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Using Model-Eliciting Activities for Primary Mathematics Classrooms

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Abstract: The mathematical process *Application* and *Modelling* has just been factored into the new mathematics curriculum. Its implication is that even children should now be involved in works of mathematical modelling. Mathematical modelling is deliberated upon in this paper with a view towards supporting the new math curriculum. An example of pupils' development of a problem representation in this paper shows evidence of their mathematizing towards a real-world solution. The use of model-eliciting activities holds promise in surfacing children's mathematical thinking and problem-solving processes as well as in helping them move beyond primitive ways of thinking.

Key words: Mathematical modelling; Model-eliciting activities; New math curriculum

Introduction

One of the major revisions made to the Singapore Mathematics Curriculum Framework (SMCF) with effect from 2007 is the factoring of mathematical processes such as *mathematical reasoning, communication and making connections*, as well as *application and modelling* (MOE, 2007) into the teaching and learning of mathematics. These new inclusions while consistent with reformed efforts are by no means a small extension to the curriculum. To promote mathematical processes such as these will require changes to pedagogical approaches where traditional ones are increasingly deemed insufficient to prepare students for the dynamic world. Successful development of the said processes requires teachers to know what the processes entail and thus enact to actualize the intended curriculum. The intent of the new curriculum, however, might elude many primary mathematics teachers because they do not know what *application and modelling* are about.

This paper will discuss mathematical modelling and what the author sees as supporting the new curriculum. An example of pupils' development of their mathematical representation through the engagement of a model-eliciting task is also explained with respect to their mathematization processes. The implications and some anticipated challenges are discussed towards kick starting the use of model-eliciting activities in making mathematical thinking visible.

Coming to Grips with Mathematical Modelling for Children and the SMCF

Prior to the 2007 revision of the SMCF, "thinking skills" and "heuristics" were taught as mathematical processes. Problem-solving heuristics such as "draw a diagram", "act it out", "work backwards", "guess and check" are some of the 11 recommended strategies (MOE, 2001, p.11) that should be called upon to use when pupils are faced with non-routine structured word problems. The ability to solve such problems has been part of the formal assessment system as well. The focus to learning these curricular mathematics suggests a problem-solving paradigm that is narrow and detached from the real world. While some may think that helping pupils to solve multi-step word problems is helping them to relate to real life contexts, that notion has recently come under scrutiny. The word problems that pupils solve do not address adequately the mathematical knowledge, processes, representational fluency, and social skills that children need to develop for the 21st century (English, 2002) and hold no regards for realistic constraints as solving problems in real-world situations does (Verschaffel, De Corte, & Borghart, 1997). The approach to solving such translational problems has remained mechanistic and behaviouristic.

While the development of conceptual understanding and mastery of skills is important, and should rightly have a place in the mathematics curriculum, the need to weave these components so as to make better use of them is to provide a platform where they can be applied in real-world situations. *Application and modelling* are seen as playing vital roles in the development of mathematical understanding and competencies (MOE, 2007, p. 14). The SMCF describes mathematical modelling as the process of formulating and improving a mathematical model to represent and solve real-world problems which involves the use of a variety of representations of data, the selection and application of appropriate mathematical methods and tools in solving real-world problems. The injection of these processes is timely and relevant considering the need to make mathematics meaningful and problem solving authentic. The brief description of mathematical modelling provided in the revised curriculum, however, is not easy for teachers to understand, more so to adopt a pedagogy that helps children develop such processes. The term *mathematical modelling* might be the first time a majority of primary school teachers are encountering it. To some primary school teachers, it would vaguely bring back memories of the times they would have done some form of mathematical modelling in higher level math used for heat-transfer systems or statistical models in pre-university or undergraduate courses. Likely, most primary school teachers would associate it to the use of the model-drawing approach. Many teachers probably do not know what it entails. The above reasons should come as no surprise because mathematical modelling has very much been seen as reserved for secondary school years (Galbraith, Blum, Brooker, & Huntley, 1998) and above and that there is the

assumption that primary school children are not capable to develop their own models and sense-making systems for managing complex situations (English, 2006).

