INFUSING THINKING SKILLS THROUGH THE USE OF GRAPHIC ORGANIZERS IN PRIMARY MATHEMATICS TO ENHANCE WEAK PUPILS' LEARNING

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Abstract: A popular topic for local mathematical research is investigating the factors underlying difficulties encountered by weak pupils in word problem solving. With the emphasis on infusing thinking skills into English, Mathematics, Science and Social Studies in primary classes, there is now an urgent need to look for alternative ways of helping weak pupils to learn thinking skills through mathematics word problem solving. Thinking strategies such as the use of graphic organizers that build thinking skills have been successfully used to teach English, Science and Social Studies to slow learners. The organizers have helped pupils to decompose problems into smaller parts for easier understanding, to organize information into schemata and to establish links between the schemata. This paper attempts to show that weak pupils in primary school could be helped to learn and think in mathematics classes through the use of graphic organizers while solving word problems. The thinking processes illustrated are part-whole, sequencing, comparing and contrasting, decision making and predicting.

Introduction

A favourite research topic in Mathematics is to investigate the underlying difficulties encountered by primary pupils in solving word problems. As English is not the mother tongue of most Singaporean pupils, language has long been suspected to be the principal culprit. But is it the only source of trouble in word-problem solving?

If we were to analyse the essential steps involved in word-problem solving they involve

1) the ability to read;
2) the ability to comprehend;
3) the ability to translate information in English into Mathematical information, and
4) the ability to sequence the mathematical information into steps in solving the problem.

In fact, according to Newman (1983), difficulty in problem solving may occur at one of the following points, (1) reading, (2) comprehension, (3) strategy know-how, (4) transformation, (5) process skill, and (6) solution. In a study by Kaur (1995), Singapore pupils experienced problem solving difficulties such as (1) lack of comprehension of the problem posed, (2) lack of strategy knowledge, and (3) inability to translate the problem into a mathematical form.

Children in primary classes are at the Concrete Operational Stage of cognitive development according to Piaget (1926, 1928, 1932, 1951). At this stage, children between ages of 7 to 11, become capable of various operations but only with concrete things. If information is concrete, comparisons can be made accurately. They are even capable of handling classification schemes involving complex logical ideas if these ideas are explicitly shown on paper to them.

Studies have shown that children who are weak in handling abstract ideas have problems in drawing relationships between discrete pieces of information and hence unable to form schemata. With the use of concept maps, mind maps, hierarchical charts and other graphic organisers, teachers have been able to concretise relationships between concepts for these children. Comprehension and learning have improved by leaps and bounds with the use of graphic organizers in Science, English and Social Studies.

This paper attempts to illustrate that it is also feasible to help primary pupils to solve word problems through the use of graphic organisers. The graphic organisers will assist comprehension by relating the different pieces of vital information in a concrete format.

The Use of Graphic Organizer to Facilitate Comprehension and Transformation

As was pointed out in the introduction, solving of word-problem involves not just the ability to read, but also the ability to comprehend and translate information in English into Mathematical structures.

It is not unusual, for example, for pupils to say "I don't understand the problem", "I don't know what to do", "I don't know what the question is talking about", even though they may be able to read the question. There are, of course, several contributing factors, such as pupils' self-concept of their mathematical ability, or pupils may lack motivation to tackle the problem. Whatever the contributing factor might be, it is helpful to assist pupils to cross the first hurdle of their word-problem solving process - comprehending the problem. Without a strategy to help them to comprehend word problems, the locus of the problem solving process will always remain with the teacher; there is no ownership of the problem on the pupils' part. In such a scenario, the pupils do not have a problem but the teacher! On the other hand, when pupils feel equipped with a strategy to help them comprehend word problems, there is a greater chance for them to make an attempt to make sense of the problem. This would help to shift the locus of the problem from the teacher to the pupils - i.e. there is a desire on the part of the pupils to want to solve the problem.

At the same time, it is also not uncommon for pupils to feel a sense of loss even after they have comprehend the problem. Transforming information written in English in a word problem to mathematical concepts helps pupils to evaluate the mathematical resources they have to handle the problem situation. Only when pupils could feel confident that they have sufficient resources to handle a problem situation would a desire to solve a problem be translated from frustration to problem solving.

It is thus desirable to provide strategies for our pupils to comprehend and translate information in a word problem into mathematical concepts.

Comprehending the information in a word problem involves making sense of the structure of a mathematical word problem. A mathematical word problem generally consists of two components - the known ("given") and the unknown ("find"). Hence, by getting our pupils to list down the given information and to state what is to be found would aid our pupils in gaining a better understanding of the problem situation. As for translating information in a word problem into mathematical concepts, it requires pupils to make an effective connection between the understood information and the repertoire of mathematical knowledge that they have acquired from their mathematics learning. As the saying goes "a picture tells a thousand words", so getting pupils to draw pictures/diagrams could help them to make sense of and see relationships between the known's and unknown's in the problem situation. This could serve as a bridge for them to then discriminate and select mathematical skills and knowledge necessary for tackling the word problem - which are the topic(s) that I could draw concepts, ideas and skills from and what are the, if there is any, formulae that could assist me in solving the problems.

