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SINGAPORE PRIMARY SCHOOL TIMSS DATA: WHOLE NUMBERS, FRACTIONS AND PROPORTIONALITY

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Abstract

As has been widely reported, Singapore outperformed all other countries in Mathematics for the Third International Mathematics and Science Study (TIMSS) and this is an achievement that reflects positively on all components of the educational system. An analysis of the performance of Singapore students on the Whole Numbers and Fractions and Proportionality items shows some clear trends and provides implications for teaching. The general conclusions are: (1) Primary four students outperform primary three on all items; (2) On most items Singapore students outperform their corresponding International cohort; (3) Girls outperform boys on Whole Numbers but the results are mixed for Fractions; and (4) Boys tend to leave questions blank more often than girls.

Finally, the analysis of the data also shows that students have more difficulty with items that require comprehension of the concepts involved than items which are purely routine, and this implies that there needs to be an increased focus on activities designed to foster understanding.

As has been widely reported, Singapore came top in Mathematics for the Third International Mathematics and Science Study (TIMSS) (Research and Testing Division, 1997; Suh and Oorjitham, 1996; Newsweek, 1996). Certainly this is an achievement that reflects positively on all components of the educational system. The National Report for Singapore (Population 1) (Research and Testing Division, 1997) suggested some factors, such as coherence in the curriculum coverage, working ethos of teachers, students home environment, premium placed by parents on education, and so on that could have contributed to the success of Singapore students. It is not the purpose of this article to reiterate details that have been published elsewhere. Rather this is generally a descriptive report that the authors hope will make readers aware of some of the specifics and the implications

of the results. In this article, the authors examine the performance of primary three and primary four students in the content categories of Whole Numbers and Fractions and Proportionality.

The TIMSS Data

The primary school sample consisted of approximately 14 500 students from primary three and primary four. Table 1 contains the exact numbers by level and sex.

Table 1 : Sample

	n (girls)	n (boys)	Total
Primary 3	3467	3749	7216
Primary 4	3442	3826	7268
Total	6909	7575	14 484

There were a total of 102 mathematics items in the test (Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzalez, E. J., Kelly, D. A., and Smith, T. A., 1997), distributed between 8 booklets, with each student answering only one booklet. The items were a combination of 79 Multiple choice (42 of which were released), 15 short answer (all of which were released) and 8 extended response (all of which were released). The following topics were covered: Whole numbers (25%); Fractions and Proportionality (21%); Measurement, estimation and number sense (20%); Data representation, analysis and probability (12%); Geometry (14%); and Patterns, relations and functions (10%). The following analysis is based *solely on the released items* (Website: http://wwwcsteep.bc.edu/timss; IEA: TIMSS, 1997).

The whole number data

There were a total of 16 released items on whole numbers (10 multiple choice, 5 short answer and 1 extended response). The data shows some clear trends.

(a) *Primary 3 versus Primary 4.* As would be expected, primary four students performed better than primary three on all the items. Students in primary four have more exposure to the basic concepts and applications associated with whole numbers (see Table 2).

Table 2 : Performance on whole number items *

Item	Primary 3 (%)	Primary 4 (%)	Grade 3 (%) (International)	Grade 4 (%) (International)
I3	43	62	46	57
I4	89	92	74	84
I9	90	91	50	71
J4	51	73	30	45
J9	79	84	64	73
K2	91	95	67	84
L7	41	61	33	49
M3	67	77	53	63
M6	71	80	57	70
M8	84	92	76	86
S2	86	92	44	64
T2	63	71	29	43
U5	86	90	63	77
V2	65	83	30	48
V3	74	82	48	62
V4a	25	29	16	24
V4b	50	69	31	48

* All figures show the percentage of students responding correctly.

(b) *Singapore versus the International cohort.* Except on one item at the primary three level (item I3) the Singapore students outperformed the International cohort. On the one item where this was not true the performance was roughly equal. Thus the performance of Singapore students is consistently better than the International cohort through the whole spectrum of whole number items (see Table 2).

(c) *Performance of boys versus girls.* While no formal analysis was done on the difference in performance between boys and girls in Singapore, the girls outperformed the boys on 15 of 16 items at primary three and 14 of 16 at primary four (see Table 3). The differences are not large, but the trend implies that at this level girls do consistently better than boys on whole number problems.