In our local curriculum, mathematical modelling has been subsumed under the topic Differential Equations for H3 Mathematics in Pre-University Mathematics. Now the new curriculum states that *application and mathematical modelling* "should be part of the learning at all levels" (MOE, 2007, p. 14), primary school children included. What could be the reason for having mathematical modelling as a process component for these younger learners? One could say that the changing societal landscape is witnessing more of the need to be able to manage complex systems of information (English, 2004) and therefore the primary school environment is deemed to be where children should begin to acquire a meaningful development of these processes and skills (Jones, Langrall, Thornton, & Nisbet, 2002). Taking a leaf from BlomhØj (2004), his views are based on pragmatic and long term societal interests; modelling experiences are viewed to bridge the gap between students' real-life experiences and mathematics, providing the means to describe and understand real-life situations, and motivating them to learn mathematics. He also sees it as a means to develop competencies for setting up, analyzing and criticizing models that are important in highly technological societies, and for making decisions that is imperative for the maintaining and further development of democracy.

Mathematical Modelling, Models and Problem Solving

What then is mathematical modelling seen in the light of the new curriculum with respect to how children are to be engaged? Preschoolers who play pretend by using a banana as a telephone or using counters for direct modelling to solve early number problems involving grouping and separation are said to display representational mapping of an entity to represent things of the real world; they are showing signs of modelling (Lehrer & Schauble, 2003). Likewise, the model-drawing method popularly used in our local primary schools to solve word problems is an example of direct modelling. From a model and modelling perspective, modelling that entails direct mapping between the structure of the problem situation and the structure of a symbolic expression usually has one way of interpreting the problem and they border on the traditional understanding of mathematical modelling (English, 2003). As well, Lehrer and Schauble (2003) consider traditional direct modelling as private representations and they fall short of the forms of representations and inscriptions that are constructed or adopted as conventions within a community to support interdisciplinary practices of communication, mobility, combination, selection, emphasis and the like.

The mathematical modelling of today is the bridging of mathematics as a way of making sense of the physical and social world, and mathematics as a set of abstract formal structures (Greer, 1997). Authentic contexts are the platforms for sense-making where children can situate their reasoning in coming up with real world solutions. In mathematical modelling, the starting point is a real-world problem or situation, and it is the process of representing such problems in mathematical terms in an attempt to find solutions to the problems (Ang, 2001). It is characterized by flows of events between problems in the real world and mathematical world with the need for model interpretation, formulation, and refinement towards real-world solutions (Ang, 2006). Mathematical modelling in the current context is thus is seen as a shift in focus from finding a solution to a particular problem to creating a system of relationships that is generalizable and reusable. In comparison with traditional direct modelling which seeks to find a definite answer, today's notion of mathematical modelling is seen as the process of edging towards a solution where sometimes an exact answer does not exist or is even beyond reach (Ang, 2001).

To engage children in mathematical modelling, a class of problems that depict realistically complex situations known as model-eliciting activities can be used to confront children with the need to develop models through expressing, testing, and refining their mathematical thinking. The models that children produce, according to Doerr and English (2003) are defined as "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar systems" (p. 112). They can be seen as external representations (e.g., a graph, a table) while also taking on the form of internal conceptual representations during the modelling process. These representational systems can be seen as overlapping, interdependent and interacting (Lesh & Doerr, 2003). An illustration of the modelling process is featured in Figure 1.

