} \) [1]

(b) \( \angle ABC + \angle BAD = \underline{______}^\circ \) [1]
    \( \angle ADC + \angle \underline{______} = 180^\circ \) [1]

(c) \( AB // \underline{______} \) [1]
    \( AD // \underline{______} \) [1]

(d) \( AB = \underline{______} \) [1]
    \( AD = \underline{______} \) [1]

A2.
(a) Measure and write down the length of each side of parallelogram ABCD in centimetres. [2]

[Note: The file is available by request.]

(b) Give a reason why ABCD is also a rhombus. [1]

A3. Given that BE = 2.85 cm, find the area of the rhombus ABCD by using CD as the base. [2]
A4. If a quadrilateral is a rhombus, then its area can also be found using

\[
\text{Area of rhombus} = \frac{\text{product of diagonals}}{2}
\]

Given that \(AC = 5.46 \text{ cm}\) and \(BD = 3.34 \text{ cm}\), use this formula to compute the area of the rhombus ABCD.
Leave your answer correct to 3 significant figures.

Is your answer the same as your answer in question A3? [2]

A5. The actual base area of the SIS is 14 592 \(\text{m}^2\). Express the base area in

(a) hectares (1 hectare = 10 000 \(\text{m}^2\)). [2]
Leave your answers correct to 3 significant figures.

(b) \(\text{km}^2\). [2]
Leave your answers correct to 3 significant figures.

*A6. The roof of the SIS resembles the figure below, called a pyramid. (The concept of pyramid will be formally introduced in Secondary two.)

The volume of the figure can be found using the following formula.

\[
\text{Volume of the figure} = \frac{1}{3} \times \text{base area} \times \text{height}
\]

Given that the area of the base is 14 592 \(\text{m}^2\) and the height \(h = 25.8 \text{ m}\), calculate the estimated volume of the roof of the SIS. [1]
*A7.  
(a) Method 1 and Method 2 shown below can be used to estimate the volume of the roof. Which method will you use? Give a reason for your answer. [1]

**Method 1**
Volume of roof = Volume of pyramid

**Method 2**
Volume of roof = Volume of $v_1$ + Volume of $v_2$ − Volume of $v_3$, where $v_1$, $v_2$, and $v_3$ are pyramids. (Note: $v_3$ is the upper part of $v_2$)

(b) Suggest other more accurate method you can think of to estimate the volume of the pyramid roof. [2]

**Task B**

B1. Consider the following rhombus ABCD. Fill in the blanks to justify that

Area of rhombus = $\frac{1}{2} \times$ product of diagonals

Area of $BCD = \frac{1}{2} \times BD \times ( \ )$

Area of $BAD = \frac{1}{2} \times BD \times ( \ )$

Area $ABCD = $ Area $BCD$ + Area $BAD$

$= \frac{1}{2} \times BD \times ( ) + \frac{1}{2} \times BD \times ( )$

$= \frac{1}{2} \times BD \times ( ) + \frac{1}{2} \times BD \times ( )$

$= \frac{1}{2} \times BD \times ( )$

$= \frac{1}{2} \times$ product of diagonals
B2. Consider the following rectangle ABCD.

(a) Find length × breadth. [1]
(b) Compute $\frac{1}{2} \times \text{product of diagonals}$. [1]
(c) By comparing (a) and (b), determine if the following formula works for finding the area of any rectangle. [1]

\[
\text{Area} = \frac{1}{2} \times \text{product of diagonals}
\]

*(d) For students who are strong in mathematics: Does the formula "Area = $\frac{1}{2} \times \text{product of diagonals}$" work for all quadrilaterals? Justify your answer. [3]

B3. Consider the following quadrilateral ABCD with perpendicular diagonals. Fill in the blanks to justify that

Area of quadrilateral = $\frac{1}{2} \times \text{product of diagonals}$

\[
\begin{align*}
\text{Area of } ABD &= \frac{1}{2} \times BD \times (\, ) \\
\text{Area of } CBD &= \frac{1}{2} \times BD \times (\, ) \\
\text{Area of } ABCD &= \text{Area } ABD + \text{Area } CBD \\
&= \frac{1}{2} \times BD \times (\, ) + \frac{1}{2} \times BD \times (\, ) \\
&= \frac{1}{2} \times BD \times (\, + \, ) \\
&= \frac{1}{2} \times BD \times (\, ) \\
&= \frac{1}{2} \times \text{product of diagonals}
\end{align*}
\]
B4. Consider the following quadrilateral ABCD where the diagonals AC and BD are perpendicular.

(i) Is it true that “area of quadrilateral $= \frac{1}{2} \times \text{product of diagonals}”$? (Explore using IT tools or other methods.)

[Note: The file is available by request.]

(ii) If it is true, justify mathematically. (Hint: Use a similar method as in B3.)

B5. Based on B1 to B4, which of the following conditions will guarantee that the formula below is applicable for quadrilateral?

- A. The diagonals are of the same length.
- B. The diagonals are perpendicular.
- C. The diagonals bisect each other.
- D. None of the above.

Area of quadrilateral $= \frac{1}{2} \times \text{product of diagonals}$

Task C

C1. Given any quadrilateral ABCD, a rectangle WXYZ (may not be unique) can be constructed to box-up the quadrilateral, such that each vertex of the quadrilateral lies on one side of the rectangle, as shown in the figures below.

Case 1

Case 2

Case 3

(a) Is the formula below applicable to each of the three cases above?

Area of ABCD $= \frac{1}{2} \times l \times b$

[Note: The file is available by request.]
(b) True / false question. [4]

i. If AC//XY, then area of ABCD = \( \frac{1}{2} \times l \times b \).

ii. If BD//WX, then area of ABCD = \( \frac{1}{2} \times l \times b \).

iii. If AC//XY and BD//WX, then area of ABCD = \( \frac{1}{2} \times l \times b \).

iv. If area of ABCD = \( \frac{1}{2} \times l \times b \), then AC//XY.

* (b) For students who are strong in mathematics:

Make a conjecture about the condition on the diagonals so that the formula Area of ABCD = \( \frac{1}{2} \times l \times b \) applies. Justify your conjecture. [2]

(c) For the cases that meet the condition identified above, fill in the blanks below:

i. Area of ABCD : Area of XYZW = \[ \text{ } : \text{ } \] [1]

ii. Area of ABCD = ( \[ \text{ } \times Area of XYZW ) [1]

*C2. For the quadrilateral given in C1 Case 3, draw a rectangular box to box-up ABCD, so that

Area of ABCD = \( \frac{1}{2} \times l_1 \times b_1 \),

where \( l_1 \) and \( b_1 \) are length and breadth of the box drawn. [3]

Task D: Simple Application of Finding Area of Quadrilaterals

Scenario:

The organisers of a singing contest are considering a new judging rubrics for the competition. This is to give the audience a clearer idea of the contestants’ overall performance and to reduce the chances of having a tie in the total score.

Under the current judging system, the contestants are judged by the following criteria: (a) appearance, (b) singing capability, (c) stage performance and (d) popularity. The score for each criterion ranges from 0 to 10 inclusively. The total score for each contestant is between 0 and 40 points.

For example, one contestant W was given the score for each criterion as follows:

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appearance</td>
<td>9</td>
</tr>
<tr>
<td>Singing Capability</td>
<td>5</td>
</tr>
<tr>
<td>Stage Performance</td>
<td>2</td>
</tr>
<tr>
<td>Popularity</td>
<td>10</td>
</tr>
</tbody>
</table>

For the cases that meet the condition identified above, fill in the blanks below:

i. Area of ABCD : Area of XYZW = \[ \text{ } : \text{ } \] [1]

ii. Area of ABCD = ( \[ \text{ } \times Area of XYZW ) [1]

*C2. For the quadrilateral given in C1 Case 3, draw a rectangular box to box-up ABCD, so that

Area of ABCD = \( \frac{1}{2} \times l_1 \times b_1 \),

where \( l_1 \) and \( b_1 \) are length and breadth of the box drawn. [3]
Overall score of contestant W under current system = 9 + 5 + 2 + 10 = 26

In the new proposed judging rubrics, the four individual scores of contestant W are plotted as four points A, B, C and D as shown below:

The overall score of contestant W is the area of the quadrilateral determined by the four points.
(Recall that the area of the quadrilateral with perpendicular diagonals $= \frac{1}{2} \times \text{product of diagonals}$)

Overall score of contestant W under the proposed system

$= \frac{1}{2} \times \text{product of diagonals}$

$= \frac{1}{2} \times (10+5) \times (9+2)$

$= \frac{1}{2} \times 15 \times 11$

$= 82.5$

**D1.** Contestant X was given 5 points for appearance, 8 points for singing capability, 9 points for stage performance and 7 points for popularity.

**Contestant X**

(a) Find contestant X’s overall score under the current judging system. [1]

(b) Find contestant X’s overall score under the new proposed judging system (based on the area of the quadrilateral ABCD). [1]

**D2.** What is the maximum overall score a contestant can achieve under the new proposed judging system? [1]
D3. Suppose contestants Y and Z received the following scores.

<table>
<thead>
<tr>
<th>Contestant Y</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Appearance</td>
<td>10</td>
</tr>
<tr>
<td>Popularity</td>
<td>6</td>
</tr>
<tr>
<td>Singing Capability</td>
<td>8</td>
</tr>
<tr>
<td>Stage Performance</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contestant Z</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Appearance</td>
<td>8</td>
</tr>
<tr>
<td>Popularity</td>
<td>7</td>
</tr>
<tr>
<td>Singing Capability</td>
<td>9</td>
</tr>
<tr>
<td>Stage Performance</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) Who has a higher overall score under the current judging system? Show your working. [2]

(b) Who has a higher overall score under the new proposed judging system? Show your working. [2]

D4. Suppose contestants P and Q received the following scores:

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contestant P</td>
<td>Contestant Q</td>
</tr>
<tr>
<td>Appearance</td>
<td>6</td>
</tr>
<tr>
<td>Singing Capability</td>
<td>1</td>
</tr>
<tr>
<td>Stage Performance</td>
<td>4</td>
</tr>
<tr>
<td>Popularity</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Using the scores given, draw quadrilaterals represent to contestants P’s and Q’s scores on the chart above. [2]

(b) Compute contestant P’s and Q’s overall scores under the new proposed judging system. [2]
D5. A contestant was awarded 20 points under the current judging system but 0 point under the new proposed judging system. Do you think this is reasonable? Suggest a way to improve on the new proposed judging system.

[2]

D6. Give one advantage and one disadvantage of

(a) the current judging system;

(b) the new proposed judging system.

[4]

D7. Suppose a fifth criterion (dress sense) is added to the new proposed judging system. Suggest a way to adjust the proposed judging system to include this criterion.

[2]

Glossary

Conjecture
The forming of a guess about something based on how it seems and not on proof.

Criterion (plural criteria)
A standard which you use to judge, decide about or deal with something.

Perpendicular
At an angle of 90° to a given line or surface.

Pyramid
A solid object with a polygonal base and planar triangular sides which slope toward each other and meet to form a point at the top.

