Reflections from Teaching Secondary Mathematics Experience

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Initial Reflection

Learning occurs when an individual reflects on his or her experiences. As a classroom mathematics teacher in the 1990s, I found the experience utterly overwhelming. For many years, I had taught mathematics to secondary school pupils. I reflected on the lessons and jotted down any crucial incidents in my notebook. When a day went well, I left the classroom with my heart singing and cheerfully made an effort to remember about all that had happened. On the other hand, when a day was unpleasant, I left feeling lousy, discouraged, perplexed and miserable. Unpleasant days could happen for numerous reasons. At times, a lesson that I thought was well planned, interesting, challenging and proper resulted in confusion, boredom or even worse turmoil. In other occasions, the class was particularly rowdy or out of tune with each other for numerous reasons, including that one pupil was disturbed and everyone was affected. On those days, I mused on events happening in the classroom and wondered if the effort was worth it.

When I write about my thoughts and ideas, I am engaging in an internal dialogue as I try to express in my own words how the experience or ideas is applied to my own reality. Writing as a tool for reflecting helps me to organize my thoughts about issues related to the teaching of mathematics. Writing also creates an opportunity for me to look back at my thoughts and reflect on my growth. I tend to ask the following questions as I reflect:

• What have I done?
• What does this mean to me?
• How do I come to accept these opinions and practices? Are there any reasons?
• How can I do these things differently?
• Are there any theories supported my responses and strategies?
Reflections from my Classroom Experience.

Establishing the structure for classroom learning at the beginning of the school year has a real payoff.

I used the first week to assist pupils understand what it means to solve mathematical problems in my lesson. Actually, I spent three weeks assisting them learn and understand my expectations in several areas: (a) the procedures in completing their mathematics homework, including which exercise book to do their homework and when to hand in their homework and a system of filing their handouts; (b) focus on solving problems and explain their thinking in all situations; (c) the significance of solving non standard problems and perverse even if they had great difficulty; (d) how to work individually, with a partner and in small groups; (e) nominated a mathematics representative to perform any administrative duties such as purchasing materials that were required for the lessons.

Examining the scheme of work for planning and provide handouts for productive learning.

Each mathematics topic last from two to four weeks, I was able to teach 10 to 14 topics each year. Preparing 10 to 14 topics meant many big planning times for me. Each of the topic included whole-class lessons; a menus of independent activities, some for pair work and some for individuals, assessments and homework assignments.

Providing different challenges for mathematically competent pupils stretch them to their fullest potential.

When working on independent activities, high ability pupils can select what they want to explore and the amount of time they spend on each activity. I have always believed that it is crucial to provide choices in and control over their learning. A menu of independent activities allows me to do so. Also when pupils are working independently – individually or cooperatively – I am able to monitor and meet their needs that were related specifically to them and their particular work.

Homework can be a useful vehicle to inform parents about their child's mathematics learning.

With a set of assignments complied for the year, I sent home a "Mathematics Homework" note to inform the parents about the purpose of the assignments and suggested how they might participate. I made approximately one home assignment each week. Feedback from the parents was positive. During the yearly Parent-Teacher meeting, some parents expressed that the problems assigned were challenging and they enjoyed solving the problem together with their children.

It is possible to emphasis pupils'attention on basic facts and formulae. At the same time, maintain an emphasis on problem solving.

Knowing the basic facts and formulae is essential for secondary pupils as it allows them to be more flexible with their mathematics estimates and manipulations. However, I do not want to promote rote learning but rather incorporate appropriate activities which promote the development of conceptual understanding. It is pertinent to teach strategies to enable them to deal with abstract mathematical concepts and to see various possibilities and alternatives in tackling mathematical problems.

Modelling for pupils how to represent their thinking concretely and symbolically in a variety of ways assists pupils to learn how to communicate their thinking mathematically.

In the discussion of the problem “Find two numbers whose sum is 67 and whose difference is 3”, John offered to draw two bar diagrams with first longer bar representing the bigger number and shorter bar representing the smaller number. He further extended the shorter bar to make it the same length as the longer bar. He pointed out that longer bar
represented one part. Two parts will represent\[67 + 3 = 70\] and hence one longer bar will be 35 which was the answer for the bigger number.