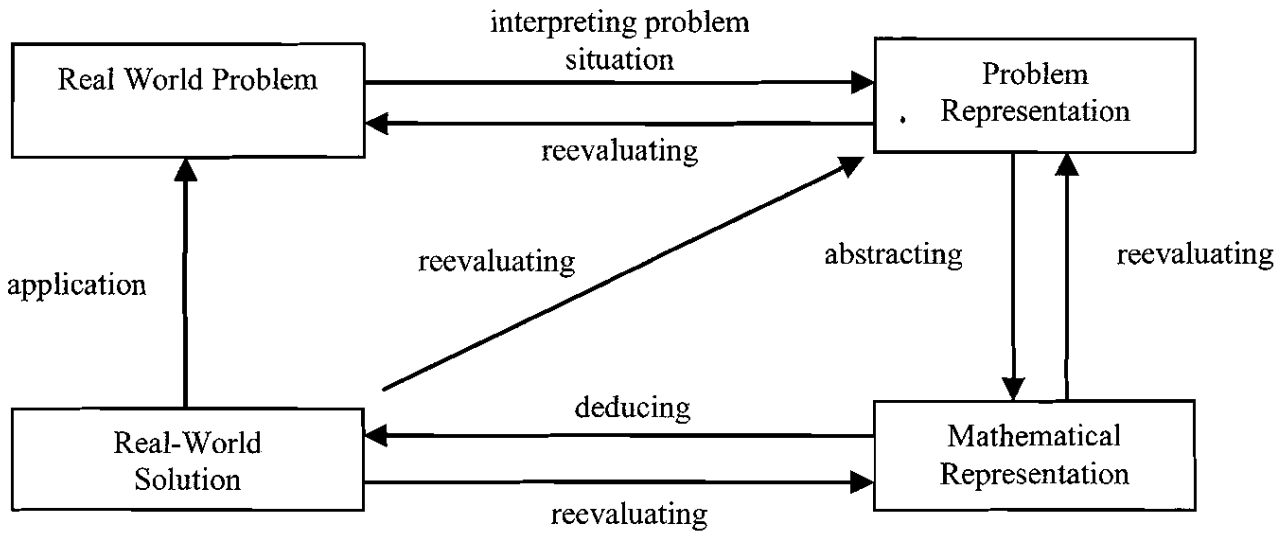


Figure 1. A general mathematical modelling process

In this paper, the modelling process is based on the assumption that the model-eliciting activity is carried out by small groups of problem solvers. A near real-world or case-based problem is first presented to pupils to make sense of it. As they interpret the problem to understand it, they discuss what the problem situation means to them through posing questions, clarifying the problem context and goals as well as relating to their prior knowledge. The discussion should lead them to identify variables and their relationships to work on and thus build a representation of the problem that they conceive. The pupils will engage in a process of developing their models and this can be seen as the interplay of their exercise of domain-specific knowledge, mathematical reasoning, metacognitive thinking and problem-solving skills towards creating a meaningful mathematical representation. Throughout, the process is not linear. The pupils' ways of thinking about the givens in the information are tested and revised iteratively (Lesh & Harel, 2003). They evaluate their thinking within components and reevaluate between components. The mathematical work in these iterative cycles is termed as *mathematization*. Doerr and English (2003) describe the mathematization process as including multiple cycles of interpretation, descriptions, conjectures, explanations and justifications that are iteratively redefined and reconstructed as learners interact with others. Mathematization can also be seen as a form of modelling involving specialized languages, symbols, graphs, pictures, materials or other notation systems to make representations (Lesh & Doer, 2000). Through the mathematizing, pupils deduce a solution after they are satisfied that their model can be adapted whenever similar problem conditions are imposed. The model or the external representation can be in the form of a table, graph, an equation (notation system) or an artifact to justify their representations.

The modelling process can also be likened to a problem-solving activity. Traditional problem solving holds the view that a certain heuristic approach is adequate to bridge the givens and the goals. In comparison, the modelling approach can be regarded as a content-independent problem-solving process where standard applied heuristics might not work because of the messiness of the data in real-life problem situations. In model-eliciting activities, the essence of problem solving becomes finding ways to mathematically interpret meaningful situations through multiple modelling cycles of getting from givens to goals (Lesh & Doerr, 2000). The iteration of trial procedures between givens and goals in order to succeed a solution would see the problem solvers move from givens to goals to test their hypotheses, refine their results and improve their solutions (Lesh & Doerr, 2003).

Reasoning, Communication and Connections

What about *reasoning, communication and connections* now made explicit in the mathematical process of the SMCF? If there is a means to enable a high engagement of mathematical reasoning, communicating mathematical ideas, and making connections, then the promotion of model-eliciting activities would require pupils to do just that as part of mathematization in the mathematical modelling process. In real-world situations, problem solving is seldom limited as a mission for an individual to work on because of issues, ambiguities and the lack of complete information. Working collaboratively towards solving cognitively demanding problems requires pupils to analyze mathematical situations and construct logical arguments, use mathematical language to express mathematical ideas and arguments, and make linkages between mathematical ideas. Engaging pupils in model-eliciting activities could enact this aspect of the SMCF.