Quadrilateral
A flat shape with four straight sides that do not cross one another.

Justify
Show that a statement is right or reasonable.
**General Description of Task**

This task involves topics in Geometry.

Task A requires students to identify the properties of parallelograms and rhombuses, use a geometry software to verify that a given figure is a rhombus and to estimate the volume of the roof of the Singapore Indoor Stadium.

The first two questions seeks to establish that the base of the roof is a parallelogram, more specifically, a rhombus. The next few questions involve computations on geometrical shapes. The last question requires students to compare two methods of estimating the volume of the pyramid roof of the Singapore Indoor Stadium.

Task B is a guided exercise on writing the proof for “area of quadrilateral (with perpendicular diagonals) = \( \frac{1}{2} \times \) product of diagonals”. It also requires students to consider whether the formula works for a special case when the quadrilateral is non-convex.

Task C can be considered as a follow-up to Task B, where students are guided to find the area of any quadrilateral (the diagonals need not be parallel), using a method the project team named “Box method”.

Task D requires students to judge whether a new suggested scoring rubrics for the Singapore Got Talent contest is appropriate. They will apply the formula they have learned in task B to calculate the areas of quadrilaterals in this task.

Some questions indicated by asterisk (*) are quite challenging and may be suitable for the high ability group only.

**Mathematics Content Covered**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Content covered</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry and Measurement</strong></td>
<td>• Conversion of units.</td>
</tr>
<tr>
<td></td>
<td>• Angles properties of triangles and quadrilaterals.</td>
</tr>
<tr>
<td></td>
<td>• Volume and surface area of composite solids.</td>
</tr>
<tr>
<td><strong>Proof</strong></td>
<td>• Write mathematical proofs.</td>
</tr>
</tbody>
</table>
Solutions

Task A

A1. (a) \( \angle ABC = \angle ADC \) or \( \angle CDA \) \[A1\]
     \( \angle BAD = \angle BCD \) or \( \angle DCB \) \[A1\]

(b) \( \angle ABC + \angle BAD = 180^\circ \) \[A1\]
     \( \angle ADC + \angle BCD \) or \( \angle DAB = 180^\circ \) \[A1\]

(c) \( AB//CD \) or \( DC \) \[A1\]
     \( AD//BC \) or \( CB \) \[A1\]

(d) \( AB = CD \) or \( DC \) \[A1\]
     \( AD = BC \) or \( CB \) \[A1\]

A2. (a)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>3.20</td>
</tr>
<tr>
<td>BC</td>
<td>3.20</td>
</tr>
<tr>
<td>CD</td>
<td>3.20</td>
</tr>
<tr>
<td>AD</td>
<td>3.20</td>
</tr>
</tbody>
</table>

A2. (b) It is also a rhombus because all the four sides of the parallelogram are equal. \[A1\]

A3. \( CD = 3.2 \) cm
    \( BE = 2.85 \) cm
    \( \text{Area of parallelogram ABCD} = 3.2 \times 2.85 \text{ cm}^2 \) \[M1\]
    \( = 9.12 \text{ cm}^2 \) \[A1\]

A4. \( \text{Area of ABCD} = 5.46 \times 3.34 \div 2 \text{ cm}^2 \)
    \( = 9.1182 \text{ cm}^2 \)
    \( = 9.12 \text{ cm}^2 \) (3 S.F.) \[A1\]

Yes, the answer is the same as that in A3. \[A1\]

A5. (a) Actual area
    \( = \frac{14592}{10000} \) ha \[M1\]
    \( = 1.4592 \) ha \[A1\]
    \( \approx 1.46 \) ha (3 S.F.) \[A1\]

A5. (b) Actual area
    \( = \frac{14592}{1000 \times 1000} \) \text{ km}^2 \[M1\]
    \( = 0.014592 \) \text{ km}^2
    \( \approx 0.0146 \) \text{ km}^2 (3 S.F.) \[A1\]

* A6. Volume of the pyramid roof of the SIS
    \( = \frac{1}{3} \times 14592 \times 25.8 \) \text{ m}^3
    \( = 125491.2 \) \text{ m}^3 \[A1\]

* A7. (a) For estimation purpose, method 1 is better because the computation is easier, \[A1\]
    OR
    for precision and accuracy, method 2 is better. \[A1\]

* A7. (b) Dividing the roof into more than 2 pyramids can give a more accurate estimation of the volume of the pyramid roof. \[A2\]
**Task B**

**B1.** Area of $\text{BCD} = \frac{1}{2} \times \text{BD} \times (\text{CE})$  

Area of $\text{BAD} = \frac{1}{2} \times \text{BD} \times (\text{AE})$ \hspace{1cm} [A1] 

Area $\text{ABCD} = \text{Area BCD} + \text{Area BAD}$  

$= \frac{1}{2} \times \text{BD} \times (\text{CE}) + \frac{1}{2} \times \text{BD} \times (\text{AE})$  

$= \frac{1}{2} \times \text{BD} \times (\text{CE} + \text{AE})$ \hspace{1cm} [A1] 

$= \frac{1}{2} \times \text{BD} \times (\text{AC})$ \hspace{1cm} [A1] 

$= \frac{1}{2} \times \text{product of diagonals}$ 

OR  

$\frac{1}{2} \ \text{AC}$ in the first six blanks and $\text{AC}$ for the last blank. \hspace{1cm} [A3] 

**B2.** (a) Area of $\text{ABCD} = 12 \times 5 \text{ cm}^2 = 60 \text{ cm}^2$ \hspace{1cm} [A1] 

(b) $\frac{1}{2} \times \text{product of diagonals}$ 

$= \frac{1}{2} \times 13 \times 13$  

$= 84.5$ \hspace{1cm} [A1]  

$\neq 60$ 

(c) The formula does not work for rectangles. \hspace{1cm} [A1] 

*(For students who are strong in mathematics: The formula works for kites, squares, but not for rectangles, because only the area of a kite or square can be calculated as sum of areas of two triangles whose bases are on one diagonal and the sum of two heights are equal to the other diagonal.)* \hspace{1cm} [A3] 

**B3.** Area of $\text{ABD} = \frac{1}{2} \times \text{BD} \times (\text{AE})$ 

Area of $\text{CBD} = \frac{1}{2} \times \text{BD} \times (\text{EC})$ \hspace{1cm} [A1] 

Area of $\text{ABCD}$  

$= \text{Area ABD} + \text{Area CBD}$  

$= \frac{1}{2} \times \text{BD} \times (\text{AE}) + \frac{1}{2} \times \text{BD} \times (\text{EC})$  

$= \frac{1}{2} \times \text{BD} \times (\text{AE} + \text{EC})$ \hspace{1cm} [A1] 

$= \frac{1}{2} \times \text{BD} \times (\text{AC})$ \hspace{1cm} [A1] 

$= \frac{1}{2} \times \text{product of diagonals}$ 

**B4.**  

(i) Yes, it is true. \hspace{1cm} [A1] 

(ii) Area of $\text{ABD} = \frac{1}{2} \times \text{BD} \times \text{AE}$ 

Area of $\text{CBD} = \frac{1}{2} \times \text{BD} \times \text{CE}$ \hspace{1cm} [M1] 

Area of $\text{ABCD}$  

$= \text{Area ABD} - \text{Area CBD}$  

$= \frac{1}{2} \times \text{BD} \times \text{AE} - \frac{1}{2} \times \text{BD} \times \text{CE}$ \hspace{1cm} [M1] 

$= \frac{1}{2} \times \text{BD} \times (\text{AE} - \text{CE})$  

$= \frac{1}{2} \times \text{BD} \times \text{AC}$  

$= \frac{1}{2} \times \text{product of diagonals}$ \hspace{1cm} [A1] 

**B5.** B \hspace{1cm} [A1]
**Task C**

**C1.(a)** No.

The formula only applies to case 1 and 2.

**Case 1**

Area of ABCD

\[ \text{Area of ABCD} = \frac{1}{2} \times \text{product of diagonals} \]

(since diagonals are perpendicular, from Task 4B)

\[ = \frac{1}{2} \times BD \times AC \]

\[ = \frac{1}{2} \times WX \times WZ \]

(since the diagonals of ABCD are parallel and equal to WX and WZ respectively)

\[ = \frac{1}{2} \times l \times b \]

**Case 2**

Area of ABCD

\[ = \text{Area of ABD} + \text{Area of CBD} \]

\[ = \frac{1}{2} \times BD \times DW + \frac{1}{2} \times BD \times DZ \]

(with the same base BD)

\[ = \frac{1}{2} \times BD \times (DW + DZ) \]

\[ = \frac{1}{2} \times WX \times WZ \]

(BD is parallel and equal to WX and the sum of the heights of triangle ABD and CBD is equal to WZ)

**C1.(b)**

i. True  [A1]

ii. True  [A1]

iii. True  [A1]

iv. False  [A1]

**C1.(b)** The condition may be that at least one diagonal of quadrilateral ABCD is parallel to one pair of opposite sides of rectangular XYZW.  [A1]

\[ = \frac{1}{2} \times l \times b \]

**C1.(c)**

i. 1:2  [A1]

ii. \( \frac{1}{2} \)  [A1]

**C2.**

**Step 1:**

Draw two straight lines parallel to BD which pass through A and C respectively or draw two straight lines parallel to AC which pass through B and D respectively.  [A1]

**Step 2:**

Draw two straight lines perpendicular to the above two parallel lines which pass through B and D respectively or draw two straight lines perpendicular to the above two parallel lines which pass through A and C respectively.  [A1]

**Step 3:**

The four lines form a rectangle and the four intersecting points are denoted by W, X, Y, Z. Then WXYZ is the required rectangle.  [A1]
**Task D**

**D1.(a)** $(7 + 8 + 9 + 5) \text{ points} = 29 \text{ points} \quad [A1]

**D1.(b)** Overall capability of contestant A

\[ \text{Area of Figure ABCD} = \frac{1}{2} \times AC \times BD \]

(from Task B4, product of diagonals divided by 2, since diagonals intersect at right angles)

\[ = \frac{1}{2} \times 14 \times 15 \]

\[ = 105 \text{ points} \quad [A1] \]

**D2.** The maximum score = $\frac{1}{2} \times 20 \times 20$

\[ = 200 \quad [A1] \]

**D3.(a)** Contestant Z will have a higher score.

Score of contestant Y under existing system

\[ = 5 + 5 + 5 + 5 = 20 \quad [A1] \]

Score of contestant Z under existing system

\[ = 10 + 10 + 2 + 2 = 24 \quad [A1] \]

**D3.(b)** Contestant Y will have a higher score.

Score of contestant Y under the proposed judging system

\[ = \frac{1}{2} \times 10 \times 10 = 50 \quad [A1] \]

Score of contestant Z under the proposed judging system

\[ = \frac{1}{2} \times 20 \times 4 = 40 \quad [A1] \]

**D4.(a)**

\[ \text{Score of contestant P under the proposed judging system} = \frac{1}{2} \times 10 \times 2 = 10 \quad [A1] \]

**D4.(b)** Score of contestant Q under the proposed judging system

\[ = \frac{1}{2} \times 14 \times 0 = 0 \quad [A1] \]

**D5.** No, this is unreasonable.

The proposed judging system can be improved if the score in each of the four dimensions can be non-zero. \[ A1 \]
### Teaching Notes

#### Task A

**A1.**

**Pre-requisite**
- Properties of parallelograms.

**Objectives/Rationale**
- To assess knowledge of the properties of a parallelogram.

**Instructional notes/Follow-up actions**
- If necessary, demonstrate one example on the board before students complete the task.

