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# SINGAPORE PUPILS MATHEMATICAL KNOWLEDGE AT THE END OF FIRST SCHOOL YEAR

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**Abstract:** As part of the International Project on Mathematical Attainment (IPMA), the complete cohorts of Primary One pupils from three schools in Singapore were tested for their mathematical knowledge at the end of the school year. A test comprising of 17 items was used to measure pupils' knowledge of numbers less than 100, shapes, fractions, mass and money. The same group of pupils was tested on the first seven items of the test at the beginning of the school year. In this paper, the presenters will share with the participants the mathematical attainment of the pupils at the end of their first year of school and also discuss the performance of the pupils on the seven items which pupils attempted at the beginning and end of their school year.

## Introduction

Singapore joined the International Project on Mathematical Attainment (IPMA) \* in January 1999. This project is a longitudinal and international one (Burghes, 1998a). At present Brazil, China Czech Republic, England, Finland, Holland, Hungary, Ireland, Japan, Poland, Russia, Singapore, South Africa, Ukraine, United States of America and Vietnam are participating in the project. The aim of this project is to monitor the mathematical progress of children from their first year of compulsory schooling throughout primary school. It hopes to study the various factors that affect that progress, with the ultimate aim of making recommendations at the national and international level for good practice in the teaching and learning of mathematics.

## The Study

### Sample

A total of 1016 pupils studying in three primary schools in Singapore are participating in the IPMA. The subjects are entire cohorts of Primary One pupils studying in these schools in 1999. Table 1 shows the distribution of the subjects by school and gender.

Table 1  
Number of Pupils by School & Gender

School	N(girls)	N(boys)	N(total)
1	191	-	191
2	180	212	392
3	194	239	433
Total	565	451	1016

## Instrument

At the end of the school year in 1999, these pupils were tested on their knowledge of numbers less than 100, shapes, fractions, mass and money. The test (Test 2) administered to the pupils comprised of seventeen items. It was constructed by the IPMA team (Burghes, 1998b) in UK. At the beginning of the school year the same group of pupils were tested on their knowledge of numbers (only the first seven items of the test [Test 1] were administered) and the results have been reported in a paper by Kaur, Koay , Yap & Burghes (1999).

The table below outlines the topics and objectives of the test items.

Table 2  
Topics & Objectives of Test Items

Items	Topics	Objectives
1	Numbers from 0 to 10	Given part of a set, complete the set to show the given cardinal number.
2	Numbers from 0 to 10	Identify a number on the number line.
3	Numbers from 0 to 10	Complete the number bonds with missing sums, differences, addends or minuends.
4a 4b*	Numbers less than 20	Compare and order numbers. Identify the odd numbers.
5	Shapes Ordinal numbers	Identify the basic shapes. Count from left and from right.
6	Basic addition and subtraction number facts	Find the sums and differences and locate them on a given number line.
7	Numbers less than 20	Complete a number pattern by counting on in threes, counting backwards in twos or counting on in fours.
8	Numbers less than 100	Count in tens and fill in the missing tens on a number line.
9	Numbers less than 100	Add and subtract with and without renaming. Part e has the missing addend while Part f has the missing minuend.
10	Numbers less than 20	Find the product, quotient, dividend or a factor in a number sentence.
11	Numbers less than 100	Complete a number series using rules such as plus 6, plus 7 and multiply by 3. The missing numbers are either at the end of the given series (Parts a, b and c) or at the beginning (Part b).
12*	Money and measures:	Solving 2-step word problem involving cents (addition and subtraction).
13*	Mass	Make up a given weight less than 20kg.

14	Money and measures	1-step division word problem involving dollars (realistic problem).
15	Number less than 20	1-step division word problem.
16*	Fraction	Show fraction of a set.
17*	Number less than 40	2-step word problem (involve algebraic thinking or thinking backwards).

\* denotes item objective / content not in Primary One mathematics curriculum

### Data Collection & Analysis

In a school all the primary one pupils were given the test scripts at the same time. Their teachers read through the test prepared on overhead transparencies, question by question, allowing sufficient time for pupils to answer the question before proceeding to read the next item. Teachers were allowed to paraphrase the items so that pupils' performance was not affected by language competency. The whole exercise took about 30 – 40 minutes.

The test scripts were scored by the researchers. The answers were marked either right or wrong. A score of one was given to a correct response and zero for an incorrect response. The maximum test score was forty. The Minitab statistical software was used to analyse the data. The mean and standard deviation of the test score, histogram and item difficulty indices were obtained.