Rationale for and the Roles of Mathematical Modelling in the New Curriculum

English (2006) provided six arguments as the rationale to include modelling in any reformed school curriculum. They make a fitting conclusion to the above discussion on making sense of mathematical modelling in the new curriculum. The author is of the opinion that these six arguments can also be seen in the light of the roles that mathematical modelling play in the mathematics classroom:

1. The benefit of using authentic problem situations to provide not only real world contexts but deriving real world solutions.
2. The opportunity for model exploration and application where pupils can build, consolidate and refine their conceptual systems.
3. The opportunity for multiple interpretations and approaches which enables others to scrutinize, to interpret, and re-interpret the problem information.
4. The opportunity for social development especially when pupils have a shared responsibility to problem resolution.
5. It calls for multifaceted end products whereby pupils can adopt various modes of representations suggesting representational fluency.
6. The opportunity for optimal mathematical development where they engage in important mathematical processes.

An Example of a Model-Eliciting Activity

The inadequacy of problem solving in traditional problem-solving context has been highlighted earlier. There is a need to relook at mathematical problem solving in the

light of helping children transfer their domain-related mathematical knowledge and problem-solving skills to out-of-school situations (Hiebert er al, 1996). Instructional designs of problems therefore should accommodate for more than just calculations. They should include, as in modelling activities, observations of patterns, testing of conjectures, and estimation of results (Schoenfeld, 1992). The role of the context in mathematical modelling should frame the problem to develop the mathematics such that the realistic contexts can enable concept formation, facilitate model representation, and provide a wider range of utility (de Lange, 1987). In considering instructional design, mathematical modes of cognition such as those mooted by the National Research Council (NRC, 1989) for reformed mathematics classroom instruction, namely, *modelling, optimization, symbolism, inference, logical analysis,* and *abstraction*, should be embraced. The author believes that the development of economic models is appropriate to advance the said modes of cognition. Economic models deal with economic concepts like profit, cost, price, maximization, equilibrium, recognition of variables, product of linear relations, and relations between variables that would allow students to develop a mathematical and economic model approaching the concepts mentioned (Aliprantis & Carmona, 2003). By engaging in such model-eliciting activities, pupils are also learning about economic literacy, a concept covered in Social Studies, thus broadening the scope of disciplines towards greater meaningfulness in problem solving. In this section, the author shares an example of an economic model-eliciting activity that he has developed to underscore the mathematization involved.

The Hiring Problem

The Hiring Problem is shown in Figures 2a and 2b. Data about cleaning, painting and moving services are provided for pupils who are on a mission to hire workers to renovate the school within the constraints spelt out. In developing the model, the pupils need to consider and compare productivity indices that will suggest making informed hiring decisions that are value-for-money.

The work of five Primary 6 pupils from a neighbourhood school who were involved in this type of model-eliciting activity is described in this section. The author has considered the instructional approach as a problem-based learning (PBL) approach because the model-eliciting tasks in many ways befit the characteristics of the type of tasks suitable for use in a PBL context. The other essential features of a PBL setting include the presence of the teacher-facilitators to provide scaffolding and the pupils working in small collaborative groups. To highlight the mathematization and mathematical thinking that surfaced during the model-eliciting activity, significant features of the evolution of the pupils' interpretations and representations are described in the vignettes.

The Hiring Problem

MISSION

Your group is in charge of hiring some workers to help clean, paint, and move furniture in the school. These workers must complete the job within 4 days.

CONDITIONS

1. **You can hire only from one company once, and you have to accept all the number of workers for that company.**
2. **You have a Worksite Supervisor who can only supervise at most 12 workers per day, so you try to hire as many as 12 per day. Assume that each worker to be hired works the same amount of time, and produces the same amount of work per hour.**
3. **You need at least: 14 workers for moving furniture, 14 workers for painting, and 14 workers for cleaning within the 4 days.**

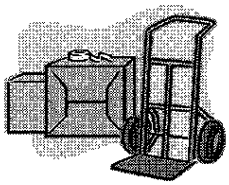
PRESENTATION

You have to present your case to your class. Show in full detail (with different solution options) how you arrive at hiring the workers. Show your productivity index and use it to make your decision.