**A2.**

**Pre-requisites**
- Properties of parallelograms and rhombuses.
- Use of IT tools (GeoGebra).

**Objectives/Rationale**
- To assess knowledge of properties of a rhombus and the relationship between parallelogram and rhombus.
- To provide opportunity to use IT tool to verify that a given figure satisfies a set of criteria and to measure the dimension of a given figure.

**A3.**

**Pre-requisite**
- Area of a parallelogram.

**Objectives/Rationale**
- To assess ability to calculate the area of a parallelogram.

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**D6. (a) (Advantage)**

It is easy to compute/understand and more conventional. [A1]

**(a) (Disadvantage)**

It is more likely for contestants to end up in ties. [A1]

**(b) (Advantage)**

It is less likely for contestants to end up in ties. [A1]

**(c) (Disadvantage)**

It is possible to have a 0 overall score. [A1]

Accept any reasonable answer. For example, the second method is more appropriate because it not only displays the strength and weaknesses of the contestant, it also shows the overall balance or all-roundedness of the contestant’s ability. It also reduces the possibility of a tie.

**D7.** Accept any reasonable answer.

Example 1: Divide the plane into five “axes” to represent the five dimensions, and the area represents the score. [A2]

Example 2: Take the fifth criterion as the height so that the representation can be a pyramid. We can get the score by using the formula for the volume of pyramid which is \( \frac{1}{3} \times \text{base area} \times \text{height} \). [A2]
Instructional notes/Follow-up actions

- Explain the need to find the base and corresponding height correctly. As this parallelogram ABCD is also a rhombus, students are likely to find the wrong heights but still get the right answer simply because all the heights in rhombus are the same. Discuss this property of a rhombus.

A4.

**Pre-requisite**
- Area of a rhombus.

**Objectives/Rationale**
- To assess ability to use an alternative formula to calculate the area of a rhombus.

A5.

**Pre-requisites**
- Conversion of $m^2$, hectares and $km^2$.
- Significant figures.

**Objectives/Rationale**
- To assess ability to convert units.

*A6.*

**Pre-requisite**
- Substitution of values into formula.

**Objectives/Rationale**
- To assess ability to use the formula to calculate the volume of a pyramid.

*A7.*

**Pre-requisite**
- Concept of approximation.

**Objectives/Rationale**
- To provide students with informal experience in approximating the volume of an irregular shape.
Task C

C1. and C2.

Pre-requisites
- Area of triangle and rectangle.
- GeoGebra.

Objectives/Rationale
- To develop investigative skill using a geometry software to verify areas of shapes.

Instructional notes/Follow-up actions
- This investigation is quite challenging. The focus is on the investigation process and not to get students to memorise another formula.
- In each of the 3 cases, students need to verify whether \( \frac{1}{2} \times l \times b = \text{area of quadrilateral} \), where \( l \) and \( b \) are sides of the rectangular box created. The area of the quadrilateral is given by the software.
- Highlight that area of ABCD given by the software may not be exactly the same as the students’ computation, due to rounding off errors and software limitations.
- For case 1, the diagonals of the quadrilateral are parallel to \( l \) and \( b \) respectively. Points A and D can be moved freely (independent points). Points B and C are dependent points. Point B is dependent on point D, so it can be moved only horizontally. Point C is dependent on point A, so it can be moved only vertically. The formula holds in this case.
- For case 2, one of the diagonals of the quadrilateral is parallel to one side of the rectangular box created. Points A, C and D can be moved freely. Point B is dependent on point D, so it can be moved only horizontally. The formula holds in this case.
- For case 3, the diagonals of the quadrilateral are not parallel to the sides of the rectangular box created. All points can be moved freely. Note that the formula does not hold in this case.
- Question C1b is designed to help student summarise and consolidate what they have learnt in question C1a.
- To stretch the better students, ask students to think about which condition is strongest and which conditions are equivalent, among parts i, ii and iii. Condition iv is the converse of condition i. Students can use the template for C1 case 3 to find a counter-example.
- C2 is a challenging task and may take around an hour. It can be done as a project work.
- Students can open the template for Case 3, click on “view ➔ navigation bar for construction steps ➔ play” to view how the template is constructed.

Task D

D1. – D4.

Pre-requisites
- Basic number operations.
- Ability to follow instruction.

Objectives/Rationale
- To expose students to new ways to think about a familiar context (award scores in competition).
- To assess ability to use the formula for area of a quadrilateral discovered in task B.
- To understand that it is helpful to examine the extreme value of a formula (D2).
### Instructional notes/Follow-up actions
- Revise the formula found in task B.
- To calculate area, only positive numbers are used. For **D2**, the maximum area will be obtained when the students substitute the biggest possible value for each criterion.

### D5. – D7.

#### Pre-requisite
- Ideas from task **D1** to **D4**.

#### Objectives/Rationale
- To provide opportunity for students to make judgment and explain their decision.

### Instructional notes/Follow-up actions
- For **D5**, students need to notice that when a contestant gets a score of zero for one or more of the categories, the shape obtained is no longer a quadrilateral (could be a triangle or a line).
- Some students may not be comfortable with explaining their answer because the reasons can be rather simple and direct. For example, a possible answer could be that the first method is more appropriate because it is easy to compute. Provide encouraging feedback.
- Allow students to discuss and bounce ideas around, before coming up with a solution.
- Show examples of good solutions to the class and discuss acceptable or unacceptable solutions taken from the class (do not display names of students whose work is not acceptable).
Linear Graphs:

Money
Money
Money
Money

Mr. Lim is currently a sales agent working at ABC Company. He comes across a job recruitment advertisement by XYZ Company selling the same products that offers different remuneration packages. He wonders if he should take up the position in XYZ Company.

In this task, you will help him make decisions using linear graphs.
1. The gross monthly salary of a sales agent under Package A offered by XYZ Company consists of two components: (1) a basic salary of $1200, and (2) a commission of 20% on the total sales.

If \( y \) is the gross monthly salary of a sales agent and \( x \) is the total sales, then

\[
y = 0.2x + 1200
\]

(a) Draw the graph of \( y = 0.2x + 1200 \) for \( 0 \leq x \leq 20000 \) [3]

(b) The gross monthly salary of a sales agent under Package B is represented in the graph given below.

(Note: \( y \) is the gross monthly salary and \( x \) is the total sales.)

(i) Write down an equation of the line in the form \( y = mx + c \). [2]

(ii) Fill in the missing numbers that appear in the advertisement for Package B. [2]

(c) If Mr. Lim decides to join XYZ Company for a higher salary, which salary package should he choose if he can achieve an average sales of

(i) $4000 per month. [1]

(ii) $8000 per month. [1]

Show your working for each question.

2. The graph below represents Package A and B mentioned in Questions 1 and 2.

(a) Label each line with either Package A or B. [1]

(b) Mr Lim notes that Package A should be chosen if the total sales is below $T$.

From the graph, find the maximum value of \( T \). [2]
3. Are the following graphs possible and logical representations of remuneration packages? Explain your answer for each graph.

(Note: $y$ is the gross monthly salary and $x$ is the total sales.)

(a) ![Graph](image)

(b) ![Graph](image)

(c) ![Graph](image)

4. Some other companies have remuneration packages as represented in the graphs below. Describe these packages in your own words. These packages may consist of basic salary, commission or bonus.

(Note: $y$ is the gross monthly salary and $x$ is the total sales.)

(a) ![Graph](image)

(b) ![Graph](image)
5. Each graph in question 4(a) to 4(c) can be represented by one of the following equations.

- \( x = 4000 \)
- \( y = \begin{cases} \frac{2}{15} x + 3000, & x < 15000 \\ \frac{2}{15} x, & x \geq 15000 \end{cases} \)
- \( y = 4000 \)
- \( y = \begin{cases} 0.1x + 1000, & x < 10000 \\ 0.3x - 1000, & x \geq 10000 \end{cases} \)
- \( y = \begin{cases} 2 \frac{1}{15} x, & x < 15000 \\ 2 \frac{1}{15} x + 3000, & x \geq 15000 \end{cases} \)
- \( y = \begin{cases} 0.1x - 1000, & x < 10000 \\ 0.3x + 1000, & x \geq 10000 \end{cases} \)

Match each graph with the correct equation.

(a) The equation for graph in question 4(a) is ____________

(b) The equation for graph in question 4(b) is ____________

(c) The equation for graph in question 4(c) is ____________

Note: The white and black dots in the graph have the same meaning as explained in (c).
General Description of Task

This task is a challenging one that combines linear equation with everyday arithmetic. The graph of the situation consists of different line segments and this concept is not in the secondary mathematics syllabus. It is included here to make the context of remuneration more authentic.

Mathematics Content Covered

<table>
<thead>
<tr>
<th>Topic</th>
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<tbody>
<tr>
<td>Percentage</td>
<td>• Solve problems involving percentage.</td>
</tr>
<tr>
<td>Coordinate geometry</td>
<td>• Interpret and find equation of a straight line graph in the form ( y = mx + c ).</td>
</tr>
<tr>
<td>Everyday arithmetic</td>
<td>• Basic salary, commission and bonus.</td>
</tr>
<tr>
<td>Applications of maths in practical situations</td>
<td>• Interpret and use graphs in practical situations.</td>
</tr>
</tbody>
</table>

Note that the graph may consist of different line segments and “broken” points. This part of the question may be more suitable for upper secondary level.

Solutions

1. (a) [A3]
   ![Graph](image)

   (b) i. \( y = 0.15x + 1500 \) [A2]
   (b) ii. The basic salary of package B is $1500. The commission of package B is 15% of the total sales. [A2]

(c) i. Package A:
   \[
   y = 0.2x + 1200 \\
   = 0.2(4000) + 1200 \\
   = 800 + 1200 \\
   = 2000
   \]

   Package B:
   \[
   y = 0.15x + 1500 \\
   =0.15(4000) + 1500 \\
   = 600 + 1500 \\
   = 2100
   \]

   He should choose **Package B**. [A1]
(c) ii. Package A:
\[ y = 0.2x + 1200 \]
\[ = 0.2(8000) + 1200 \]
\[ = 1600 + 1200 \]
\[ = 2800 \]

Package B:
\[ y = 0.15x + 1500 \]
\[ = 0.15(8000) + 1500 \]
\[ = 1200 + 1500 \]
\[ = 2700 \]

He should choose Package A. [A1]

2. (a) [A2]

(b) From the graph, maximum value of \( T = 6000 \) [A2]

3. (a) No. The downward sloping graph suggests that staff achieving higher sales will receive lower salary (may eventually be negative). [A2]

(b) Yes, the graph is a possible representation of a salary package. The staff does not receive a basic salary. The staff needs to achieve minimum sales of $3000 before he starts to earn commission. [A2]

(c) No. The graph suggests that for sales of $10,000, the income can be any positive value, and for other amount of sales, we do not know what is the corresponding income. [A2]

4. (a) The staff earns a fixed basic salary regardless of the sales amount achieved. [A2]

(b) For sales more than 10 000, the commission per unit sales is higher. [A2]

(c) On top of the basic salary and commission, the staff also earns a $3000 bonus when the sales is $15 000. [A2]

(d) For sales less than $5000, the income per month is $1000; for sales $5000 \leq y < 10 000$, the income per month is $2000$; for sales $10 000 \leq y < 15 000$, the income per month is $3000$ and for sales \( y > $15 000 \), the income per month is $4000 + a commission. [A2]
5.