## Results & Discussion

### Overall Performance

As shown in Figure 1, the distribution of the test scores is skewed to the left thereby showing that the majority of the pupils could do most of the items in the test easily. The mean score of the test is 30.27 with a standard deviation of 5.65. The modal score is 33 and the median score is 31.

Percent

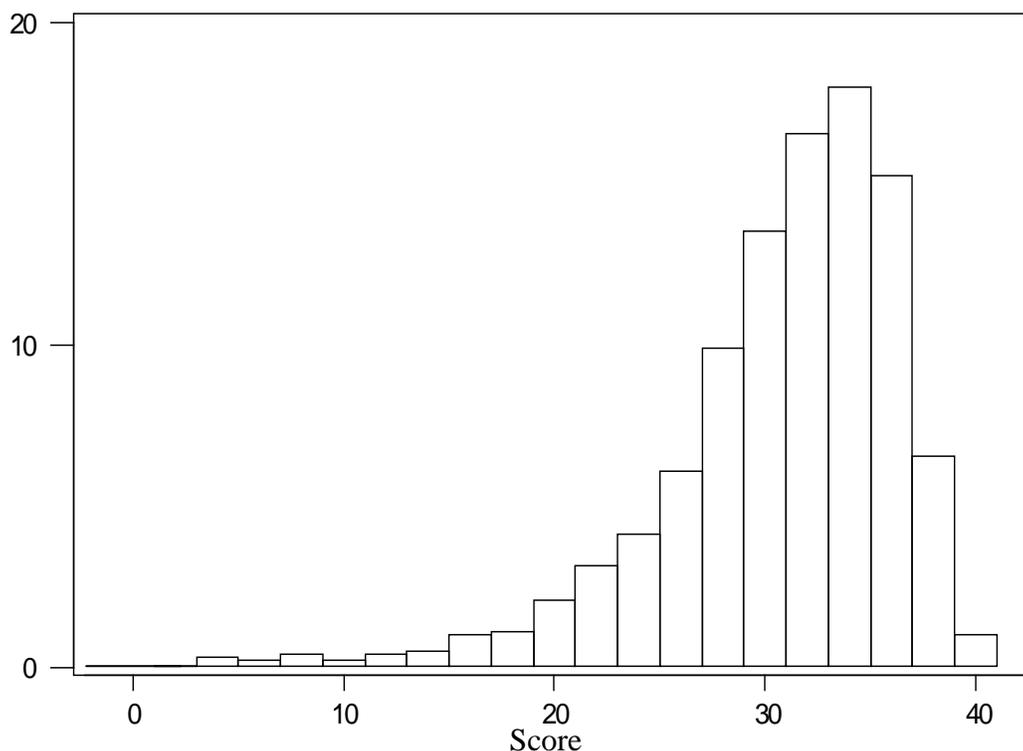


Figure 1: Frequency distribution of Test 2 scores

### Performance of Pupils on the Items of the Tests

Table 3 shows the difficulty index of each item (% correct based on the entire sample) in both the tests.

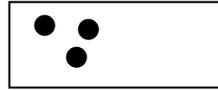
Table 3  
Difficulty Indices of Test Items

Item	Test 1 (%)	Test 2 (%)
	Beginning of 1999	End of 1999
1	87.89	97.17
2	87.11	91.98
3a	99.02	98.92
3b	96.95	98.73
3c	98.33	98.14
3d	94.49	96.09
3e	93.50	96.58
3f	91.04	95.60
4a	69.49	80.65
4b	22.93	35.19
5a	82.48	86.71
5b	75.98	86.51
5c	90.35	96.87
6a	87.60	97.26
6b	85.43	96.38
6c	79.63	96.58
6d	62.40	88.86
7a	57.58	88.95
7b	57.38	85.92
7c	45.28	80.84
8	-	94.62
9a	-	90.91
9b	-	93.06
9c	-	84.16
9d	-	64.71
9e	-	68.62
9f	-	50.34
10a	-	88.17
10b	-	74.78
10c	-	76.03
10d	-	70.67
11a	-	66.96
11b	-	41.35
11c	-	1.17
12	-	47.51
13	-	72.43
14	-	32.55
15	-	75.76
16	-	29.23
17	-	9.97

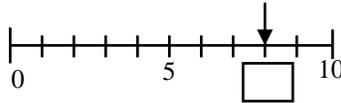
A qualitative discussion of the performance of pupils on the test items follows:

Counting up to 10

1. Complete the picture so that it has 7 dots.



2. What is the number shown?



For both tests pupils found items 1 and 2 very easy, although number lines are either seldom used or explained explicitly in primary mathematics classroom.