Figure 2a. The Hiring Problem

CLEANING SERVICES					
Company	A	B	S	D	E
No. of workers	4	2	6	3	5
Cost ($)	160	76	270	120	175

PAINTING SERVICES (Paints Provided)					
Company	F	G	H	R	J
No. of workers	3	6	7	4	5
Cost ($)	114	240	315	160	210



MOVING SERVICES					
Company	K	L	Q	N	T
No. of workers	7	4	3	6	4
Cost ($)	245	160	135	228	140

Productivity index is calculated at the end as follows:

(Total No. of workers ÷ Total cost) x 100 {give index to 3 decimal places}

Productivity index is for you to determine if you are getting value for money. The larger the index, the better.

Figure 2b. The Hiring Problem Data Sheet

An Interpretation of the Pupils' Initial Development of Problem Representation

The engagement in the model-eliciting activity commenced right after the teacher had briefly introduced the pupils to the task. The early period of problem solving saw the pupils try to make sense of the variables as they deliberated on the task. They were focused on their goals and they were trying to develop a conceptual representation by considering the maximum number of workers as the first cut.

[1] S4: We have to find the perfect combination.

[2] S5: Also need to find the productivity index.

[3] S1: You only get as much as you pay.

[4] S3: Not necessary. It depends on costs and the number of workers. So our group is in charge of some workers to clean, paint and move furniture in the school. But they must complete the job within 4 days. Within 4 days (reiterating the condition)

[5] S5: Within 4 days. That means 12, 4. (referring to 12 workers and 4 days), Minimum 4.

[6] S2: 48. Can.

[7] S3: That means we do 12, 12, 12, 6 (sets a benchmark of 42 because of 14 workers x 3)

[8] S3: So what we have to do is ...(interrupted by S5)

[9] S5: Must calculate the cost for each company.

[10] S3: So you have to get a combination. Do cleaning, everything ...can we write it here? You know something, we can make them the same.

[11] S4: This is the cheapest (as he examines the task sheet)

[12] S3: How do you know? Have you calculated? This is two only (referring to the two workers in company B)

[13] S4: Yeah, two only.

[14] S2: That's why it's the cheapest!

[15] S5: Cost, that means for each worker right?

[16] S3: The cost, is it per worker or the whole thing?

Interpretation of the vignette: S4 and S5 (1 and 2) initiated the goals the group would like to achieve. Further interpretation of the problem was provided by S3 as he identified the variables *cost* and *number of workers* to have an effect on their goals (4). The initial interpretation of the problem task to frame combinations of workers within 4 days was quickly conjectured by S5 and S3 although both had different combinations for the fourth day (5 and 7). The difference in combinations was not discussed as yet. What followed then was the clarification of initiatives and problem representation. S3 initiated to make something "the same" (10) as in trying to have a common base to use for comparison purpose while S4 concluded that the cheapest workers came from company B (11). This prompted S3 to query how S4 arrived at the answer (12) but this was not pursued further although the reasoning was only partially true. Both S4 and S5 took it literally that the fewer the number of workers, the lesser the cost (13 and 14) without checking if the rates were the same for the other companies. Then S5 and S3 began to wonder what *cost* meant, whether the cost referred to the cost of each worker or the entire lot of workers from a company (15 and 16). It can be seen that at this initial stage, the ideas discussed was task-oriented with a focus to get on with the job and the discourse was characterized by interpreting the problem task, initiating ideas, and developing an initial conceptual model that showed glimpses of the emergence of a mathematical model. Monitoring one another's inputs was at a superficial level, lacking in deeper probing.

An Interpretation of the Pupils' Intermediate Development of Problem Representation

The group was more certain that they needed to come up with appropriate combinations that were value-for-money. The modelling process was not so straightforward. They continued to check with one another if what they were thinking mathematically fitted the conceptual representations they had in mind.

[41]	S5:	You know, for cleaning, E is the cheapest (after working out some calculations)
[42]	S1:	Then we choose E.
[43]	S5:	Second cheapest is B
[44]	S1:	Why don't choose all E?
[45]	S5:	No. We can only hire once, once from each company (referring S1 to the task sheet)

[46] S5: So third cheapest...these two are the same price (A and D), so these two are the third cheapest. Then we got to do these (referring to the painting and moving services)

(S3 does the recording)

[47] S2: How many workers do we need per day?