(a) \( y = 4000 \) \[A1\]

(b) \( y = \begin{cases} 0.1x + 1000, & x < 10\,000 \\ 0.3x - 1000, & x \geq 10\,000 \end{cases} \) \[A2\]

(c) \( y = \begin{cases} \frac{2}{15}x, & x < 15\,000 \\ \frac{2}{15}x + 3000, & x \geq 15\,000 \end{cases} \) \[A2\]

Teaching Notes

1. and 2.

Pre-requisites
- Linear equations \((y = mx + c)\) and linear graphs.
- Percentages.

Objectives/Rationale
- To draw linear graph based on given equation.
- To write an equation based on a given linear graph.
- To compare the contexts of two given linear graphs.

Instructional notes/Follow-up actions
- Revise the pre-requisite, if necessary.
- Since large values are involved, help students to think about suitable scales for the axes.
- To help students who have difficulty with large values, change the question by using smaller values, e.g., \(y = 0.2x + 12\).
- For Q2, the maximum value of \(T\) is obtained at the point of intersection. Explain why this is so.

3. and 4.

Pre-requisite
- Linear graphs.

Objectives/Rationale
- To interpret linear graphs made up of different line segments.

Instructional notes/Follow-up actions
- For each graph, ask students to think about the following questions by referring to specific points on the graph:
  - What happens when the sale is zero?
  - Is the gradient positive, negative or zero? What does it mean?
  - What happens at the point where two different line segments meet?
- If necessary, begin with small values and plot the graphs accurately on graph paper. This will help students to read off specific values and discuss the meanings.

5.

Pre-requisite
- Equation of linear graph.

Objectives/Rationale
- To relate graph to equation.

Instructional notes/Follow-up actions
- Most students have not been taught graph that is made up of different line segments and equation consisting of 2 or more parts. This has to be taught using simple examples.
Angles and Parallel Lines:

Three Rockstars On The Wall

A school principal, Mr. Wong has a poster of "The Three Rockstars" and would like to turn it into a large painting on one of the school’s walls. He has assigned Siew Leng, Adrian, Arif and you to do this task.
1. One of the ways to enlarge or transfer a picture that you want to draw and paint is to use the "grid method". (Read more about the grid method at http://www.art-is-fun.com/grid-method.html.)

(a) The size of "The Three Rockstars" on the poster measures 49 cm by 49 cm. If the wall selected is 8 metres long and 4.9 metres high,
   i. what is the length of one side of the biggest square picture you can paint on the wall? [1]
   ii. what is the ratio of the area of the poster to that of the biggest square picture found in (a)? [2]

(b) The principal would like to position the picture at the centre of the wall. You are given the following materials: chalk, metre ruler, a roll of string, pencils, big protractor, big pair of compasses, sellotape, scissors and a ladder. Describe how you can find the center point of the wall using some or all of these materials. [2]

(c) Below is the conversation between Siew Leng, Adrian and Arif regarding the number of squares to draw on the poster and wall using the grid method. How many squares do you think is appropriate for this task? What is your reason for this? [2]
Besides using the grid method for this painting task, do you have any other way(s) of drawing the picture of "The Three Rockstars" (accurately) on the wall? [2]

2. The figure below shows one of the squares/grids of the picture. To accurately draw an outline of the picture on the wall, it is important to mark/plot the points on these straight lines.

(a) Plot and label the points A, F and G and draw the lines as shown in Figure 1A. (Some points are plotted and labeled) [2]

[Note: The file is available by request.]

(b) Are $\angle$BEC and $\angle$CED complementary angles? How did you find out? [2]
3. Figure 2 shows a close-up of the “guitar strings” from the poster.

(Note: The file is available by request.)

(a) By drawing a line across these “guitar strings” and labeling the angles, show how you check that line 1 and line 6 are parallel to each other. [3]

(b) Looking at the picture below, determine if the horizontal lines are parallel. [1]

General Description of Task

This task involves topics in ratio, angles and parallel lines. Students also develop IT skills in measuring lengths and angles using a geometry software.

Question 1(a) requires students to write ratio in the simplest form. Question 1(b) allows students to think creatively and critically on how to find the center point of a rectangular wall. Question 1(c) and (d) assess students’ reasoning skills.

Question 2 assesses students’ ability to construct a geometrical figure as well as to measure angles by making use of the online tools.

Question 3(a) requires students to draw a transversal and to formulate that corresponding angles are equal as a test of parallel lines. Question 3(b) is an optional question which allows students to create an optical illusion creatively using the available online tools.

Mathematics Content Covered

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<tr>
<td>Ratio</td>
<td>• Write a ratio in its simplest form.</td>
</tr>
<tr>
<td>Angles and parallel lines</td>
<td>• Complementary and supplementary angles.</td>
</tr>
<tr>
<td></td>
<td>• Angles formed by parallel lines and a transversal:</td>
</tr>
<tr>
<td></td>
<td>corresponding angles, alternate angles.</td>
</tr>
<tr>
<td></td>
<td>• Construction of simple geometrical figures.</td>
</tr>
</tbody>
</table>
## Solutions

**1.**

(a) i. \[ 4.9 \text{ m or } 490 \text{ cm} \] \[ \text{[A1]} \]

(a) ii. Area of poster = \( 49 \times 49 \text{ cm}^2 \)
\[ = 2401 \text{ cm}^2 \]

Area of biggest square picture
\[ = 490 \times 490 \text{ cm}^2 \]
\[ = 240000 \text{ cm}^2 \] \[ \text{[M1]} \]

The ratio is \( 2401 : 240000 = 1:100 \). \[ \text{[A1]} \]

(b) There are several answers to this question.
Any reasonable answers that state the procedures of using the available tools to find the center will be correct.
For example, the student can describe the steps:
- Use the sellotape and paste the tip of the string at one corner of the wall.
- Pull the string diagonally to the other corner of the wall.
- Do the same for the other corner to create an 'x'. The center point of the wall is where the two strings meet.
- Mark with a chalk or pencil. \[ \text{[A2]} \]

(c) Reasonable answers will be 49 squares (7 by 7) as the picture will be 490 cm by 490 cm on the wall if the scale is 1:10. Other answers can be acceptable if the student can substantiate with good reasons and some calculations. \[ \text{[A2]} \]

(d) One of the ways is to print a copy of the poster on transparency. Then use an overhead Projector (OHP) and project the image onto the wall. Then draw the outlines of the picture. \[ \text{[A2]} \]

**2.**

(a) The points will be marked on the IT platform. \[ \text{[A2]} \]

(b) They are not complementary as the sum of the 2 angles is not 90°, even though it appears so. I can find out by measuring \( \angle \text{BED} \). \[ \text{[A2]} \]

(c) It is a straight line. I found out by measuring the angle. It is 180°.

It is not a straight line. I found out by measuring the angle. It is not 180°.
(Note: The answer will depend on how the student uses the online protractor which sometimes gives 181° or 182° depending on how they placed the points.) \[ \text{[A2]} \]

**3.**

(a) Students will need to draw a transversal on the IT platform. They will need to formulate that corresponding angles are equal or alternate angles are equal. \[ \text{[A3]} \]

(b) Yes. The horizontal lines are parallel. \[ \text{[A1]} \]
Teaching Notes

1. (a), (b), (c) and (d)
   Pre-requisite
   • Simplify ratio.

   Objectives/Rationale
   • To assess ability to simplify ratio.
   • To develop general problem solving skills through applying the “grid method”.

   Instructional notes/Follow-up actions
   • Demonstrate with a picture in the class how the “grid method” works. A OHT may be required and it will take some time to complete. Use a simple diagram.

2. (a), (b) and (c)
   Pre-requisites
   • Complementary angles and angles on a straight line.
   • Basic IT skills.

   Objectives/Rationale
   • To use IT skills to construct lines, line segments and angles.

   Instructional notes/Follow-up actions
   • Demonstrate how to use the software.

3. (a) and (b)
   Pre-requisite
   • Properties of angles with parallel lines.

   Objectives/Rationale
   • To show that two lines are parallel by measuring the corresponding angles (or alternate angles).
   • To gain experience with optical illusions.

   Instructional notes/Follow-up actions
   • Due to limitation of the software, the corresponding angles may not be exactly equal. Discuss this limitation.

   • Highlight the two different statements to students:

   (1) If two lines are parallel, then their corresponding angles are equal.
   (2) If the corresponding angles are equal, the two lines are parallel.

Statement (2) is called the “converse” of statement (1), although it is not necessary to mention the term “converse”.

   • Ask students to explore more optical illusions from the website http://www.optillusions.com and create their own examples. Set this as a project.
When to Retire?

Mr Tan is a taxi driver living in his three-room HDB flat with his wife and a 7-year-old child. He is the only member of his family who is working. He is 35 years old now and hopes he can have the same standard of living as now after retirement. He started working as a taxi driver when he was 27 years old.

Mr Tan also wants to retire as early as possible. He needs help to decide when he can retire.

In this task, you are going to help Mr Tan's family come up with a financial plan so that Mr Tan can realise his dream.
Task A

Retirement without his child’s educational spending

In this task, assume that the monthly expenditure of Mr Tan’s family remains the same throughout the years.

Below is the monthly expenditure of Mr Tan’s family.

<table>
<thead>
<tr>
<th>Item</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groceries</td>
<td>600</td>
</tr>
<tr>
<td>Bills</td>
<td>200</td>
</tr>
<tr>
<td>Entertainment</td>
<td>150</td>
</tr>
</tbody>
</table>

A1. Assume that Mr Tan can save $12,600 a year.

(a) How much does he spend in a year?
(b) Find his income in a year (annual income).
(c) Find his average monthly income.

A2. If Mr Tan can earn 17.5% more than the amount stated in A1,

(a) Find the increase in his average monthly income.
(b) Find the increase in his average monthly saving.
(c) Do you think your answers in part (a) and (b) are the same? Why or Why not?
(d) Find the percentage increase in the amount he can save per month. Give your answer correct to one decimal place.

A3. Assume that he can save x% more than the amount given in A1, which is $12,600 a year.