Table 3 : Performance of Singapore Students (Whole Numbers) – Boys versus Girls *

Item	Primary 3**		Primary 4**	
	Girls	Boys	Girls	Boys
I3	50	44	67	56
I4	92	86	94	91
I9	92	87	93	90
J4	47	54	73	74
J9	81	78	87	82
K2	92	89	96	93
L7	42	40	60	61
M3	69	65	79	75
M6	73	70	81	79
M8	85	82	94	89
S2	87	84	92	91
T2	64	63	72	70
U5	59	53	56	47
V2	66	65	84	82
V3	76	72	82	81
V4a	26	25	33	29
V4b	51	50	72	67

* Number of boys and girls varies between items and levels. There were between 900-950 boys and 800–850 girls writing any individual item at a specific level.

** All figures show the percentage of students responding correctly.

(d) *Leaving answers blank.* In most cases Singapore students answered the questions. There were relatively few blank answers, but again the data shows that girls are less likely to leave an answer blank than boys.

Above is a brief summary of the whole number data. However, while Singapore students did well, this does not mean that all students did well on all items. The next section of the paper will discuss some of the specific items, with a particular focus on items where students did not perform extremely well. It should be noted that there is no attempt to provide a detailed analysis of each item. Rather the focus is on general groups of items that might indicate a potential difficulty faced by students that could have implications for teaching.

Results on some specific whole number items

The following items have a very high level of correct responses, in the region of 90%. These items were usually classified by TIMSS as performing routine procedures.

I4. What is 3 times 23?

A. 323
B. 233
C. 69
D. 26

I9.
$$\begin{array}{r} 6000 \\ -\ 2369 \\ \hline \end{array}$$

A. 4369
B. 3742
C. 3931
D. 3531

M8. What is the largest number?

A. 2735
B. 2537
C. 2573
D. 2753

S2. Here is a number sentence

$2000 + \square + 30 + 9 = 2739$

What number goes in the □ to make this sentence true?

Of more interest are items where the level of performance was lower. Three such items are given below.

J4. 25 X 18 is more than 24 x 18. How much more?

A. 1
B. 18
C. 24
D. 25

L7. In which pair of numbers is the second number 100 more than the first?

A. 199 and 209
B. 4236 and 4246
C. 9635 and 9735
D. 51 863 and 52 863

T2. What is the smallest whole number that you can make using the digits 4, 3, 9 and 1? Use each digit only once?

Correct responses on these three items varied from a low of 41 % by primary three students on item L7 to a high of 73 % on item J4 by primary four students. These items were classified by TIMSS as using complex procedures or solving problems.

A careful inspection of these items seems to indicate that students need to have an understanding of place value to find the correct answers. They are not like M8 and S2, which also involve place value but are of a very routine nature. Since the responses to the items were in a written form and there was no interviewing, the implication that the difficulty is place value is only speculative, but appears reasonable in light of the data.

The only item where the performance was below 30 % was on part a of item V4 i.e. (V4a).

V4. In a game, Mysong and Naoki are making addition problems. They each have four cards like these.

2	3	4

The winner of the game is the person who can make the problem with the largest answer.

Mysong placed the cards like this.

$$\begin{array}{r} 4\ 3 \\ +\ 2\ 1 \\ \hline \end{array}$$

Naoki placed the cards like this.

$$\begin{array}{r} 3\ 1 \\ +\ 2\ 4 \\ \hline \end{array}$$

Who won this game? _____

How do you know? _____

_____

The level of correct response reported was 25% for primary three and 29% for primary four. However, if we include answers in which Mysong was indicated to be the correct answer, but where, as far as can be ascertained from the reports, unacceptable explanations were given, these percentages rise to 88% and 92% respectively. It seems reasonable to conclude that the difficulty might be language based.

What are the implications of these results for teaching?

Implications of the findings for teaching whole numbers

As the results show Singapore students did well. They can perform routine operations better than their International counterparts. The finding that students have difficulty with items that seem to require an understanding of place value means that there needs to be a shift towards a greater emphasis on the concept of place value, per se, and away from drill and practice in the classroom. Such a change is consistent with the present goals of mathematics education, which places less emphasis on rote learning and more on relational understanding (Skemp, 1976).