Addition and subtraction (less than 10)

There are six incomplete number sentences in item 3.

- |                                       |                                       |
|---------------------------------------|---------------------------------------|
| a) $2 + 3 = \underline{\hspace{2cm}}$ | b) $4 - 1 = \underline{\hspace{2cm}}$ |
| c) $3 + 4 = \underline{\hspace{2cm}}$ | d) $4 + \underline{\hspace{1cm}} = 9$ |
| e) $8 - \underline{\hspace{1cm}} = 3$ | f) $\underline{\hspace{1cm}} + 7 = 7$ |

Generally, most Primary One pupils (over 90% in Test 1 and over 95% in Test 2) were able to complete correctly the given addition and subtraction number sentences. They found it more difficult to find the missing number to the left of the equal sign, especially when the missing number is zero.

Compare and order numbers less than 20

4.  $12, 7, 15, 4, 1, 10, 18$

- (a) Write these numbers in order of increasing size  
(b) Circle all the **odd** numbers.

The difficulty indices of 4(a) and 4(b) are 69.5% and 22.9% respectively in Test 1 and 80.65% and 35.19% respectively in Test 2. Pupils seemed to have difficulty ordering 7 numbers simultaneously at the beginning of the school year. However, their performance improved by the end of the year. Their difficulty may be language related. That is, inability to comprehend the phrase 'in order of increasing size'. Pupils are more familiar with a phrase like 'arrange the numbers in order. Begin with the smallest.' as found in their Primary One textbook. In addition, the memory load of the task may contribute to the item difficulty. Among those pupils who answered this item incorrectly, most could order the first five numbers correctly (1, 4, 7, 10, 12, 18, 15) while others could only manage to order the first three numbers correctly (1, 4, 7).

The concepts of odd and even numbers are not taught in the Primary One. Hence, only 22.93% and 35.19% of the pupils were able to circle the odd numbers in Test 1 and Test 2 respectively.

Ordinal numbers and identification of shapes



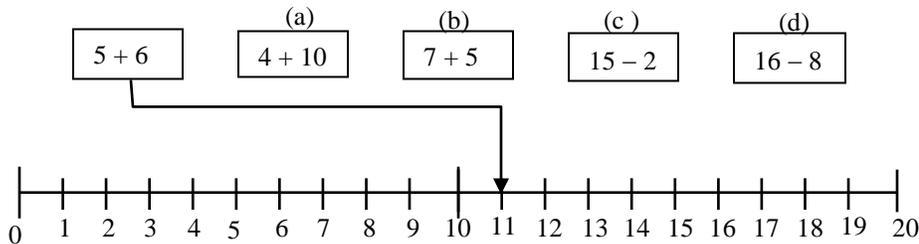
- (a) Write the letter A on the third shape from the left.
- (b) Write the letter B on the fourth shape from the right.
- (c) Write the letter T on any triangle.

In Test 1, 82.48% of the pupils were able to locate positions from the left while 75.98% of them were able to locate positions from the right. 90.35% of them were also able to recognise triangle from given shapes such as squares, circles and pentagons.

By the end of the year, 96.87% of the pupils could recognise shapes correctly while about 86% of the pupils could locate positions from the left or the right.

Addition and subtraction (Less than 20)

6. Show with an arrow the answer to each sum. The first one has been done.



As expected, pupils found the four tasks increasingly more difficult. At the beginning of the school year, 87.6% of the pupils were able to add ones to ten. 85.43% were able to add 2 single digits with sum more than 10. Pupils found it relatively more difficult to carry out subtraction. 79.63% of the pupils were able to subtract without renaming while only 62.40% of them were able to subtract with renaming.

By the end of the year, though their performance improved significantly in all items (97.26%, 96.38%, 96.58% and 88.86%), they still found subtraction with renaming most difficult. It must be noted that column addition and column subtraction are not taught in Primary One. The pupils are expected to use number bonds to carry out these types of computation in class.

Skip counting

- 7. What is the next number?
- (a) 3, 6, 9, 12, \_\_\_\_\_

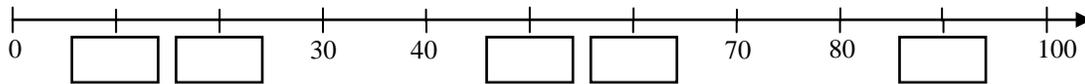
- (b) 20, 18, 16, 14, \_\_\_\_\_  
 (c) 2, 6, 10, 14, \_\_\_\_\_

Only about half of the pupils were able to answer these questions correctly in Test 1. The difficulty indices are 57.6%, 57.3% and 45.3% respectively. It seems that pupils found it easier to count on by twos and count back by threes than count on by fours.