The Problem Wheel shown in Figure 1 helps to put these ideas diagrammatically.

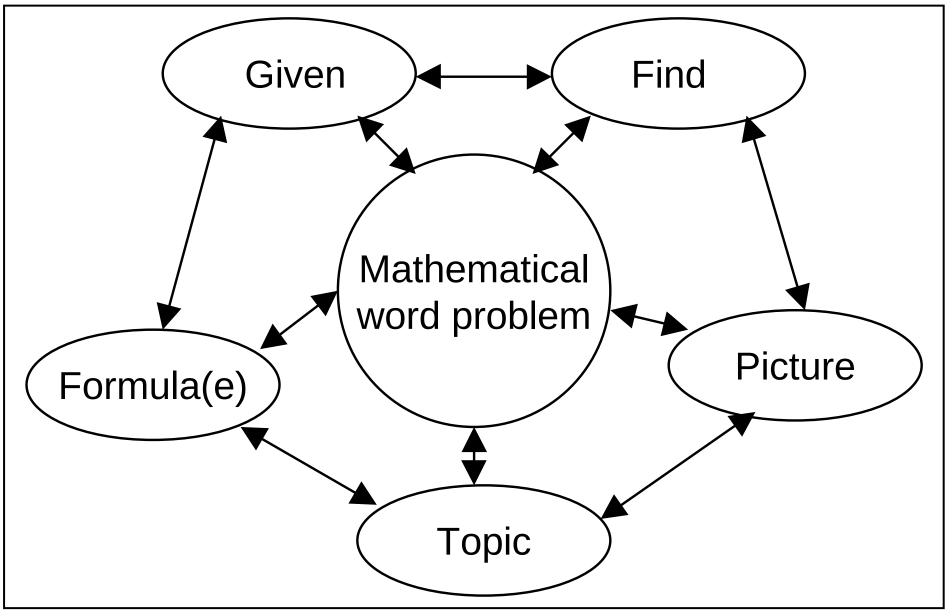

Figure 1: Problem wheel

The ideas are depicted as a wheel with double headed arrows to convey the message that the steps are not to be perceived linearly though sometimes they may occur as such. More often than not, pupils go through the various components of the wheel non-sequentially. They would possibly going back to earlier components of the wheel to revise the information gathered and translated as they move round the 'Problem Wheel' to gain a better understanding of the word problem and trying to translate the information into mathematical concepts. The interactivity of the various components reflects the dynamics involve in solving word problem.

However, such a wheel only allows us, the teachers, to see the relationship of the various components, it may not be comfortable nor practical for pupils to go into drawing of such a wheel whenever they encounter a word problem. A more practical approach towards implementing such a schema for our pupils to put into practice may be to encourage pupils to list the various components in the form of a graphic organiser as they embark on solving word problem, such as that shown in Figure 2 following:

Given	:
Find	:
Picture	:
Topic(s)	:
Formula(e)	:
Solution :	

Figure 2: Graphic organiser corresponding to the problem wheel

The disadvantage, though, of doing a listing is that pupils might perceive the steps as being sequential, and could therefore be uncomfortable to revise their understanding as they proceed through the various components of the wheel. Thus it is important that teachers should highlight and encourage pupils to view their understanding and translation of the information in the word problem into mathematical concepts as non-sequential and incremental in nature.

Furthermore, to help pupils in recalling the various components of the wheel, it might be helpful to encourage pupils to come out with their own mnemonic for these steps. Unless teachers could develop one that pupils could relate to, for example "Goals For Playing The Football", pupils may end up remembering the mnemonic but not the components of the Wheel!

Once pupils have acquired a good understanding of the word "problem", and are able to translate the information into mathematical concepts, they are then in a better state of readiness to perceive the relationships that exist between the various information components presented in the problem situation. This will help pupils to tap on appropriate strategies to solve the problem.

Arithmetic Word Problems

As mentioned in the introduction, lack of strategy knowledge is another difficulty that our pupils faced when they solved word problems. This often stems from pupils' inadequacy in establishing relationships between the various information components presented in a problem situation. The use of graphic organizers may enable pupils to better co-ordinate these various information components of a word problem and provide a visual cue to the relationships that exist, moving them a step closer to solving the problem.

As arithmetic constitutes the bulk of the primary mathematics curriculum in Singapore, it is most appropriate and relevant to use it as a platform for discussion here.

The Semantic Structures of Arithmetic Word Problems: A Classification Of Arithmetic Word Problems

Marshall (1995) has identified five basic situations that are present in arithmetic word problems. The five basic situations are Group, Change, Restate, Compare and Vary. The situations are described and illustrated using word problems found in local textbooks.

A Group situation is a static situation where parts form a whole. It is otherwise known as part-part-whole situation. A problem that involves a Group situation is shown in Figure 3.