Peter had a different idea. He said that he could make a guess of the two numbers such that the difference is 3 and then check if the sum of the two numbers is 67. To ensure that the difference between the two numbers is 3, Peter began with the number 0 and 3. In this case, the difference is 3 but the sum is not equal to 67. Peter made another guess of two numbers. Peter repeated the process for three times to get the answer. He called this method trial and error. I then said to the class that Peter's approach of solving the problem was termed as 'guess-and-check' which required it to guess a number and then to check whether the constraint was satisfied and John's approach was using diagrams and using bars to represent number which was a semi concrete representation. Although the standard strategy of solving this type problem is to transform the problem statements into two algebraic equations, pupils should also be encouraged to use varied strategies to solve problems and to seek alternative solutions to problems.

**Modelling effective use of worked examples**

**assisted pupils to recognise underlying similarities between problems**

When I first began to use worked examples as a teaching tool, many of my pupils had difficulty using them effectively. It was obvious that many perceived mathematics as merely solving a series of problems with little attention for conceptual understanding. Although I instructed the pupils to spend time examining each example until they understood it, pupils often ignored the examples that were given and went straight to the practice problems that were assigned, even when they did not understand how to tackle the problems. If they did not know how to solve a problem instantly, many would skip it or rely on me for guidance.

As one of my aims in using worked examples was to assist pupils to be more independent learners and relied less on me and more on themselves, I found it useful in the beginning of the year to model a strategy for using worked examples. To show such a strategy, a worked example was followed by a similar problem or two were placed on the overhead projector or
the whiteboard. I asked the class to (1) study the example for understanding in a step-by-step manner; (2) make some points about what was shown in the example, for instance, “Now I understand. A linear equation is formulated”; (3) go on to the practice problem when the example was understood, referring back to the example when necessary; and (4) check the final answer in the practice problem, relating it back to the example if necessary. Not all pupils would always use this approach but it provided some steps for the pupils to begin.

Probing for more ideas and further reasoning, learning to allow wait time for pupils to collect their thoughts and giving pupils many opportunities to verbalise their thinking are techniques that support learning.

When asked the pupils to think about one particular fact, like “(a + b)"", each time a pupil offered the result, I would ask if anyone had another idea. Pupils would think of interpreting the expression both algebraically and geometrically. I would provide some prompts if they were any difficulties along the way. Then I would wait. I made a deliberate effort to learn to talk less in the class and have the pupils talk more. The payoff had been great. Pupils soon learned that I would wait for their responses. When pupils gave an answer, I would be sure to ask them for an explanation and then follow with the same question: “Does anyone have another idea?”

Taking the time to encourage pupils to write in mathematics class enable them to construct new knowledge and to clarify their understanding.

I have long been convinced that writing could give pupils an opportunity to clarify their understanding of a concept or topic. I have seen evidence of how writing has pressed pupils to organized and clarify their thoughts. In addition, writing offers an excellent way to assist pupils to create new knowledge by reflecting on previously learned mathematics and linking it to the new information. Writing is a general life skill that all pupils must learn and I believe that writing ought to be integral to all of pupils’ learning including their mathematics learning.

Although I had learnt the basic of getting the pupils to write, I found some pupils reluctant to write during mathematics class. They have the belief that mathematics class involves only solving problems. It can be quite discouraging. I decided to create writing prompts to solicit their written response. Though their write up may be brief, it enables me to assess the level of understanding and identify any inaccuracies.

Some Parting Thoughts

No matter how skillful I am as a mathematics educator and how deeply I understand the process of learning, I know that the many variables in teaching make the job extraordinarily difficult. Becoming a better mathematics teacher is not a straightforward endeavour. Collaborating, reflecting and revising programmes can assist teachers to think smarter about teaching and minimize the negative effects of the tension inherent in becoming a better mathematics teacher. From the pupils’ perspective, I need to consider the impact I have on the pupils’ education. I must passionately believed in the innate and latent qualities of pupils, no matter how unpromising they may have seemed. Look at each pupil as an individual who is capable and willing to learn. Create an atmosphere where the pupils are encouraged to ask questions, not made them feel stupid. Be patient and remind myself that the pupils are not experts. If they were, they would not be in my mathematics class. Becoming a mathematics teacher is more than classroom teaching. In short, I would want to be a mathematics teacher who through my words and actions says, “I care”.