By the end of the year, pupil performance on all three items improved markedly (88.95%, 85.92% and 80.84%). However, pupils found counting by fours relatively more difficult than counting by twos and by threes.

### Counting up to 100

8. Fill in the missing numbers on the number line.



By the end of year 1, about 95% of the pupils were able to count in tens and write the numbers correctly on the number line.

### Addition & subtraction (Numbers less than 100)

9. Fill in the missing numbers.

(a)  $27 + 12 = \square$

(b)  $35 - 3 = \square$

(c)  $15 + 17 = \square$

(d)  $46 - 18 = \square$

(e)  $73 + \square = 99$

(f)  $43 - \square = 27$

Pupils are taught addition and subtraction with and without renaming within 100 in Primary One. As expected, they found items involving renaming (items c, d and f), missing addend (item e) and missing subtrahend (item f) difficult. The percentages of correct answers are 91%, 93%, 84%, 65%, 69% and 50% for items (a) to (f) respectively. Item f is undeniably the most difficult item as it involves both renaming and missing subtrahend. Besides careless errors and random errors, the common errors found may be categorised as follows:

- Incomplete knowledge of the algorithm - pupils had partial knowledge of the algorithm, they missed one or more steps in the algorithm. e.g,

$$\begin{array}{r} 1^1 5 \\ + 17 \\ \hline 22 \end{array} \qquad \begin{array}{r} 4^1 6 \\ - 18 \\ \hline 38 \end{array}$$

- Improvised algorithm - pupils followed the algorithm without understanding. They 'invented' their own algorithm based on familiar technique and past experiences. e.g.

$$\begin{array}{r} 35 \\ -3 \\ \hline 32 \end{array}$$

$$\begin{array}{r} 46 \\ -18 \\ \hline 32 \end{array}$$

$$\begin{array}{r} 43 \\ -27 \\ \hline 24 \end{array}$$

These pupils subtracted column by column and were unable to subtract 8 from 6. Their past experiences directed them to subtract a smaller number from the larger number, hence, they subtracted 6 from 8.

- Incorrect basic number facts - pupils had not mastered the basic number facts in addition and subtraction. e.g.,

$$\begin{array}{r} 34^10 \\ -18 \\ \hline 23 \end{array}$$

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$$\begin{array}{r} 35 \\ -3 \\ \hline 31 \end{array}$$

$$73 + 23 = 99$$

$$27 + 12 = 38$$

Multiplication & division (Numbers less than 20)

10. Fill in the missing numbers.

(a)  $8 \times 2 = \square$

(b)  $14 \div 2 = \square$

(c)  $15 \div \square = 3$

(d)  $6 \times \square = 18$

In Primary One, multiplication sign is introduced in the textbook but not the division sign. The pupils are not required to write division sentences. They understand multiplication as repeated addition and division as sharing and grouping. However, division as the reverse operation of multiplication is not emphasised.

Hence, it is not surprising that pupils in this study performed relatively poorly on items (b), (c) and (d) than (a). The percentages of correct responses for items (a), (b), (c) and (d) were 88%, 78%, 76% and 71% respectively. To deal with the unfamiliar sign in the number sentences, some pupils seemed to resort to finding the sum or difference of the two given numbers, the two operations that they were more competent with.

- $8 \times 2 = 10$ ,  $14 \div 2 = 16$ ,  $15 \div 18 = 3$  and  $6 \times 24 = 18$
- $8 \times 2 = 16$ ,  $14 \div 2 = 12$ ,  $15 \div 12 = 3$  and  $6 \times 12 = 18$

Another common error is due to incorrect multiplication facts. E.g.

$$2 \times 2 = 4$$

$$3 \times 2 = 8$$

$$4 \times 2 = 10$$

$$5 \times 2 = 12$$

$$6 \times 2 = 14$$

$$14 \div 2 = 6$$

Number sequence (Numbers less than 100)

11. Fill in the missing numbers.

(a) 31, 37, 43, ,

(b) , 12, 19, 26,

(c) 3, 9, 27,

The percentages of correct answers for the above items are 67%, 41% and 1% respectively. Pupils appeared to have derived the rule by examining only the first two numbers in the sequence. In Primary One, pupils have limited experience with number patterns involving multiplication or division. Hence, only 1% of the pupils got item (c) correct. Compared to item (a), pupils' performance in item (b) is poor. This may be due to the fact that both the first and the last numbers in this sequence are missing and therefore pupils had to count backwards as well as count on. Consequently, the task demanded a higher processing load on memory of these pupils. Besides the random errors, the most common set of incorrect responses is 7, 12, 19, 26, 33. Pupils were able to find the common difference in the sequence but instead of subtracting 7 from 12, they placed 7 as the response.