An activity in which students are encouraged to model a number such as 54 in as many different ways as possible using number blocks and then discuss the alternative representations will stress place value. Later this can be replaced with larger numbers using either physical or pictorial models. This is illustrated in Figure 1.

Figure 1: Making 54 using blocks

The students are presented with number blocks and are asked to make 54 in as many ways as possible. Some examples are given below.

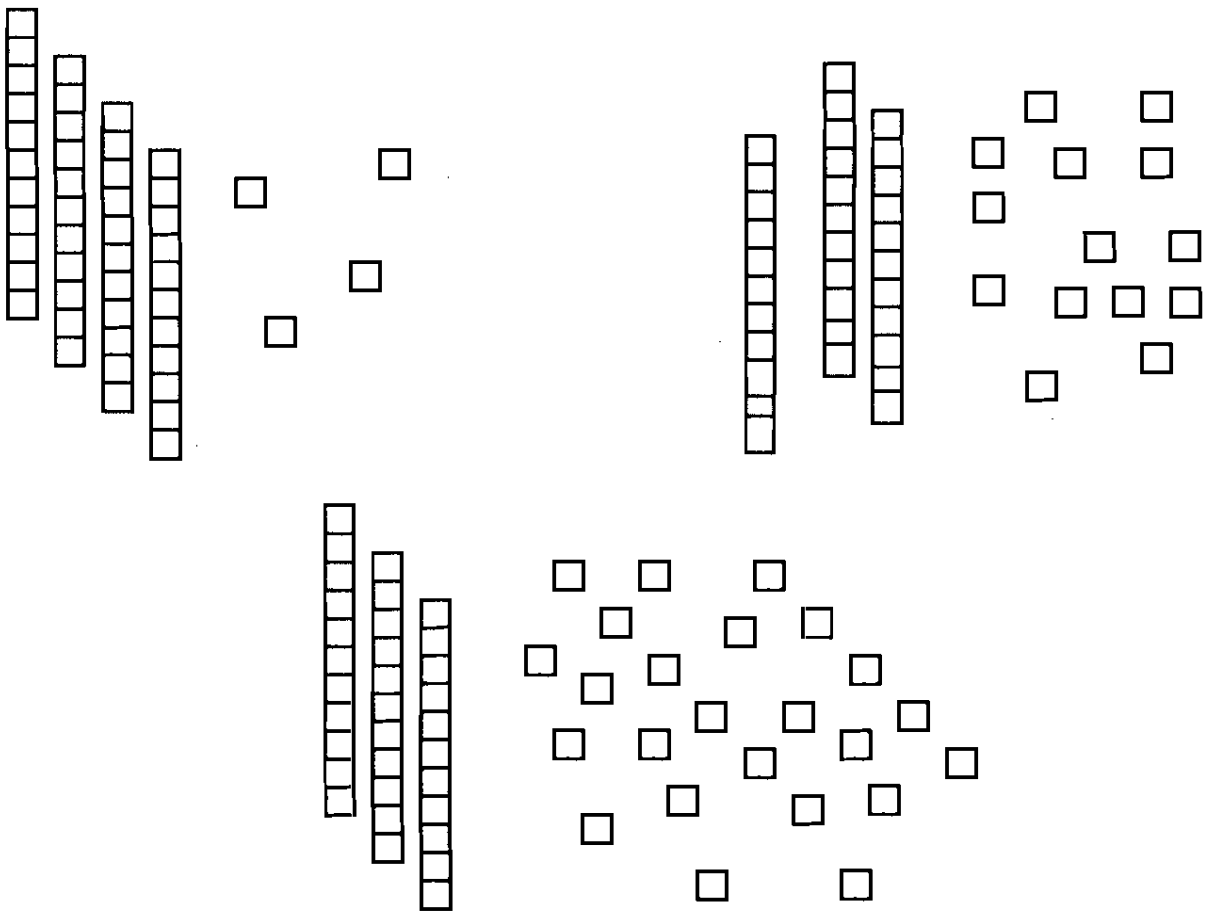
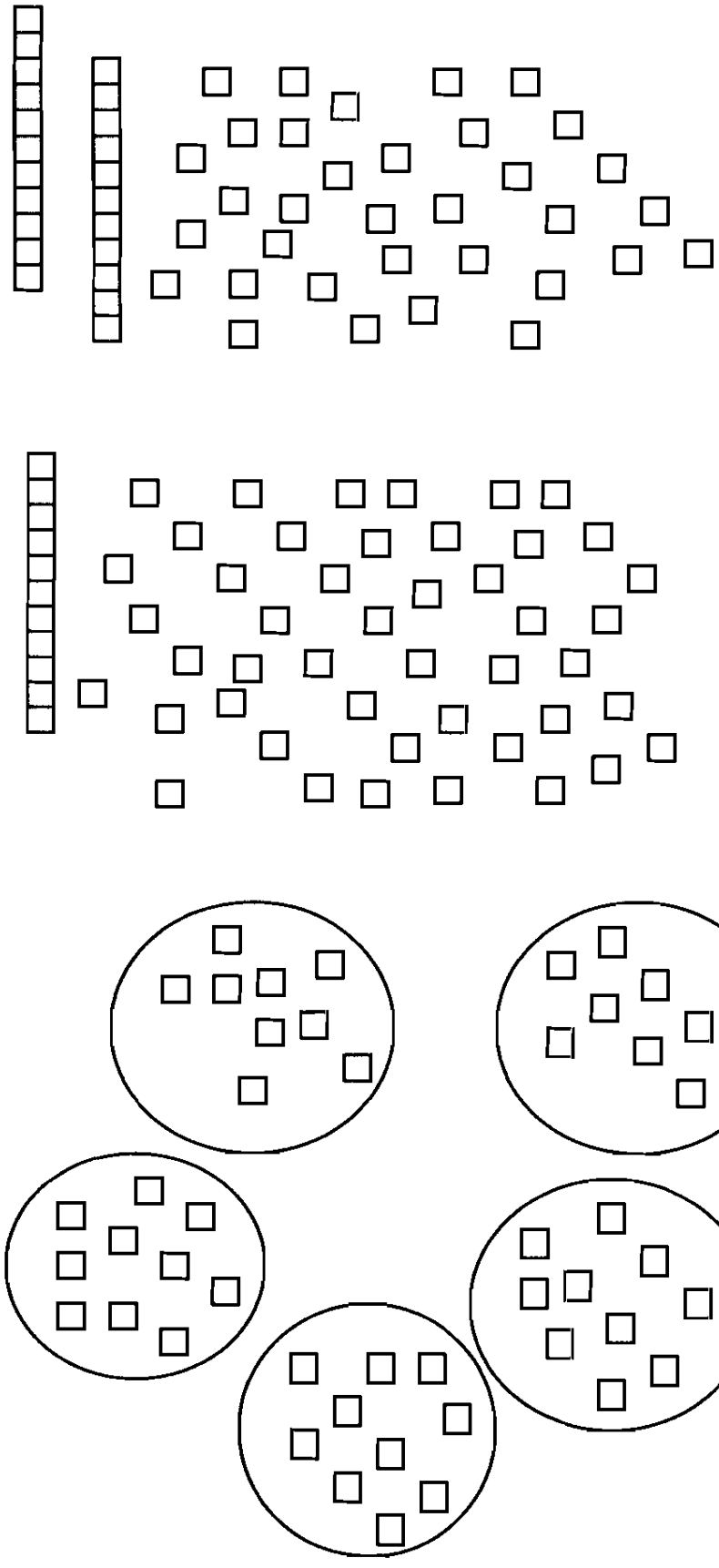