In this article, I have tried to make my beliefs accessible and useful to others by identifying what I experienced. I feel more analytical and less emotional but still deeply committed to growing as a mathematics educator and searching for ways to support and improve pupils' learning.

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THE CHANGES

The old
- Lists of the top 50 Express stream schools and top 40 normal stream schools were drawn up. For both lists, the schools where ranked according to how well their students performed in the O levels.
- To get this ranking, say, for the Express Stream schools, where students complete their secondary education in four years, the youngsters’ scores for their first language and best five subjects were added up and divided by the total number of students to derive the school’s average score, right down to the decimal point.
- The ranking introduced in 1992 lead to rumbles that it made schools even more examination-oriented and led to teachers, students and parents fretting over a school’s exact position on the list and whether it moves up or down from year to year.
- The School Excellence Model – under which schools can win awards for developing their students in non-academic areas – was introduced in 1999, but the awards were not included in the ranking tables.

The new
- The ranking tables have been renamed the School Achievement Tables this year. Schools are still ranked according to their students’ performance in the O levels, but instead of using the exact academic score, schools are banded according to groups with similar results.
- The new tables also display, just as prominently, how many awards the schools have won in value-added academic areas as well as non-academic fields, such as the arts and sports.
- The schools offering the integrated programmes are left out of the academic ranking list, as their students do not do the O levels and move straight on to the A levels or the equivalent.

The reason
- The new School Achievement Table send the strongest signal yet the academic excellence is no longer the sole measure of success. More importantly, they transform the closely watched annual list into a tool to develop all-round education.

The old
- Only independent schools and autonomous schools could use other criteria besides the PSLE results to admit Secondary 1 students. Independent schools could admit up to 10 per cent of their intake using these criteria and autonomous schools up to 5 per cent.
- Students admitted using such criteria had to have scores no lower than 10 points below the school’s PSLE cut-off.
- Students who were admitted based on other criteria were taken in only after most of the places have been given to those students accepted on their PSLE scores.

The new
- Independent schools can enrol up to 20 per cent of their Secondary 1 intake using discretionary criteria in 2006; autonomous schools can do so for up to 10 per cent of their intake.
- By 2006, mainstream secondary schools can admit up to 5 per cent of their Secondary 1 intake who do not meet their PSLE cut-off but can help the school develop their niche areas, subject to the Ministry of Education’s approval.
- Schools can take in youngsters using discretionary criteria before the PSLE results are released.

The reason
- To give schools more flexibility in admitting students and to recognise children with a range of abilities beyond examination results.
The old
- Students had to do the GCE O levels before going on to the A levels.

The new
- Students in the integrated programme offered in a handful of schools can skip the O levels. They can do a six-year programme starting at Secondary 1 or a four-year scheme starting from Secondary 3. Some schools offer both.

School offering six-year programmes:
- Anglo-Chinese School (Independent) and Dunman High, where students stay all six years in the same school
- Raffles Institution and Raffles Girls', whose students go to Raffles Junior College in the fifth year
- Chinese High and Nanyang Girls' High, whose students join Hwa Chong JC in the fifth year

Schools with four-year programmes:
- Anglo-Chinese School (Independent), Dunman High, Raffles Institution, Raffles Girls', Chinese High and Nanyang Girls' High
- Unlike at other schools, IP students at ACS (I) take the International Baccalaureate diploma which is equivalent to the A levels

The gifted programme:
- The Ministry of Education will still offer its four-year Gifted Education Programme (GEP), leading up to the O levels, in three schools: ACS (I), Dunman High and Victoria School. This programme is an extension of the primary school GEP, from Primary 4 to 6, which was started in 1984 and seeks to stretch the intellect and creativity of the intellectually gifted identified through aptitude tests.
- Integrated programme schools also have their own six-year gifted education programme, not run by the ministry.

The reasons
- Instead of slogging for the O levels, students can have time to develop their "intellectual curiosity" and try things outside the syllabus, like field trips, camps and attachment programmes to universities and companies.
- This allows them to experience a broad-based education, and equips them with the skills to deal with the solve real-world problems.
The Changes

The old
Normal (Technical) students:
- About 15 per cent (7,000) of all students enter Secondary 1 enter this stream.
- Students in this stream are prepared for a technical-vocational education with the Institute of Technical Education.
- Their core subjects are English, mother tongue, mathematics and computer applications. Compared to other streams, they take simpler examinations geared to test their fundamentals.
- At the end of their fourth year, they take the N-level examinations.
- If they do well enough in Secondary 1 or 4, they can transfer to Normal (Academic) stream to do Secondary 2 or 5 respectively.