(a) Find the percentage increase in the amount he earns in that year in terms of x.
(b) Write down an algebraic equation in terms of x if his saving at the end of the year is $17,640.
(c) Solve the equation and find the value of x.
(d) Explain your answer in (c).

A4. From A3(b), we know that he can save $17,640 a year. Mr Tan is expected to live until 75 years old, and he wants to retire when his saving is enough for the rest of his life.

(a) When he is y years old, how much will his saving be since he started working at 27 years old? Give your answer as an algebraic expression in terms of y.
(b) How much money will he spend for the rest of his life from y years old to 75 years old?
(c) Write down an algebraic expression in terms of y assuming that when he is y years old, his saving equals to the money he needs for the rest of his life.
(d) Do you think he can retire when he is 50 years old? Why or why not?
Task B

Retirement with his child's educational spending

In this task, assume that the monthly expenditure of Mr Tan's family remains the same throughout the years. From Task A, we know that the monthly expenditure of Mr Tan's family is $950 if he does not spend money on his child's education. In this part, you will help Mr Tan to make a decision on whether he can retire when he is 50 years old if he also spends money on his child's education.

B1. Mr Tan spends $150 per month for his child's education. If he earns $29 040 a year,
   (a) Find his new expenditure in a year.
   (b) Find his new saving in a year.
   (c) What is the ratio of his new expenditure to his new saving?

B2. Mr Tan is expected to spend money for his child's education when he is 34 years old to 50 years old, not including the period after 50 years old. Let \( u \) be the amount of money he spends on education per month.

   (a) Find his expenditure per year during the period when his child is studying in terms of \( u \).
   (b) Using the annual income given in B1 which is $29 040,
      i. Find his saving before he starts to pay for his child's education.

ii. Find his saving till the age of 50 when his child is studying in terms of \( u \).
iii. Find his total saving in terms of \( u \) when he is 50 years old.

B3. If he spends \( u \) per month for his child's education from 34 years old to 50 years old, not including the period after 50 years old, find the total amount of money he will spend for the rest of his life if he is 50 years old. Recall that he is expected to live until 75 years old.

B4. Assume that his saving at 50 years old equals to the total amount of money he needs for the rest of his life.

   (a) Write down an algebraic equation in terms of \( u \).
   (b) If he retires when he is 50 years old, do you think he can spend more than $150 per month for the education of his child? Show your working.

B5. Do you have any suggestions to help Mr Tan to retire as early as possible?
SOLUTIONS

TASK A

A1.  
(a) $600 + $200 + $150 = $950  
$950 \times 12 = $11 400

(b) $12 600 + $11 400 = $24 000

(c) $24 000 \div 12 = $2000

A2.  
(a) $2000 \times 17.5\% = $350 more a month.

(b) $2000 + $350 - $950 - ($2000 - $950) = $350 more a month. Or the students may use answer from part (c) to get $350.

(c) Both answers are the same because the expenditure remains at $950 a month.

(d) $\frac{350}{2000 - 950} \times 100\% = 33.3\%$

Thus, the percentage increase in the amount he saves is 33.3%.

A3.  
(a) $12 600 \times x\% = $126x

(b) He earns $126x more a year.

The percentage increase in the amount he earns is

\[ \frac{126x}{12 \times 2000} \times 100\% = \frac{126x}{24000} \times 100\% = \frac{21}{40} x\% \]

(c) $(100\% + x\%) \times 12 600 = 17 640$

\[ x + 100 = 140 \]

(d) $x = 40$

(e) He needs to save 40% more to reach $17 600.

A4.  
(a) $(y - 27) \times 17 640$

\[ = 17 640y - 476 280 \]

His saving is

\[ $(17 640y - 476 280)$.\]

(b) $(75 - y) \times 950 \times 12 = 855 000 - 11 400y$. The total money he will spend for the rest of his life is $(855 000 - 11 400y)$.

(c) $17 640y - 476 280 = 855 000 - 11 400y$.

\[ 29 040y = 1331 280 \]

(d) From part (c) we have

\[ 29 040y = 1331 280 \]

\[ y = 45.8 \]

Since $y < 50$, we conclude that Mr Tan can retire when he is 50 years old.
When to Retire?

Task B

B1.
(a) \((\$950 + \$150) \times 12 = \$13\,200\) per year.

(b) \(\$29\,040 - \$13\,200 = \$15\,840\)

(c) \(\frac{\$13\,200}{\$15\,840} = \frac{5}{6}\).

B2.
(a) \((\$950 + \$u) \times 12 = \$12\,u + \$11\,400\)

(b) 

i. \((34 - 27) \times \$17\,640 = \$123\,480\)

ii. \((50 - 34) \times (\$17\,640 - 12\,u) = \$(282\,240 - 192\,u)\)

iii. \$(123\,480 + 282\,240 - 192\,u) = \$(405\,720 - 192\,u)\)

B3. \((\$950 \times 12) \times (75 - 50) = \$285\,000\)

B4.
(a) \$(405\,720 - 192\,u) = \$285\,000\)

(b) From (a) we have
\[192\,u = 120\,720\]
\[u = 628.75\]
Since \(u > 150\), we conclude that he can spend more than \$150 per month for the education of his child.

B5. To retire as early as possible Mr Tan should get additional income from various sources.
Statistics and Percentage:

Which Mobile Plan?

Mrs Lim wants to change her child’s mobile subscription plan. You are going to help Mrs Lim make a decision so as to save cost based on her child’s typical phone usage in 2010.
Task A

Her child's phone bill in 2010

A1. Below is a pie chart showing her child's phone bill paid in August 2010.

(a) What is the angle representing the charge of incoming calls?
(b) If the charge for incoming calls was $24.00, what was the charge for sending SMS?
(c) What percentage of the total August bill was for outgoing calls?
(d) What was the charge for outgoing calls?

A2. The bar chart below shows her child's phone bill in 2010.

(a) The total bill in August was $96. If the bill in July was 6.25% less than that in August, find the bill in July.
(b) If the bill in November was $2 less than in July, find the bill in November.
(c) The charge for outgoing calls in July was 12.5% more than that in August. Find the percentage of the charge for outgoing calls to the total bill in July.
(d) In every month, the charge for sending SMS is the same as in A1(b). From the bar graph, in which month did the SMS bill have the highest percentage of the total bill of that month?
Task B

Which mobile plan?

Mrs Lim would like to save cost. First, she will need to find out her child’s typical phone usage in 2010.

Then using this information, she will look at the rates of different mobile plans.

B1. The pie chart below shows the typical usage of her child’s phone for a month in 2010. The phone bill was $96.

<table>
<thead>
<tr>
<th>Phone Charges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global SMS</td>
</tr>
</tbody>
</table>

(a) The local incoming or outgoing call is 8¢/min.
   i. Find the charge for local incoming calls.
   ii. Find the time her child spent on local incoming calls.
   iii. Find the charge for local outgoing calls.
   iv. Find the time her child spent on local outgoing calls.

(b) The charge for sending one local SMS is 5¢.
   i. Find the charge for sending local SMS.
   ii. Find the number of local SMS sent.

(c) The charge for sending one global SMS is 15¢.
   i. Find the charge for sending global SMS.
   ii. Find the number of global SMS sent.

B2. The table below shows the rates of a particular mobile subscription plan. For example, in the $25.68 monthly subscription plan, the subscriber is entitled to free incoming calls, 100 minutes of free local outgoing calls and 500 free local SMS. Any additional usage will be charged according to the local talk time, local and global SMS rates.

<table>
<thead>
<tr>
<th>All Day Free Incoming calls Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Subscription</td>
</tr>
<tr>
<td>Free local outgoing calls</td>
</tr>
<tr>
<td>Free Local SMS</td>
</tr>
<tr>
<td>Local talk time</td>
</tr>
<tr>
<td>Local SMS</td>
</tr>
<tr>
<td>Global SMS</td>
</tr>
</tbody>
</table>

(a) Answer the following questions based on your answers in B1 and the $25.68 monthly subscription plan.
   i. Find the time of the local outgoing calls that is not covered by the free local outgoing calls.
   ii. Find the charge for making the local outgoing calls.
   iii. Find the number of local SMS that is not covered by the free local SMS.
   iv. Find the charge for sending the local SMS.
   v. Find the charge for sending the global SMS.
   vi. Find the total bill.
(b) Answer the following questions based on your answers in B1 and the $48.15 monthly subscription plan.

i. Find the time of the local outgoing calls that is not covered by the free local outgoing calls.

ii. Find the charge for making the local outgoing calls.

iii. Find the total bill.

(c) Which monthly subscription plan should Mrs Lim choose in order to save cost for her child’s typical phone usage? Give your reason.

B3. Mrs Lim came upon a prepaid card X. The price of the prepaid card X is $28. For this price, the card will give $30 international balance and $100 local balance. Local incoming or outgoing calls and local SMS sent will be charged to the local balance while global SMS sent will be charged to the global balance. The table below shows the charges using the prepaid card X. Do you think Mrs Lim should choose this card instead of the monthly subscription? Give your reason.

<table>
<thead>
<tr>
<th>X Card Rates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Voice &amp; Video Calls</td>
<td></td>
</tr>
<tr>
<td>Per min (whole day)</td>
<td>8¢</td>
</tr>
<tr>
<td>Per local SMS</td>
<td>5¢</td>
</tr>
<tr>
<td>Per global SMS (Free 50 SMS a day after sending 5 SMS in the same day)</td>
<td>15¢</td>
</tr>
</tbody>
</table>
**Solutions**

**Task A**

**A1.**

(a) $25\% \times 360^\circ = 90^\circ$

(b) $\frac{162^\circ}{90^\circ} \times 24 = 43.20$

(c) $\frac{360^\circ - 162^\circ - 90^\circ}{360^\circ} \times 100\% = 30\%$

(d) $\frac{30\%}{25\%} \times 24 = \frac{6}{5} \times 24 = 28.80$

The charge for outgoing calls is $28.80.

**A2.**

(a) $(100\% - 6.25\%) \times 96 = 90$

The bill in July was $90.

(b) $90 - 2 = 88$

The bill in November was $88.

(c) Total bill in July was $90.

The charge of outgoing calls in July is:

$\frac{360^\circ - 162^\circ - 90^\circ}{360^\circ} \times 100\% = 36\%$

(d) October.

**Task B**

**B1.**

(a)

i. $25\% \times 96 = 24.$

ii. $\frac{24}{0.08} \text{ min} = 300 \text{ min}.$

iii. $30\% \times 96 = 28.80.$

iv. $\frac{28.8}{0.08} \text{ min} = 360 \text{ min}.$

(b)

i. $40\% \times 96 = 38.40$

ii. $\frac{38.40}{0.05} \text{ SMS} = 768 \text{ SMS}.$

(c)

i. $5\% \times 96 = 4.80$

ii. $\frac{4.8}{0.15} \text{ SMS} = 32 \text{ SMS}.$

**B2.**

(a)

i. $360 \text{ min} - 100 \text{ min} = 260 \text{ min}$

ii. $260 \times 16.05\text{¢} = 4173\text{¢} = 41.73\text{¢}$

iii. $768 - 500 = 268 \text{ SMS}$

iv. $268 \times 5.35\text{¢} = 1433.8\text{¢} = 14.34$

v. $32 \times 16.50\text{¢} = 528\text{¢} = 5.28$

vi. $41.73 + 14.34 + 5.28 + 25.68 = 87.03$

(b)

i. $360 \text{ min} - 300 \text{ min} = 60 \text{ min}$

ii. $60 \times 16.05\text{¢} = 963\text{¢} = 9.63$

iii. $9.63 + 14.34 + 5.28 + 48.15 = 77.40$

(c) Mrs Lim should choose the $48.15 subscription plan, because under this subscription the total bill she pays will be smaller.