Figure 1 (… cont'd)



The fractions and proportionality data

There were a total of 12 released items on fractions and proportionality (6 multiple choice, 2 short answer and 4 extended response). As with the whole number data, there are some clear trends.

(a) *Primary 3 versus Primary 4.* As would be expected, primary four students performed better than primary three on all the items. On two items (I2 and M5) the performance of primary three is way below that for primary four. An analysis of both items shows they involve decimal representation of a fraction, which is not taught in primary three (Curriculum Planning Division, 1995) and, consequently, the low level of performance in primary three might be expected (see Table 4).

Table 4 : Performance on fractions and proportionality items *

Item	Primary 3 (%)	Primary 4 (%)	Grade 3 (%) (International)	Grade 4 (%) (International)
I2	8	90	21	39
I5	49	60	42	53
I8	57	71	46	54
J7	91	94	42	61
K9	55	74	27	37
M5	23	80	33	40
S4	41	62	30	46
U2	62	76	41	57
S3	44	60	12	26
T4a	25	41	10	21
T4b	21	35	6	15
U3a	56	73	44	61
U3b	46	68	28	45
U3c	77	85	61	73
V1	46	58	13	26

* All figures show the percentage of students responding correctly.

(b) *Singapore versus the International cohort.* On most items the Singapore students outperformed the International cohort (see Table 3). The only items where the performance of the Singapore students' performance was lower than that of the International cohort were I2 and M5 at primary three. As was indicated in (a) above this topic is not part of Singapore's primary three mathematics curriculum.

It is worth noting that on U3, which involves rate, a topic that is not formally taught in Singapore at primary three or primary four, that Singapore students did relatively well and better than their International counterparts. However, unlike translating decimals to fractions, this is a topic that students are likely to have some familiarity with from daily life.

U3. Maria and her sister Louisa leave home at the same time and ride their bicycles to school 9 kilometers away,

Maria rides at the rate of 1 kilometer in 10 minutes. How long will it take her to get to school?

Answer: _____ minutes

Louisa rides at a rate of 2 kilometer in 3 minutes. How long will it take her to get to school?

Answer: _____ minutes

Who arrives at school first?

Answer: _____

It seems reasonable to conclude that except on items that are not part of the mathematics curriculum and where students are unlikely to have had out of school experience, the performance of Singapore students versus the International cohort is consistently better through the whole spectrum of fractions and proportionality items.

(c) *Performance of boys versus girls.* The comparison of the performance of boys and girls produced different results from those obtained for whole

numbers. While the girls still outperformed the boys on approximately two-thirds of the items at primary four, the boys outperformed the girls on approximately one-half of the items at primary three (see Table 5). As was the case with whole numbers the differences are not large, but the trend is different from that with whole numbers in that while girls do consistently better at primary four this is not true for primary three.

Table 5 : Performance of Singapore Students (Fractions and Proportionality) – Boys versus Girls *

Item	Primary 3**		Primary 4**	
	Girls	Boys	Girls	Boys
I2	7	9	92	88
I5	44	53	59	60
I8	55	59	74	67
J7	93	87	95	93
K9	55	55	74	74
M5	20	25	84	77
S4	39	44	63	60
U2	63	61	74	77
S3	47	42	63	58
T4a	25	24	41	40
T4b	23	20	36	35
U3a	51	60	71	76
U3b	44	48	65	70
U3c	80	74	76	78
V1	48	44	62	54

* Number of boys and girls varies between items and levels. There were between 900-950 boys and 800 – 850 girls writing any individual item at a specific level.

** All figures show the percentage of students responding correctly.

(d) *Leaving answers blank.* As with whole numbers there were relatively few blank answers, but again the data shows that girls are less likely to leave an answer blank than boys.

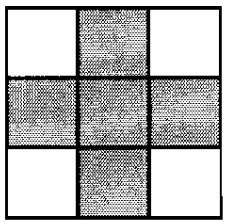
Again the overall data shows that Singapore students did well relative to the International cohort, but the general level of performance was lower than that

for whole numbers (see Tables 2 and 4). The authors will now focus on some specific items.

Results on some specific fractions and proportionality items

An item such as J7 has a high level of success. Similarly, once the concept has been taught, students perform well on an item such as I2.

J7. Part of the figure is shaded.



What fraction of the figure is shaded?

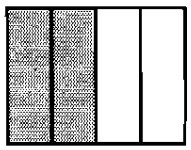
A. $\frac{5}{4}$ B. $\frac{4}{5}$ C. $\frac{6}{9}$ D. $\frac{5}{9}$

I2. 0.4 is the same as

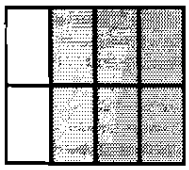
A. four
B. four tenths
C. four hundredths
D. one-fourth

Of more interest are items where the level of performance was lower. Two such pictorial items follow.

