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SOME PRINCIPLES AND GUIDELINES FOR DESIGNING MATHEMATICAL DISCIPLINARY TASKS FOR SINGAPORE SCHOOLS

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1. About SMAPP

The Singapore Mathematics Assessment and Pedagogy Project (SMAPP) is part of a broader research initiative on "Pedagogy and Classroom Processes" in the research framework of the National Institute of Education, Singapore. The overarching goal of SMAPP is to enhance the quality of mathematics teaching and learning by developing formative and summative assessment practices, making these assessment practices an integral part of teaching and learning, and broadening student learning outcomes through authentic tasks based mathematics assessments. Its focus areas include developing new assessment tasks, teacher learning, and the integration of IT in mathematics instruction.
Conceptual Framework

SMAPP Disciplinary Tasks

- Creative and critical thinking
- Application skills
- Meta-cognition
- Positive attitudes in mathematics
- Communication and modelling skills
- Logical reasoning

Students’ learning

Desired outcomes

Development and Implementation of Tasks

Mathematical content

- Singapore’s mathematics syllabus
- Focuses on core mathematics knowledge and skills
- Tailored to school’s scheme of work

Teachers’ teaching

Features

Contextualised
Developmental/formative
Multi-staged
Open-ended, problem-solving
ICT-based/digitalized

http://niesmapp.servy.net/teamsite/index.html
Research questions (Impact Issues)

• What are the changes in instructional practices (in particular the assessment practices and teaching methods) that have been brought about by the introduction of the new assessment approach?

• What are the effects of the new assessment approach on student learning outcomes in terms of their academic achievement, mathematical problem-solving ability and their attitudes towards mathematics and learning of mathematics?

• How does teacher’s teaching behaviour change?

• What is the impact of the new assessment approach on teachers’ beliefs/perceptions of mathematics, assessment and pedagogy?
2. What is a good quality disciplinary task?

In order to create a proper disciplinary task, we need to define what we mean by a good quality disciplinary task.
A special feature of the disciplinary tasks that distinguishes them from the traditional assessment problems is their contextualized content. We use local events, places, data or commonly encountered names and terms to make the scenario more realistic and relevant to students’ daily life.
“Aziz and Bryan are planning to drive to **Malacca** during the June holidays. Aziz lives in **Ang Mo Kio Avenue 1** and Bryan in **Bedok South Avenue 1**. On the way from their homes to Malacca, they plan to meet at the **Woodlands Checkpoint** and again at the rest point in **Yong Peng**. You are to help in the planning of the trip by working through Tasks A …”

The following picture is obtained from Google Maps. It shows the **Marina Reservoir** near the **Singapore River** and the **Kallang River**. In this task, you are to estimate the area of the Marina Reservoir and make some calculations based on your estimation.”
Real and relevant data

The data used in the problems should be realistic and obtained from reliable sources. Fictitious data should be avoided as far as possible. Real data provide students with a realistic sense of how mathematics can be applied in the real world.
“According to national statistics, water consumption in Singapore is about 1,262,000 m³ gallons a day in 2008. Assuming Singapore maintains these rates of water consumption, how long will the supply of water (as approximated in B3) from the Marina Reservoir last?”

“Given that the size of an A0 paper is 0.841 m by 1.189 m, find the area of an A0 paper.”
Curriculum connection

The developed tasks will be used by the school as an embedded assessment. It is important that the tasks are connected to the school curriculum. It is also necessary to consider when the tasks in the semester will be implemented and make sure the students have all the core prerequisite content knowledge. It is thus necessary to list the prerequisite knowledge for doing the tasks. Furthermore, the terms and definitions used in the tasks must be the same as those given in national syllabus.
“Pre-requisite/Content are: Concept of average speed; knowledge of computation and conversion involving distance, speed and time.”

----- the prerequisite for the task Malacca Trip:

In the following question, originally students were asked to solve the inequality, but later they are just required to form an inequality, as they still have not learned the solution of inequality at that stage.

“If Bryan wants to reach Yong Peng before 09 55, what should his average speed be? Write down an inequality in v using your answer from part (a).”
Multiple competencies and content knowledge assessment

One advantage of disciplinary tasks (especially the complex ones) is in assessing the multiple mathematical competencies of students and their comprehensive abilities to apply what they learn in the classroom. The task problems should then be designed to serve these purposes.

(i) understanding the problems ; (ii) constructing mathematical models ; (iii) computation and reasoning; (iv) communication using appropriate representations and means.
The following are the competencies assessed in *Malacca Trip*: (i) basic skills involving speed, distance and time; (ii) ability to represent a situation (*x* minutes before 07 30) mathematically using algebra; (iii) ability in calculations and solving algebraic equations; (iii) ability to translate scenario-based situation into algebraic expressions; (iv) ability to use the correct inequality sign to formulate linear inequalities; (v) ability to formulate linear equations and linear inequalities.
Experience enriching

Attempting a disciplinary task may also provide chances for students, besides practicing and learning mathematics skills and knowledge, to gain some social experiences and learn more about the environment and society. The enriching experience will also increase the students’ motivation and interest for attempting such task.
For instance, by going through the task *Water Water Water!*, students will have a better understanding of how precious the water is to Singapore; which are the seven reservoirs and their locations in Singapore; their sizes and roles in supplying water.