**B3.** Mrs Lim should choose the pre-paid card, because with the price of $28, the local balance and the international balance can cover the bill.
Outing to the Zoo

Jasmine, her mother, and her grandmother live at Yishun Avenue 7. They plan to visit the Singapore Zoological Gardens and Night Safari on a Saturday. They plan to go to the zoo by bus. They also plan to go home by taxi.

In this task, you will make calculations to help them plan their journey.
Task A

Time taken for the trip

In this task you are to find the time taken to travel to the zoo by bus for the family.

First, they need to take Bus 171 from bus stop X near their house.

Then, they will need to alight at bus stop Y along Mandai Road to take Bus 927 to the zoo.

The distance from bus stop X to bus stop Y is 14 km. The distance from bus stop Y to the zoo is 6 km.

A1.

(a) Fill in the blanks based on the information given above.

(b) If Bus 171 travels at an average speed of 30 km/h, find the time taken to travel from bus stop X to bus stop Y.

(c) If Bus 927 travels at an average speed of 20 km/h, find the time taken to travel from bus stop Y to the zoo.

(d) Find the total time that they will spend on both buses.

(e) The family boards Bus 171 at 09 00. When they reach bus stop Y, Bus 927 has just left. Using your answer from (d) above, do you think they can reach the zoo at 10 00? Show your working.

(Hint: Refer to the frequency table of Bus 927.)

The table below shows information about Bus 927:

<table>
<thead>
<tr>
<th>Frequency (from)</th>
<th>06 30 – 08 30</th>
<th>08 31 – 16 59</th>
<th>17 00 – 19 00</th>
<th>After 19 01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choa Chu Kang Interchange</td>
<td>17 – 18 minutes</td>
<td>17 – 25 minutes</td>
<td>17 – 25 minutes</td>
<td>27 minutes</td>
</tr>
</tbody>
</table>

A2. Suppose Bus 171 travels at an average speed of \(x\) km/h and Bus 927 travels at an average speed of 20 km/h.

(a) Write an algebraic expression for the total time spent on both buses in minutes.

(b) If they have to reach the zoo by 10 00, what is the smallest possible value of \(x\), assuming that they board Bus 171 at 09 00 and that there is no waiting time involved? Show your working.

The table below shows information about Bus 171:

<table>
<thead>
<tr>
<th>Frequency (from)</th>
<th>06 30 – 08 30</th>
<th>08 31 – 16 59</th>
<th>17 00 – 19 00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yishun Interchange</td>
<td>10 – 15 minutes</td>
<td>9 – 15 minutes</td>
<td>10 – 15 minutes</td>
</tr>
</tbody>
</table>

A3. Using your answer in A1(d) and the tables provided, what is the latest time that they should be at bus stop X to make sure that they can reach the zoo by 10 00?
Task B

Transport fees

In this task, you are to find the total fare for the trips. Jasmine, her mother and grandmother are planning to take buses to the zoo. Jasmine uses a student EZ-Link card. Her mother holds an adult EZ-Link card and her grandmother is using a senior citizen EZ-Link card.

The following tables show the bus fares.

### Child/Student

<table>
<thead>
<tr>
<th>Distance</th>
<th>Fare Per Ride (cent)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EZ-Link card</td>
<td>Cash</td>
<td></td>
</tr>
<tr>
<td>Up to 3.2 km</td>
<td>36</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>3.3 – 4.2 km</td>
<td>41</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>4.3 – 5.2 km</td>
<td>46</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>5.3 – 6.2 km</td>
<td>51</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>6.3 – 7.2 km</td>
<td>55</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Over 7.2 km</td>
<td>58</td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>

### Adult

<table>
<thead>
<tr>
<th>Distance</th>
<th>Fare Per Ride (cent)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EZ-Link card</td>
<td>Cash</td>
<td></td>
</tr>
<tr>
<td>Up to 3.2 km</td>
<td>71</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>3.3 – 4.2 km</td>
<td>81</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>4.3 – 5.2 km</td>
<td>91</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>5.3 – 6.2 km</td>
<td>101</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>6.3 – 7.2 km</td>
<td>109</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>7.3 – 8.2 km</td>
<td>115</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>8.3 – 9.2 km</td>
<td>121</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>9.3 – 10.2 km</td>
<td>125</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>10.3 – 11.2 km</td>
<td>129</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>11.3 – 12.2 km</td>
<td>133</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>12.3 – 13.2 km</td>
<td>137</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>13.3 – 14.2 km</td>
<td>141</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>14.3 – 15.2 km</td>
<td>145</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>15.3 – 16.2 km</td>
<td>149</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>16.3 – 17.2 km</td>
<td>153</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>17.3 – 18.2 km</td>
<td>157</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>18.3 – 19.2 km</td>
<td>161</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>19.3 – 20.2 km</td>
<td>164</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>20.3 – 21.2 km</td>
<td>167</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

### Senior Citizen

<table>
<thead>
<tr>
<th>Distance</th>
<th>Fare Per Ride (cent)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EZ-Link card</td>
<td>Cash</td>
<td></td>
</tr>
<tr>
<td>Up to 3.2 km</td>
<td>53</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>3.3 – 4.2 km</td>
<td>61</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>4.3 – 5.2 km</td>
<td>68</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>5.3 – 6.2 km</td>
<td>76</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>6.3 – 7.2 km</td>
<td>81</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>Over 7.2 km</td>
<td>86</td>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>

B1. They take Bus 171 for 14 km and transfer to bus 927 for 6 km. Consider the following two methods to calculate the total fare when using their EZ-Link cards.

**Method A**
Total fare is computed based on the total distance traveled.

**Method B**
When passengers alight at a bus stop, it is calculated as one trip. When they board a bus at the bus stop where they alighted, it is considered as a new trip.

(a) Using method A, find
i. Jasmine’s fare.
ii. Her mother’s fare.
iii. Her grandmother’s fare.
iv. Their total bus fare.

(b) Using method B, find
i. Jasmine’s fare.
ii. Her mother’s fare.
iii. Her grandmother’s fare.
iv. Their total bus fare.
Outing to the Zoo: Data handling and Algebra

(c) From the internet, find out which method is correct in computing their total bus fare if they use EZ-Link cards.

B2. If the family forgets to bring their EZ-Link cards and they need to pay by cash, find their total bus fare.

B3. The family plans to go home at 23 00 by taxi. The flag-down fare which is inclusive of the 1st km or less is $2.80. For every 385 m thereafter or less up to 10 km, it is $0.20.

Assume that the total distance travelled by the taxi is 9855 m.

(a) Find the taxi fare for the trip.
(b) If they did not get a taxi before midnight and the additional midnight charges is 50% of the total taxi fare, find the new taxi fare.
(c) Would they rather travel by bus or taxi home at 23 00 if they have forgotten to bring their EZ-Link cards? Give your reason.

Note to teachers: The bus fare structure has been changed since 2010.

SOLUTIONS

Task A

A1.
(a) $\frac{14}{30} \times 60 \text{ min} = 28 \text{ min}.$

(b) $\frac{6}{20} \times 60 \text{ min} = 18 \text{ min}.$

(d) $28 \text{ min} + 18 \text{ min} = 46 \text{ min}.$

(e) No they cannot.

If they were to be at the zoo by 10 00, then the total time they need should not be more than 1 hour. The time they need for waiting for bus 927 is at least 17 minutes.

Since $46 \text{ min} + 17 \text{ min} = 63 \text{ min},$ the total time they need is more than 1 hour.

A2.
(a) The time they spend on bus 171 is $\frac{14}{x} \times 60 \text{ min}.$

The time they spend on bus 927 is 18 min.
Thus the total time they spend on both buses is $\frac{14}{x} \times 60 + 18 \text{ min}.$
Outing to the Zoo: Data handling and Algebra

(b) Since the total time is 60 min (from 09 00 am to 10 00 am), we have

\[
\frac{14}{x} \times 60 + 18 \leq 60
\]

\[
\frac{14}{x} \leq \frac{42}{60}
\]

\[
x \geq \frac{14 \times 60}{42} = 20
\]

Hence, the smallest value of \( x \) is 20.

A3. Since they need to be on the buses for 46 min and there is a possibility to spend 15 min to wait for bus 171 and 25 min to wait for bus 927, the longest time they need for the trip is 86 min. Hence the latest time they should be at bus stop is at 08 34.

Task B

B1.
(a)

i. 58 cents
ii. 164 cents
iii. 86 cents
iv. \((58 + 164 + 86)\) cents = 308 cents

(b)

i. 58 cents + 51 cents = 109 cents
ii. 141 cents + 101 cents = 242 cents
iii. 86 cents + 76 cents = 162 cents
iv. \((109 + 242 + 162)\) cents = 513 cents

(c) Method A is the correct method because bus fares are computed based on total distance travelled if EZ-link card is used.

B2. \((75 + 55) + (180 + 120) + (120 + 90)\)

\= 640 cents

B3.
(a) \(\$2.80 + \left(\frac{\$0.85 - \$1.00}{\$0.85}\right) \times \$0.20 = \$7.40\)

The total fare is $7.40.

(b) \(7.4 + (50\% \times 7.4) = 11.1\)

The new fare is $11.10.

(c) They would rather take taxi than bus because the difference in cost is only $1 but the time taken to reach home will be less than taking the buses. Thus the additional $1 is worth it. Any other reasonable answers like saving $1 can be accepted.
Everyday Mathematics Test

1. The facility indices were based on the results collected from about 900 Secondary 1 Express students in 2011. Teachers may compare their students' performance against these indices, although many factors can be used to explain differences in performance.

2. Questions 9 and 10 given here are parallel but not the actual questions given to the students in the study. Hence, the facility indices for these two questions given below are only rough guides.