Normal (Academic) students:
- The Normal (Academic) stream allows students to take O levels in five years. Compared to the Normal (Technical) stream, subjects are done at a higher level but at a slower pace than that of Express stream students.
- There are about 50,000 of these students in secondary schools this year.
- After Secondary 4, students sit for the N-level examinations and then O Levels at the end of their fifth year if they do well enough.
- These students can take either mathematics or mother tongue or both as O-level subjects if they do well and sit for these papers at the end of Secondary 4.

Express and Special stream students:
- Students in these streams take the O-level examinations at the end of four years. They use their score in one language and their five best subjects to enter polytechnic or junior college.

The new
Normal (Technical) students:
- Students can cross over to the Normal (Academic) stream in any year if they perform well. But if they transfer after they have completed Secondary 2, they have to repeat that year in the Normal (Academic) stream. So a student who crosses over at the end of Secondary 3 has to repeat Secondary 3 in the academic stream.
- From 2006, students can offer a maximum of two Normal (Academic) subjects if their teachers believe they will be able to cope with the subjects. They can choose from the full range of subjects tackled in the academic stream, including literature and history, which are not offered at Normal (Technical) level.
- The syllabus for Normal (Technical) students will be revised by 2007 to involve more practice-oriented learning. This includes a choice of electives which schools are developing with the Institute of Technical Education and polytechnics to give them a grounding in their chosen line.

Normal (Academic) students:
- From 2006, students can choose to do subjects at O-level standard from a wider range that includes humanities, music and sciences, provided their school offers these subjects. They can take a maximum of two O-level subjects at the end of Secondary 4.

Express and Special stream students:
- Schools can offer students a choice of new O-level subjects.
- From 2007, Secondary 1 students can take a non-native mother tongue language – Chinese and Malay – as a third language. This means a student taking Chinese as a second language, for example, can now take Malay as a third language.

The reason
- The changes are part of an MOE plan to help students develop their strengths.
The old
- Students could earn up to eight CCA points for representing the school, combined schools or the nation.
- Students with a second co-curricular activity (CCA) received leadership and achievement points but not participation points — CCA points are needed for university admission.

The new
- Students can get up to two CCA points if the activities they suggest get the nod from their school. These suggestions can be a new CCA, an ad hoc activity or a one-off event requiring students to put in at least eight hours.
- Students’ involvement in community-based organisations, such as the Residents’ Committee Youth Club and Community Club’s Youth Executive Committee, are recognised as a second CCA.
- Taking up a sport or game, or joining a uniformed group as a second CCA can earn a student up to two points.

The old
- Students who are good in sports, arts or science and maths had to go to normal schools.

The new
- Setting up of specialist schools such as the Singapore Sports School, the National University of Singapore High School of Mathematics and Science (NUS High) and the Arts School.
- The sports school started this year, specialising in eight sports, such as track and field and badminton. It has about 140 students in Secondary 1 and 2. Students need to pass selection trials and show that they can complete the 0 levels within four or five years. After the 0 levels, students can move on to junior college or do a two-year sports science diploma course offered by the Auckland University of Technology.
- Students can also choose to skip the 0 levels and enter Republic Polytechnic after passing an entrance exam. Those who are national athletes can apply to the Nanyang Technological University (NTU) without 0 levels. NTU is looking to offer bridging courses for these students.
- NUS High starts next year with 125 Secondary 1 students and 125 Secondary 3 students. Students bypass the 0 levels and get the new NUS High School diploma, which is recognised by NUS and NTU, at the end of four or six years.
- The Arts School will open in 2007 and will take in 200 students a year from secondary 1. Students must qualify for the Express stream (that is be in the upper 60 per cent in the PSLE to apply. Others will be admitted on a case-by-case basis. After six years students will take the International Baccalaureate (W) diploma, equivalent to the A levels.

The reason
- Giving students more options and allowing them to specialise and excel in areas like sports or the arts.