[48] S3: 14, my goodness. (shows S2 the task sheet)

[49] S5: But can only hire up to 12 per day (explaining to S2)

[50] S2: But each company doesn't have 12?

[51] S3: So you have to combine.

[52] S2: One day only can hire 12? So how to get?

[53] S3: That's the problem.

[54] S1: 6, 4, 2 (Proposing number of workers for Cleaning)

[55] S5: But it looks expensive.

[56] S1: But no choice because you want to maximize for the whole day.

[57] S4: Can be 5, 3, 4 what?

[58] S5: Wait... (punching on the calculator). 445 (from 160 + 120 + 175, A, D and E, and then works on the other combination). Hey, you're right, this one is cheaper (the 445).

Interpretation of the vignette: At this stage, the group was trying to rank the companies in terms of offering the cheapest workers with the intention of selecting the cheaper ones (41 to 43, and 46). It could be seen that enquiries were made to clarify the situation and keep each other in check. For example, S1 proposed that if company E offered the best rate, then one should keep hiring from E (44). That was a logical proposal which however did not meet the conditions posed and S1 was made aware of the condition by S5 (45). Then S2 and S3 were confused over hiring the number of workers per day and the number of workers needed per service (47, 48, 50, 52 and 53). Indeed decentering of data can make it difficult for pupils to conceptualize but it helps when the more knowledgeable peers are able to offer explanation (51) and observable ideas (54 to 58). S1 and S4 were able identify two combinations to make 12 workers per day and then S5 made an informed decision after comparing their rates (58). At this juncture as seen in moves 54 to 58, the group had not thought about combinations across service for each day but

combinations within service for each day. Consequently, these ideas began to evolve and become firmer mathematical representations as the pupils gained confidence of the mathematical properties associated with the situational conditions specified.

This next vignette shows how the pupils shifted their thinking from considering making combinations within services to combinations across services.

[73] S5: Yes. E is the cheapest. E followed by B, D and A, and then S.

[74] S1: Maybe we need to have like Monday all these, Tuesday also all these, and Wednesday all these (finger sweeping across the task sheet to refer to the cleaning, painting, and moving services)

[75] S5: The thing is that we need 14, we need a good combination.

[76] S4: What good combination?

[77] S5: OK, right combination.

[78] S2: Without changing task? (noticing that S1 refers to combination across different services)

[79] S1: You have to have all the combinations (using his pen to indicate across services from the task sheet).

[80] S3: Because you can't have all the cleaning on one day, and all the painting on another day, right, as at most you can only have 12? Unless, we have this condition. I can make this up as one condition: Cleaning, painting, moving, one day (as seen from folding of fingers as in counting to show each service for each day), and then the last day a combination of all three.

Interpretation of the vignette: The pupils had ranked the companies based on the rates of workers as confirmed by S5 (73). S1 then conjectured that they needed to consider combinations across services per day instead of within services (74 and 79). This move was supported by S3 who at the same time proposed they could still have combinations within services for each day but with the condition that on the last day they consider combinations across services (80). This vignette shows that the pupils have progressed towards a more sophisticated level of thinking. They were able to decenter by having multiple perspectives as they construct different versions of representing their information and were still focused on their goals.

An Interpretation of the Pupils' Concluding Mathematical Representation

As a means to help young learners understand something about optimization, this model-eliciting task comes with a formula for calculating the productivity index (PI) to enable the pupils to compare if their selections of workers are value-for-money. The PI is to be determined after they are satisfied that they have obtained a plausible combination. This vignette shows the pupils' interpretation of the PI. The presence of the teacher at this point was to elicit the pupils' understanding of the PI.

[240]	T:	Maybe we should take a look at this productivity index. You see the numerator and the denominator. What does that tell you? (pointing to the formula in the task sheet)
[241]	S3:	That the smaller the denominator, the larger the number will be. You see, the total cost has to be as low as possible.
[242]	T:	So what about the numerator?
[243]	S3:	Numerator has to be the same throughout. It has to be 42 because it's 14, 14, 14.
[244]	T:	Are you sure?
[245]	S5:	At least 14.
[246]	S3:	Yes, we try to keep it as low as possible.
[247]	T:	OK, if I want to get a high productivity index, what must I do to the numerator, and what must I do to the denominator?
[248]	S3:	Denominator must be as small as possible and numerator as large as possible.
[249]	T:	And numerator is the number of workers.
[250]	S3:	Which means we can exceed.
[251]	S5:	Then we must maximize it
[252]	S3:	We are allowed to exceed right?
[253]	T:	Based on the conditions?
[254]	S5:	Yes, they say "at least 14 workers" so we can exceed.
[255]	T:	Yes, but per day you can only have…

[256] S5: At most 12. So maximum can be 48, because 4 days.