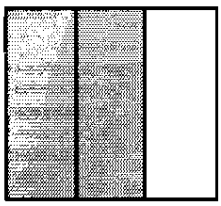
I8. Each figure represents a fraction



1



2



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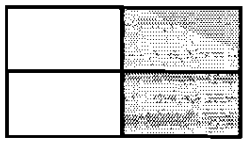


4

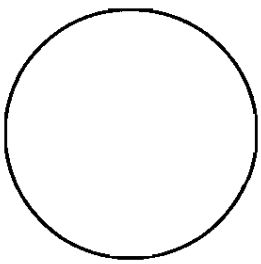
Which two figures represent the same fraction?

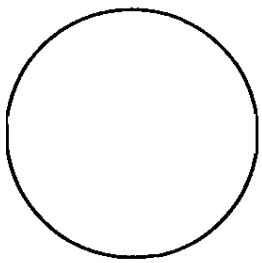
A. 1 and 2 B. 1 and 4 C. 2 and 3 D. 3 and 4

V1. Sam said that $\frac{1}{3}$ of a pie is less than $\frac{1}{4}$ of the same pie.

Is Sam correct? _____

Use the circles below to sow why this is so.





Shade in $\frac{1}{3}$ of this circle Shade in $\frac{1}{4}$ of this circle

Correct responses on these items varied from a low of 46% by primary three students on V1 to a high of 71% by primary four students on I8. These items were classified by TIMSS as using complex procedures or solving problems. A careful inspection of these items seems to indicate that students need to have an understanding of the concept of a fraction to find the correct answers. They are not like items J7 and I2, which are both routine in nature. It seems reasonable to conclude that a significant proportion of the students has difficulty with the concept of a fraction. This view would be reinforced by the students' performance on item T4.

T4. There are 10 girls and 20 boys in Juanita's class. Juanita sais that there is one girl for every two boys. Her friend Amanda said that this means $\frac{1}{2}$ of all the students in the class are girls.

How many students are there in Juanita's class.

Answer: _____

Is Juanita right? Answer: _____

Use words or pictures to explain why.

Is Amanda right? Answer: _____

Use words or pictures to explain why.

In this item the students had to explain their answers. Even in situations where the correct answer was given but either there was no explanation or an unsatisfactory explanation was included the success rate is still low. The fact that students' performance does not increase when they are asked to give explanations might be expected since students are not used to writing explanations during mathematics lessons.

Finally, the following are some comments on the students' performance on word problems. The following are two two-step word problems:

I5. Mario used 5 tomatoes to make half a liter of tomato sauce. How much sauce can he make from 15 tomatoes?

A. A liter and one half
B. Two liters
C. Two liters and one half
D. Three liters

K9. There are 54 marbles, and they are put into 6 bags, so the same number of marbles is in each bag. How many marbles would 2 bags contain?

A. 108 marbles
B. 18 marbles
C. 15 marbles
D. 12 marbles
E. 9 marbles

The most common incorrect response on item I5 was D (3 liters), in which it appears that students just divided 15 by 5. For item K9 the most common incorrect response was A (108), in which it appears that students just multiplied 54 by 2 and the second most common incorrect response was 9, in which it appears that students just divided 54 by 6. It appears that they simply manipulated the numbers in a single step.

It seems reasonable to conclude that these errors might be related to the structure of two-step word problems, rather than a specific artifact of the topic of fractions and proportionality. Two-step word problems are not introduced until

primary three and are difficult for many students. Since there is little additional work done with the particular concepts of fractions and proportionality in primary four as it relates to these problems, the additional experience the students have in primary four with two-step word problems might reasonably account for the improved performance. Effectively, the authors are arguing that the relatively low performance on these items does not indicate a difficulty with fractions or proportion, per se, but rather with two-step word problems.

What are the implications of these results for teaching?

Implications of the findings for teaching fractions and proportionality

The students have a basic idea of fractions, but for **many** students it is limited. They seem to have difficulties with items that require a deeper understanding of the concept. It appears that students need broader experiences with fractions in order to develop a deeper understanding of the concept. Activities in which they are asked to investigate, compare, describe, etc. fractions would help in overcoming this problem. For example, an activity in which students are asked to make halves, quarters, etc. in different ways and then describe the similarities and differences would help. Using a compare/contrast sheet as an organiser for thinking would aid in the implementation of this idea.

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