While in the task *Paper Recycling*, students will find detailed information of the actual sizes of different types of sheets of paper and their weights, number of trees and the amount of water needed to produce a given quantity of papers.

The actual Google map is used in the task *Malacca Trip*, so that students will have an accurate idea about the locations and distances between some of the places in Malaysia.
Scaled levels of difficulties

In our design, the first part usually consists of warming up questions, giving chance for students to familiarize themselves with the scenario and to recall the basic skills and knowledge. The parts that follow will usually consist of more difficult and challenging questions. Sometimes, open-ended questions are also included.
3. Designing the tasks
Getting started

From our experience, the crucial and difficult part is the initial stage. There are two suggested approaches:
(1) start with a topic and then make up suitable and interesting scenario;
(2) start with a rich scenario and then pose the relevant mathematics questions.
Start with a topic

Suppose we want to assess students’ ability to solve speed and related problems. After considering a few possible scenarios, we found that the one concerning a trip to Malaysia is the ideal one. For many Singaporeans, travelling to the neighbouring country Malaysia is a favourite way to spend their holidays. So, planning a trip to a Malaysia city such as Malacca provides us with a real life scenario to pose mathematics questions. The scenario is rich in the variables that we need to consider in planning a trip: departing and arriving times, resting time, etc. For example, “Aziz wants to meet up with Bryan at 07 50 at Woodlands Checkpoint, 20 km away from Ang Mo Kio Ave 1. If he travels at an average speed of 60 km/h, find his departure time from Ang Mo Kio Ave 1.”
Start with a scenario
Sometimes we can start with a scenario leading to good questions. In line with the special feature of linkage to real life described in the previous section, current affairs and local events provide a rich source for rich scenario. With a local scenario, National Education can also be incorporated into the task
In the statistics task *Up Down, Up Down* concerning Singapore aging population, the National Day Rally 2008 by Singapore Prime Minister provides the backdrop. This leads to questions on Singapore demographic issues. For example, the declining birth rate motivated questions based on the line graph of total fertility rate (TFR) shown in Figure 1.

“*There is a sharp decline in the TFR from 1972 to 1975. Search the web for possible reason(s) to explain this decline.*”
Pose warming-up questions
A typical disciplinary task is often divided into sub-tasks: Task A, Task B, and so on. Beginning questions in each sub-task should have straightforward solution. These are warming-up questions that allow students to recall the required concepts, and get familiar with the scenario. Below are some examples.
“The figures below shows the pieces used to approximate the area of the Marina Reservoir. Name the shapes of those figures.”

“The pie chart below shows the ethnic composition of residents for 2008. The total number of residents was 3,642,700. Estimate the number of Chinese residents in 2008, correct to the nearest hundred thousand.”
Develop the task

The problem/topic or scenario for a task is often selected for its richness in the sense that it allows mathematical questions of different levels of difficulty to be asked to assess students’ multiple competencies and content knowledge. After the first few warming-up questions, the next few questions are usually less straightforward.
“The diagram below shows a three dimensional view of the pool. The depth of water in Section A increases gradually from 0.9 m at the shallow end to 1.8 m at the deep end. The depth of water in Section B is 0.9 m throughout. Find the volume of water in the pool, assuming it is completely filled. Give your answer in litres.”
In the warming up questions, students are asked to find the area of each of the geometric figure which then be used to estimate the volume the reservoir.
Further tasks

“(a) On the question paper (hardcopy) provided, draw the shapes you would use (different from the diagram given in Question 1) to approximate the area of the Marina Reservoir.

(b) Based on the shapes you’ve drawn, will the measure of the area of the Marina Reservoir be exactly the same as you obtained earlier? Give a reason for your answer.”

“The bar graph below shows the amount of water consumed per person per day among households … in Singapore for four different years. The Public Utilities Board has taken a series of water conservation initiatives … targeted to reduce domestic water consumption per capita to about 155 litres per day by 2012. Based on the trend in the graph, do you think this target is achievable? Give an explanation for your answer.”
Lastly, the following points have to be considered when designing a task:

• Use plain words;
• avoid long sentences;
• make sure students will not take too much time to understand the questions;
• scaffold questions if necessary;
• where it is possible, pictures, tables or other forms of presentation should be used.
4. Conclusion
Some of the principles listed here have already been applied in our design of the various tasks for the SMAPP project. At the beginning, we created a few tasks without any existing guideline. Then when the reviewers started to review and revise them, there were a lot of disagreements and arguments, and we have to make several rounds of revision and modification before achieving an agreement. Having a set of agreed criteria and guideline helps to reduce the work of correction and revision. Furthermore, more schools may start to implement this type of assessment and they need to design new tasks by themselves. The criteria and illustration shown in this paper could help them in the future. In one training workshop, some teachers already followed these guidelines and designed some interesting tasks.
References