[257] S3: So that means we can exceed by 6.

Interpretation of the vignette: The pupils had developed a mathematical model based on 42 workers across services per day for four days. Their model was a 12, 12, 12, 6 framework, meaning having 12 workers for the first 3 days and 6 workers for the last day. The 42 workers were based on the understanding of having 14 workers per service. The presence of the teacher at that juncture served to make the pupils' understanding of the PI visible. The teacher queried what the PI meant (240). S3 had the understanding that a high PI could be obtained by making the denominator as low as possible (241), which in a sense is true. However, he maintained that the numerator remained constant (243). This is likely because the pupils have fixed a ceiling to the number of workers for each service. The teacher then queried if the numerator could have an effect on the PI (247), to which S3 responded that the numerator should be made as large as possible. Quickly, the pupils' thinking shifted to make the ceiling 48 instead of 42 (256). This vignette shows that when pupils have developed a sensible model, it would not be too difficult for them make adjustments. Using the same model allowed them to eventually come up with 3 different combinations for comparison. They obtained 2.56, 2.57, and 2.63 as their PIs respectively and selected the best combination with the PI of 2.63 (in the actual transcript but not presented here). The pupils could have gone on to discuss the PI had the teacher not visited them. The probes made by the teacher revealed that the pupils had some ideas about the PI but they were not adequate. The vignette also shows how the scaffolding provided by the teacher enabled the pupils to think more critically in order to optimize.

The group of pupils observed in this activity had started well. As early as in Move (5), S5 had thought about having a (12, 12, 12, 12) framework as the first cut while S3 proposed a (12, 12, 12, 6) framework. However, S5's idea was not pursued and instead the development centered on S3's proposal. Nonetheless, the entire model-eliciting episode saw the richness of the mathematization that the pupils were engaged in. The pupils had to communicate with one another to make their conceptual representations visible. This allowed the others to make sense of one another's representations in relation to the task as well as their own. It had provided opportunities for dispute as well as convergence of thinking. The mathematical constructs that the pupils' representations personified had also evolved along the way as they considered the multiple dimensions in the makeup of the combinations. The later part of the activity saw greater stability in their model development after having clarified goals, terms, and strategies. Their mathematical model that was

emerging had placed them in an advantageous position to make modifications to their thinking without much difficulty when the need arose.

Implications of Mathematical Modelling through Model-Eliciting Activities

When pupils work on model-eliciting activities, they elicit mathematical ideas that are embedded in the model-eliciting tasks. They develop important mathematical processes that facilitate their development of generalizable conceptual systems. They have to draw on their domain-specific mathematical knowledge and transfer such knowledge in the light of real world context to interpret, analyze, explain, hypothesize, conjecture, compare, and justify to making their thinking known. When they organize their thinking, they are developing conceptual representations of the problem situation. In the process, they continue to modify, refine and extend their conceptions. These models are constructed in meaningful ways as they involve conceptual understanding and mathematization (Lesh & Doerr, 2003). The models are seen as powerful tools for solving real world problems.

By engaging pupils in model-eliciting activities, they are also seen to be involved in problem solving since they have to move from the givens towards achieving their goals. Because the model-eliciting tasks are challenging and authentic, it is consistent with the notion that a problem should indeed pose an obstacle whereby the pupils have no immediate solution. The model-eliciting activities therefore place high demands on the pupils cognitively and require them to exercise thinking that is beyond what they normally do in solving traditional word problems. As the problem solving involves iterative modelling cycles, it also places a high demand on the pupils' metacognitive capabilities. Thus it sees the need for working collaboratively with peers and the factoring of the teacher as a facilitator. Such a pedagogic approach embraces socio-cultural aspects that are deemed as means to enhance learning in complex situations, and deviates from the traditional teacher-centered practices.