> There were 2055 people at a concert.
> 1637 of them were adults.
> How many children were there?
> (CPDD, 1999a, p35)

Figure 3: Group situation

A Change situation is a dynamic situation involving an initial state, a change and a final state. A problem that involves a Change situation is shown in Figure 4.

> A man bought 650 curry puffs for a party.
> There were 39 curry puffs left after the party.
> How many curry puffs were eaten during the party?
> (CPDD, 1999a, p22)

Figure 4: Change situation

A Restate situation is one where the same information is presented in both absolute and relative terms. A problem that involves a Restate situation is shown in Figure 5.

> 429 concert tickets were sold on Sunday.
> 64 more concert tickets were sold on Sunday than on Saturday.
> How many tickets were sold on Saturday?
> (CPDD, 1999a, p23)

Figure 5: Restate situation

A Vary situation assumes a fixed relationship between two quantities. This relationship is maintained when the number of either quantity changes. A problem with a Vary situation is shown in Figure 6.

> Cik Siti paid $18 for 3 kg of durians.
> What was the cost of 1 kg of durians?
> (CPDD, 1999a, p43)

Figure 6: Vary situation

A Compare situation is one where two or more quantities are compared. Compare situations usually occur with other situations. Figure 7 shows a problem posed by a Primary 5 pupil that contains three types of situations, Restate, Change and Compare.

> John has 36 stamps.
> Peter has 5 stamps more than John [Restate].
> John gives 3 stamps to Mary [Change].
> Who has the most stamps [Compare]?
> (Yeap & Kaur, 2000)

Figure 7: A problem that includes a Compare situation

The Use of Graphic Organizers in Establishing Relationships

In solving the problem shown in Figure 5 which has a Restate situation, a graphic organizer such as one shown in Figure 8 may be used.

64 more than	
Sunday 429	Saturday ?

Figure 8: Graphic organizer for a Restate situation

Similar graphic organizers for a Change situation and a Group situation are given in Figure 9 and Figure 10.

Change ?	
At first 650	Finally 39

Figure 9: Graphic organizer for a Change situation

Altogether 2055	
Adults 1637	Children ?

Figure 10: Graphic organizer for a Group situation

A graphic organizer for a Vary situation is shown in Figure 11.

3 kg	1 kg
$18	?

Figure 11: Graphic organizer for a Vary situation

A Classroom Scenario

One of the authors conducted a lesson with a class of Primary 5 pupils using such graphic organizers. The problem used in the lesson is shown in Figure 12.

> At a book fair, Ali bought 24 books at 3 for $5 and had $2 left.
> How much money did he have at first?
> (CPDD, 1999b, p25)

Figure 12: Problem used in the lesson

After reading the problem, the teacher asked some comprehension questions such as "Where did Ali go?", "What did he do at the fair?", "How much were the books?", "How much money did he have at first?", "How much money did he spend?", "How much money did he have finally?".

The teacher then told the class that he was going to show them yet another way to solve problems. This chart was put up on the board.

Spends	
At first	Finally

Pupils were invited to fill the chart using information from the word problem. The teacher asked for the information that was easiest to be put onto the chart. He then asked for the information that is unknown.

Spends	
At first ?	Finally $2

One pupil offered that the amount spent was also known. The teacher asked him to explain how that was done. The pupil said, "If 3 books cost $5 then 24 books cost $40." The teacher asked pupils who understood this explanation to raise their hands. As there were pupils who did not raise their hands, the teachers asked those who raised their hands to explain how the amount $40 was derived. One pupil offered this explanation, "You times $5 with 8." "Why 8?", asked one pupil. "Because 24 divided by 3 is 8." was the response. The teacher asked for another method. A pupil wrote this on the board.

3 books	$5
6	$10
9	$15
12	$20
15	$25
18	$30
21	$35
24	$40

The teacher then invited pupils to complete the chart and use it to solve the problem.

Spends $40	
At first ?	Finally $2

The teacher proceeded to ask pupils a series of 'what-if' questions.

- What if Ali had $50 at first?
- What if 3 books cost $10?
- What if 4 books cost $5?
- What if Ali had $10 after paying for the books?

Instead of asking for an answer, the teacher required pupils to predict the effect of each change. For example, the teacher asked pupils "What if Ali had $50 at first? Will he have $2 left, more than $2 or less than $2?".

Spends $40	
At first	Finally
$50	$2 (?) < $2 (?) > $2 (?)

In this lesson, graphic organizer was used to help pupils sequence information in a Change situation. Graphic organizer was also used to facilitate prediction in 'what-if' situations.

Conclusion

The paper attempts to demonstrate the versatility of graphic organizers in helping pupils to transform an abstract verbal word problem into an easy to understand and relate set of concrete concepts. The method has been tried out on pupils who have difficulties in coping with word problems with success. Graphic organizers can be used effectively for the teaching of thinking skills such as part-whole, sequencing, comparing and contrasting, decision-making and predicting in mathematics.

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