<table>
<thead>
<tr>
<th>Question</th>
<th>Context (Topic)</th>
<th>Facility Index (Mean as a Percent of Max Score)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sale (Percentage, discount)</td>
<td>71.7</td>
</tr>
<tr>
<td>2</td>
<td>Tourism (Interpretation of table and pie chart, rate)</td>
<td>64.9</td>
</tr>
<tr>
<td>3</td>
<td>Kool Biscuits: Reduced fat (Percentage)</td>
<td>61.8</td>
</tr>
<tr>
<td>4</td>
<td>Population (Interpretation of table, significant figures, rate)</td>
<td>57.4</td>
</tr>
<tr>
<td>5</td>
<td>Types of fires (Interpretation of table, percentage change)</td>
<td>54.6</td>
</tr>
<tr>
<td>6</td>
<td>Decibels (Four operations, rate)</td>
<td>40.8</td>
</tr>
<tr>
<td>7</td>
<td>Earthquake (Powers of 2)</td>
<td>36.6</td>
</tr>
<tr>
<td>8</td>
<td>Mobile plan (Rate, line graph)</td>
<td>35.5</td>
</tr>
<tr>
<td>9*</td>
<td>Hokkien char mee (Interpretation of chart, percentage)</td>
<td>26.1</td>
</tr>
<tr>
<td>10*</td>
<td>Math Olympiad (Line graph, bar graph, misuse of graphs)</td>
<td>22.2</td>
</tr>
</tbody>
</table>
1. A particular item costs $6. Shop X advertises, “buy four items for the price of three”.

   (a) How much does a customer have to pay for 4 such items in shop X?

   (b) What is the percentage discount for the customer who buys 4 such items from shop X?

   (c) Another shop Y offers, “buy three at the regular price and pay 50% for the fourth item.” Shop Z offers a voucher of 10% on the total amount paid.

   Your parents wish to buy 4 such items.

   Out of these 3 shops (X, Y, and Z), which shop gives the best deal?

   (d) Other than the amount you have to pay, what other reasons would you give to your parents to support your choice?
2. The following table shows the number of tourists who visited each of the three countries and the amount they spent while visiting the country.

<table>
<thead>
<tr>
<th>Countries</th>
<th>Number of Tourists (millions)</th>
<th>Amount spent US$ (billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Malaysia</td>
<td>23.6</td>
<td>15.4</td>
</tr>
<tr>
<td>Singapore</td>
<td>7.5</td>
<td>9.2</td>
</tr>
<tr>
<td>Thailand</td>
<td>14.1</td>
<td>15.9</td>
</tr>
</tbody>
</table>

(a) Given that 1 billion is one thousand million, write down 15.4 billion using only numbers.

(b) On average, how much did a tourist spend in Singapore? Give your answer correct to the nearest dollar.

(c) Which country is the most popular tourist destination? Explain your choice.

(d) The pie chart below shows the number of tourists visiting the three countries and another country X. How many tourists visited country X?
3. A box of Kool biscuits labelled as "reduced fat" has 30% less fat per serving than the regular variety.

(a) If the regular biscuits contain 4 g of fat per serving, how many grams of fat are there in one serving of the Kool reduced fat biscuits?

(b) If the Kool reduced fat biscuits contain 2.5 g of fat per serving, how many grams of fat are there in one serving of the regular biscuits?

(c) Food labels about fat are based on the following rules:
   - "reduced fat" if it has at least 25% less fat per serving than the regular variety;
   - "low fat" if it contains 3 g or less of fat per serving;
   - "fat free" if it contains less than 0.5 g of fat per serving.

According to these rules,
(i) are Kool biscuits correctly labelled as "reduced fat"? Why?

(ii) can you label these Kool biscuits as "low fat"? Why?
4. The following table shows the area and population of six towns in Singapore.

<table>
<thead>
<tr>
<th>Town</th>
<th>Area (sq km)</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ang Mo Kio</td>
<td>13</td>
<td>179 300</td>
</tr>
<tr>
<td>Bedok</td>
<td>22</td>
<td>294 500</td>
</tr>
<tr>
<td>Bukit Timah</td>
<td>18</td>
<td>70 300</td>
</tr>
<tr>
<td>Jurong West</td>
<td>15</td>
<td>267 500</td>
</tr>
<tr>
<td>Ubi</td>
<td>2</td>
<td>30 100</td>
</tr>
<tr>
<td>Yishun</td>
<td>21</td>
<td>185 200</td>
</tr>
</tbody>
</table>

(a) In which town is your school located?

[Note for teachers: Please change the town locations to include your school.]

(b) (i) Which town has the least population?

(ii) Express this population in words.

(iii) Round off your answer to (ii) above to 2 significant figures.

(c) Which town is the most crowded? Justify your answer.

(d) The area of Toa Payoh is 8 sq km. Using the information in the above table or otherwise, estimate its population. Show your working and explain your answer.
5. The table below shows the number of different types of residential fires in Singapore in 2009 and 2010.

<table>
<thead>
<tr>
<th>Types of Fires</th>
<th>2009</th>
<th>2010</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubbish Chute</td>
<td>1285</td>
<td>1241</td>
<td>+3</td>
</tr>
<tr>
<td>Discarded Items</td>
<td>644</td>
<td>801</td>
<td>-24</td>
</tr>
<tr>
<td>Unattended cooking</td>
<td>517</td>
<td>474</td>
<td>+8</td>
</tr>
<tr>
<td>Total number of fires</td>
<td>3549</td>
<td>3267</td>
<td>+8</td>
</tr>
<tr>
<td>Number of people injured from these fires</td>
<td>127</td>
<td>143</td>
<td></td>
</tr>
</tbody>
</table>

(a) Calculate the change (%) for the number of people injured from these fires and write it in the empty cell of the above table, including the correct “arrow” to show the change. Give your answer to the nearest whole number.

(b) What mathematical symbol or sign can you use to replace the “arrows” (∴ or ) in the table above? Explain your choice.

(c) As the chairperson of the Singapore National Fire Prevention Council, you intend to launch a Fire Safety Campaign to help Singaporeans reduce one of the three residential fires mentioned in the above table.
   Which type of residential fire would you select for the campaign and why?

(d) Jamie was puzzled by the values in the last two rows: from 2009 to 2010, there were fewer fires but the number of people injured by these fires had increased. She believed that more fires meant more people injured, so there must be some mistakes in the values.
   Do you agree with her observation? Explain your reasoning.
6. The loudness of sound is measured in decibels (dB). Noise from heavy traffic is about 85 dB and this can cause hearing damage if one is exposed to it for 8 hours or more. For every 3 dB over 85 dB, the exposure time before damage occurs is decreased by half.

(a) If the noise is 88 dB, what is the exposure time before damage occurs?

(b) John likes to listen to his music using ear-plugs at high volume of 100 dB. How long could he do this before damage occurs?

(c) Is the information useful to you? Why?
7. Singapore is lucky to be free from major earthquake. The magnitude (or strength) of an earthquake is measured by a number on the Richter scale. When the magnitude increases by 0.2, the energy of the earthquake is doubled.

When the magnitude is 1, the energy released is about 2 million Joules (J). When the magnitude is 1.2 (1 + 0.2), the energy released is doubled (2 × 2 million).

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (Joules)</td>
<td>2 million</td>
<td>4 million</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the above table for magnitudes 1.4 to 2.0.

(b) The 2004 Indian Ocean and tsunami had a magnitude of 9.2. How much energy was released? Write your answer in million Joules. Hint: Think in powers of 2.
8. A Telco company offers the following two mobile phone plans to its customers.

<table>
<thead>
<tr>
<th>Plan</th>
<th>SaverCall</th>
<th>ValueCall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Subscription</td>
<td>$25.68</td>
<td>$48.15</td>
</tr>
<tr>
<td>Free Outgoing Calls (minutes)</td>
<td>100</td>
<td>300</td>
</tr>
</tbody>
</table>

All Day Free Incoming Calls

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Local talk time (exceeding free outgoing call minutes)</td>
<td>16.05¢/ min</td>
</tr>
<tr>
<td>Free Local SMS</td>
<td>500</td>
</tr>
<tr>
<td>Local SMS (exceeding free SMS limit)</td>
<td>5.35¢</td>
</tr>
</tbody>
</table>

(a) If a customer sent 700 local SMS in a particular month, how much did he have to pay for sending these SMS under each plan? Ignore the monthly subscription.

(i) Under the SaverCall plan: ________________________________

(ii) Under the ValueCall plan: ________________________________

(b) The graph below shows the monthly charges (subscription plus calls) for different local outgoing calls (minutes) for the SaverCall plan. Ignore the SMS sent for this question. Explain why the graph from A to B is horizontal.

[Graph showing monthly charge against local outgoing calls]
(c) For this part, you have to plot the graph for the ValueCall plan on the same graph. Ignore the SMS sent for this question.

(i) Find the monthly charge (subscription plus calls) for making 400 local outgoing calls (minutes) under the ValueCall plan.

(ii) On the graph, mark the point V to show the monthly charge for making 400 local outgoing calls (minutes) under the ValueCall plan.

(iii) On the graph, plot the line to show the monthly charges under the ValueCall plan to cover local outgoing calls (minutes) from 0 minutes to 400 minutes.

(d) If you make 260 minutes of local outgoing calls per month, which is a cheaper plan? Ignore the SMS sent. Explain how you arrive at your answer by referring to the two graphs only, without further calculations.
9. The diagram below shows the amount of salt, total fat, and cholesterol in a plate of Hokkien char mee.

(a) Look at the information about “Cholesterol”. What do 210 mg and 65.8% mean?

(b) Look at the information about “Total fat”. Calculate the amount of fat you would need in one day.

(c) One piece of information in this diagram does not make sense. Identify and correct the error.

Note for teachers:
These are not exact values. Information about Singapore food and daily dietary allowance can be found at www.nutrition.com.sg/.
10. The diagram below shows some information about the number of students who were prize winners at the Math Olympiad over several years.

![Line graph showing number of prize winners and prize winners per 100 participants from 2007 to 2012.]

(a) Which statement below explains the meaning of “2.5” in the line graph?

(A) 2.5 participants were prize winners in year 2012.
(B) A person had the chance of 25% of becoming a prize winner in year 2012.
(C) 2.5 out of every 100 participants were prize winners in year 2012.
(D) One person out of 2.5 participants became a prize winner in year 2012.

Ans: __________

(b) Look at the bar graph. The bar for year 2010 is twice as long as the bar for year 2009. This shows that the number of prize winners in year 2010 was twice the number of prize winners in 2009. Is this true? Why or why not?

(c) Use the information in the above diagram to find the total number of participants in Math Olympiad in year 2009.
Marking Scheme for Everyday Mathematics Test

1.  
(a) \(6 \times 3 = \$18\) \([1]\)  
(b) \(\frac{[4 \times 6 - 18]}{4 \times 6} \times 100\%\) \([M1]\)  
   \[= 25\%\] \([A1]\)  
(c) \(Y: 6 \times 3 + (6 \times 50\%)\) \([M1]\)  
   \[= \$21\] \([A1]\)  
   
   \(Z: 6 \times 4 - (6 \times 4 \times 10\%)\) \([M1]\)  
   \[= \$21.60\] \([A1]\)  
   so \(X\) gives the best buy. \([1]\)  
(d) Any reasonable answer. \([1]\)  

2.  
(a) \(15,400,000,000\) \([1]\)  
(b) \(9.2 \times 10^9 \div (7.5 \times 10^6)\) \([M1]\)  
   \[\approx \$1226.67\]  
   \[= \$1227 \text{ (Correct to nearest dollar)}\] \([A1]\)  
(c) Any reasonable statement stated clearly, e.g., number of tourists, the number of tourists divided by size of country (no need to give exact values) or amount spent (popular for shopping). We are testing general ability to take note of information and to write clearly. \([2]\)  
   For an attempt to use information, but writing not clear. \([1]\)
(d) X is $\frac{1}{4}$ of the total  

\[
(23.6 + 7.5 + 14.1) \div 3 \text{ million} 
\approx 15.1 \text{ million}
\]

3.
(a) $4 \text{ g} - (4 \text{ g} \times 30\%)$  

\[
= 2.8 \text{ g} 
\]

(b) $2.5 \text{ g} \div 70\%$  

\[
\approx 3.57 \text{ g} 
\]

(c) (i) Yes,  

because it has 30\% (more than 25\%) less fat per serving than the regular variety.  