The Challenges for Teachers in Promoting Mathematical Modelling

To promote mathematical modelling in the primary mathematics classroom can be quite a challenge because teachers need to make sense what those processes mean, how they "look like" and how and by what means they can be developed in our children. Embracing newer pedagogies would mean the need to unlearn, relearn and enhance what counts in making pupils' mathematical thinking visible and thereby shaping beliefs and values systems. During initial sessions, teachers might not feel comfortable dealing with such problems because of the complexity involved and the openness of the discussion or solutions that may emerge. To address this concern,

teachers would need to learn to adjust by being involved and be part of the problem solving community with the pupils. The teacher could spend short periods of dedicated time with the pupils and through the rapport, help elicit responses, give feedback, and extend their thinking. Over time, when the pupils are more immersed in such settings, the facilitation can fade. Of course, getting acquainted with the problem task first would make it easier for the teacher to ease into the scaffolding role.

For a start, the teacher can provide them with simpler types of model-eliciting tasks that do not involve many variables. For example, observing the change in water level when water is poured into containers and plotting the graphs of volume against time. Whenever possible, teachers can also seize the opportunity to talk about different ways to interpret certain data to promote qualitative reasoning, or using non-examples and misconceptions to trigger cognitive conflicts. The day-to-day questioning to develop young learners' mathematical ways of knowing, according to Lamon (2003), is essential to preparing pupils on mathematical modelling. Both the teachers and pupils need such experiences before they can internalize the benefits of solving problems in more complex situations and gradually come to terms with how it matters in the cognitive and social development of the pupils.

The other challenge is to be convinced that the younger learners can and have what it takes to be involved in mathematical modelling. Chan (2007) and Chan, Neo, and Ng (2007) carried out model-eliciting activities with Primary 6 pupils from different schools and they were convinced that pupils were able to express the problem representations mathematically even though at different levels of sophistication. Perhaps uppermost in the minds of teachers is the concern whether such activities would help the pupils in their high-stake examinations. To this, the author would point out that the rationale for having model-eliciting activities is more developmental than for excelling in examinations. Moreover, the problem types (model-based problems versus exam item type) are structurally and contextually different. Developmentally, mathematical modelling could serve the teachers well when pupils encounter problems when their misconceptions and reasoning flaws surface. This would then call for the pupils to revise their ideas or have their primitive conceptions reviewed (Lamon, 2003).

Also, engaging the pupils in model-eliciting activities requires the teachers to relinquish a fair amount of control in the classroom as pupils will raise difficult questions, attempt to make assumptions that may border on the ridiculous, lose concentration and become disengaged, start a personal conflict, and so on. This is where opportunities should be seized by teachers to facilitate the problem solving situation by keeping their cognitive engagement high. It is also an opportunity for

the group members to monitor group progress, challenge assumptions, and identify the errors and misconceptions of others which according to Vygotsky (1978) leads towards learning beyond the current level of development. The model-eliciting experiences will gradually enable pupils to "make sense of the situation so that they can mathematize it themselves in ways that are meaningful to them" (English, 2006, p. 305).

Conclusion

The revision to the Process component of the SMCF suggests a more communicative and constructive approach in developing mathematical thinking. Mathematics learning has to take into consideration real-world situations, and modeling activities are seen as the catalyst to promoting mathematical reasoning and making the learning meaningful.

While this aspect of the new curriculum may indeed be new to many a primary school teacher especially when notions of mathematical modelling tend to evoke relations to higher level math, mathematical modelling can be made relevant to primary school pupils. The example of the pupils' engagement of the Hiring Problem discussed earlier suggests that pupils were able to make meaning of situations through devising combination systems and making sense and use of a formula such as the productivity index and they were developing their mathematical thinking along the modelling process. By bringing model-eliciting activities into the classroom, it allows pupils to solve problems rooted in real-world settings. It provides pupils with opportunities to develop mathematical processes that traditional problem solving approaches would not. The implementation of the new curriculum is not without new challenges in terms of trying to actualize the curriculum. The challenges as highlighted are not insurmountable. Perhaps the most difficult thing is getting it started. Convictions coupled with perseverance in adopting model-eliciting activities would help towards enculturating both teachers and pupils in meaningful problem solving via model-eliciting activities.

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