2-step word problem (Money)

12. Mary buys two pencils costing 20 ø and 25 ø.

What is her change from 50ø?

48% of the pupils were able to find the correct change in this item. The common incorrect responses include the follows:

- $20\text{ ø} + 25\text{ ø} = 45\text{ ø}$ ,
- $50\text{ ø} - 45\text{ ø} = 15\text{ ø}$  (incorrect subtraction)
- $25\text{ ø} - 20\text{ ø} = 5\text{ ø}$

Two other factors that may have contributed to the difficulty of this item are the vocabulary (e.g., costing, change) and the task demand. Two-step word problems are not commonly done during mathematics lesson in Primary One.

Combination of weights (Mass)

13. Colour the weights which together make exactly 17 kg.



5 kg      7kg      4 kg      3 kg      2 kg      2 kg

72% of the pupils were able to identify correctly the four weights that add up to 17kg. The factors that may have contributed to the difficulty of this item are:

- inability to add four numbers simultaneously, and
- inability to consider possible combinations of weights and select the appropriate one.

1-step word problem (Division)

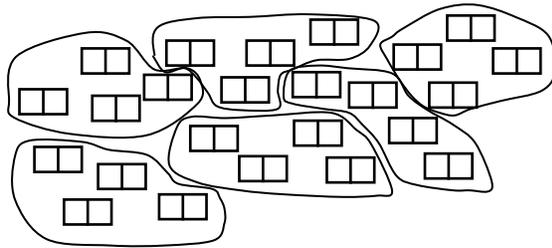
14. *Tickets cost \$4 each.  
How many can be bought for \$15?*
15. *20 cards are shared out equally among 5 children.  
How many cards does each child have?*

Although both the above items involve division, pupils performed better in Item 15 (percent correct = 76%) than Item 14 (percent correct = 33%). It may be due to the fact that the key phrase 'shared out equally' appears in the problem statement of Item 15. This key phrase implies division and pupils were able to draw diagram to find the answer. e.g, Item 15



$$20 \div 5 = 4$$

4



$$20 \div 5 = 4$$

4



4



4

In Item 14, there is no key phrase to help pupils identify the operation. In addition, it involves division with remainder, a topic not included in the Primary One mathematics

curriculum. The common responses for this question included 19 (i.e., add instead of divide), 11 (subtract instead of divide) and 4 (unable to deal with the remainder appropriately).

Fraction (part of a set)

16. *Colour in a quarter of the total number of circles.*



Although fraction of a set is taught only in Primary Four, 29% of the Primary One pupils were able to colour correctly a quarter of the given set of circles. These pupils probably applied their informal knowledge on fraction to answer this question.

2-step word problem (Numbers less than 40)

17. *Peter thinks of a number. He multiplies it by 3, takes away 2 and gets 25. What was his number?*

Only 10% of the pupils were able to find the number in Item 17 correctly. To answer this item correctly, one has to have both the mathematical language and the problem solving skills such as working backwards or guess and check. Most of the pupils who answered this item correctly did not show their working on the script. Hence, we are unable to describe in full their problem solving strategies.

### Conclusions

The performance of Primary One pupils in Test 1 show that in Singapore, children generally enter Primary One with considerable knowledge in arithmetic. The two or three years of pre-school education seems to give children a head-start in mathematics at school. They are able to count to twenty, distinguish common shapes, complete simple number sentences involving addition and subtraction within 20. They tend to use finger counting to find the answers.

After a year of formal schooling, Primary One pupils appear to do well on topics covered in their mathematics syllabus. From the Test it is apparent that they are able to

- count up to 100,
- count in tens to 100
- add and subtract within 100, without renaming better than with renaming
- do simple multiplication of number less than 20 but not division,
- complete number sequences involving addition but not multiplication,

They have difficulties

- solving 1-step word problem involving division
- solving 2-step word problems involving the other three operations,
- considering possible combinations of weights and selecting the appropriate ones,
- modelling fraction of a set graphically.

It must be noted that, the last three are not a part of the Primary One mathematics curricula.

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