(ii) Yes,  

because it contains 2.8 g of fat per serving which is less than 3 g of fat per serving.

4.
(a)  

<table>
<thead>
<tr>
<th>Town</th>
<th>Schools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ang Mo Kio</td>
<td>Chong Boon SS</td>
</tr>
<tr>
<td>Bedok</td>
<td>Bedok South SS, Damai SS, St. Anthony Canossian SS</td>
</tr>
<tr>
<td>Bukit Timah</td>
<td>Nanyang Girls’ High school</td>
</tr>
<tr>
<td>Jurong West</td>
<td>Hong Kah SS</td>
</tr>
<tr>
<td>Ubi</td>
<td>Manjusri SS</td>
</tr>
<tr>
<td>Yishun</td>
<td>Ahmad Ibrahim SS, Yishun SS</td>
</tr>
</tbody>
</table>

(Award the mark only when they answer in the correct town.)

(b)  

(i) Ubi

(ii) Thirty thousand and one hundred

(iii) 30 000
(c) Divide each population by the area to get the population per square kilometre. [M1]

<table>
<thead>
<tr>
<th>Town</th>
<th>Area (sq km)</th>
<th>Population</th>
<th>Population/Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ang Mo Kio</td>
<td>13</td>
<td>179 300</td>
<td>13 792</td>
</tr>
<tr>
<td>Bedok</td>
<td>22</td>
<td>294 500</td>
<td>13 386</td>
</tr>
<tr>
<td>Bukit Timah</td>
<td>18</td>
<td>70 300</td>
<td>3 906</td>
</tr>
<tr>
<td>Jurong West</td>
<td>15</td>
<td>267 500</td>
<td>17 833</td>
</tr>
<tr>
<td>Ubi</td>
<td>2</td>
<td>30 100</td>
<td>15 050</td>
</tr>
<tr>
<td>Yishun</td>
<td>21</td>
<td>185 200</td>
<td>8 819</td>
</tr>
</tbody>
</table>

Jurong West is the most “crowded” town based on population per square kilometre. [1]

(d) Toa Payoh is an old town like Jurong West, so use Jurong West to calculate.

Explanation [1]

\[
(267\ 500 \div 15) \times 8
\approx 143\ 000
\]

Calculation [1]

The reported figure is 150 000.

5.

(a) \((143 - 127) \div 127 \times 100\%\) [M1]

\approx 12.598\%

\approx 13\% \text{ (Round off to whole number)} [A1]

Correct arrow \(\wedge\) [1]

(b) ‘+’ to replace \(\wedge\) and ‘−’ for \(\vee\) because ‘+’ means increase and ‘−’ means decrease.

Or any system with reasonable explanation. [1]

(c) Reasonable argument using the information. [2]

Some examples:
- Rubbish chute because it is most frequent.
- Discarded items because it is increasing fast.
- No mark awarded if student does not refer to information.

(d) Clear explanation that fires may or may not injure people. [2]

If the idea is vague and not well written. [1]
Everyday Mathematics Test

6.

(a) 8 hours ÷ 2 = 4 hours  [1]

(b) 100 - 85 = 15 dB  
    15 ÷ 3 = 5  [M1]  
    8 ÷ 2^5 = \frac{1}{4}  [M1]  
    15 minutes  [A1]

(c) Any reasonable explanation like learn about protecting oneself from hearing loss.  [1]

7.

(a)  

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (Joules)</td>
<td>8 million</td>
<td>16 million</td>
<td>32 million</td>
<td>64 million</td>
</tr>
</tbody>
</table>

All correct answers.  [3]
For the correct 8 million for 1.4.  [1]
For any of the 2 remaining.  [1]
No mark should be awarded if the student begins with 6 for 1.4.  
But give 1 mark if it is multiplied by 2 later.

(b) 9.2 - 1 = 8.2  [M1]  
    8.2 ÷ 0.2 = 41  [M2]  
    Final answer: 2^{40} million  [A1]  
    2^{41} or 2^{42} appears somewhere  [1]  
    Any attempt to find patterns, but not successful.  [1]  
    Any attempt to find patterns with a few powers of 2, but not successful.  [2]

8.

(a)  

(i) $0.0535 \times 200 = $10.70  [1]  
(ii) $0.0535 \times 200 = $10.70  [1]

(b) No charge for first 100 minutes as it is in  
    the monthly subscription  [1]
(c)

(i) \((400 - 300) \times 16.05\text{¢} = \$16.05\) \[M1\]

\[
= 48.15 \text{¢} + 16.05 \text{¢}
\]

\[= \$64.20\] \[M1\] \[A1\]

(ii) (400, 64.2); must be 400; allow half a square for 64.2 \[2\]

(iii) Horizontal line joining (0, 48.15) [which is already marked on the graph] to (300, 48.15). \[1\]

Line joining (300, 48.15) to V. \[1\]

For neat and accurate graph. Allow for half a square for plotting. \[1\]

(d) Student can explain clearly the meaning of the point of intersection; or which line is above; etc. \[2\]

If explanation is in the right direction (must refer to graphs) but not well stated. \[1\]

No mark if no reference to the graphs.

9.

(a) It contains 210 mg of cholesterol \[1\]

and this is 65.8\% of daily allowance. \[1\]
(b) \(30 \text{ g} ÷ 0.462\)  
\[= 64.9 \text{ g}\]  

(c) Amount of salt.  
Because the amount is too high; it could be 1.525 g or 1525 mg.

10.
(a) C  
[1]

(b) Not true.  
Because the y-axis of the bar graph starts at 5 so the length of the bar is not representative of the number of prize winners; vertical axis does not start from zero.  
For vague idea but not well expressed.

(c) 15 prize winners and 1.5 prize winners in every 100 participants in year 2009.

\[\frac{15}{1.5} × 100\]  
\[= 1000\]
Attitudes toward Learning Mathematics Questionnaire

1. This Questionnaire has been partially validated with about 900 Secondary 1 Express students in 2011. The Cronbach’s alphas of the six scales are within acceptable ranges (0.63 to 0.90).

2. Instead of a 9-point scale, a 5-point scale may be used: 5 = strongly agree, 4 = agree, 3 = neutral, 2 = disagree, 1 = strongly disagree.

3. Negative items are indicated by asterisks. Reverse score these items as follows. If the 9-point scale is used, subtract the student’s score from 10. If the 5-point scale is used, subtract the student’s score from 6.

4. After the negative items have been reverse scored, add the scores of the respective items to obtain the score of each scale for the student. An overall Attitude score for a student can be obtained by adding his/her scores (after reverse scored) for all the items.

5. For the whole class, compute the mean for each scale and the overall Attitude scale.

6. Teachers do not have to use all the 24 items. Choose scales that are of interest. However, for the selected scales, try to use all the given items in that scale.

<table>
<thead>
<tr>
<th>Scales</th>
<th>Items</th>
<th>Cronbach’s Alphas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check solutions</td>
<td>1, 7, 13*, 19</td>
<td>0.714</td>
</tr>
<tr>
<td>Confidence</td>
<td>2, 8, 14, 20*</td>
<td>0.695</td>
</tr>
<tr>
<td>Enjoyment</td>
<td>3, 9*, 15, 21</td>
<td>0.898</td>
</tr>
<tr>
<td>Use of IT</td>
<td>4*, 10, 16, 22</td>
<td>0.693</td>
</tr>
<tr>
<td>Multiple solutions</td>
<td>5*, 11, 17, 23</td>
<td>0.633</td>
</tr>
<tr>
<td>Usefulness</td>
<td>6, 12, 18*, 24</td>
<td>0.711</td>
</tr>
</tbody>
</table>
Dear students: The purpose of this survey is to find out how Secondary One students think and feel about mathematics. Your responses will help us understand students like you better. All responses will be kept strictly confidential. Please answer **ALL** the questions as best as you can. For each question, please tick (✓) your answer. There is no correct or wrong answer to each question. Thank you for your cooperation.

Please take note of the following scale:

1 - Disagree totally  
2 - Disagree a lot  
3 - Disagree  
4 - Disagree a little  
5 - Neither Disagree nor agree  
6 - Agree a little  
7 - Agree  
8 - Agree a lot  
9 - Agree totally

<table>
<thead>
<tr>
<th>Attitudes</th>
<th>Disagree Totally</th>
<th>Agree Totally</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. When I know I have made a mistake in solving a problem, I will try to find out why.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>2. I am good at using mathematics to solve real-life problems.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>3. I enjoy doing mathematics.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>4. I do not like to use the computer to learn mathematics.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>5. I do not like to think of other ways to solve the same problem.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>6. Mathematics is important.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>7. After I have solved a problem, I will go through the solution again and check if I have made any mistakes.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>8. I am confident in solving mathematics problems.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>9. I find mathematics boring.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>10. I can learn mathematics from playing computer games.</td>
<td>O O O O O O O O O</td>
<td></td>
</tr>
<tr>
<td>Attitudes</td>
<td>Disagree Totally</td>
<td>Agree Totally</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>-------------------</td>
<td>---------------</td>
</tr>
<tr>
<td></td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>11. I often figure out different ways to solve mathematics problems.</td>
<td>O     O     O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>12. I think mathematics is useful in solving real world problems.</td>
<td>O     O     O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>13. Once I have worked out an answer to a problem, I do not check my answer.</td>
<td>O     O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>15. Overall, I have good feelings about mathematics.</td>
<td>O     O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>16. IT (Information Technology) has been helpful to my mathematics learning.</td>
<td>O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>17. I try to understand the different solutions given by my classmates.</td>
<td>O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>18. I think mathematics is useful only for tests.</td>
<td>O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>19. After I have solved a problem, I will ask myself if the answer makes sense to the given problem.</td>
<td>O     O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>20. I am not good at giving reasons in mathematics.</td>
<td>O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>21. Solving mathematics problems is fun to me.</td>
<td>O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>22. Mathematics software (e.g., graphing) helps me to learn mathematics.</td>
<td>O     O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>23. After I have solved a problem, I will look for other methods to solve it.</td>
<td>O     O     O     O     O     O</td>
<td></td>
</tr>
<tr>
<td>24. Mathematics helps me to understand reports and advertisements about prices, sale, percentages etc.</td>
<td>O     O     O     O     O     O</td>
<td></td>
</tr>
</tbody>
</table>