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MATHEMATICS TEACHING IN A SECONDARY ONE CLASSROOM IN SINGAPORE

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Abstract

The TIMSS Videotape Classroom Study compared the teaching of eighth-grade mathematics in Germany, Japan, and the United States. The study provided rich and contextual information about what went on inside the classes, as well as compared actual mathematical teaching methods across the countries. The findings served to stimulate and focus discussions of teaching practices among educators, policymakers and the public as one strives to understand the processes leading to learning in classroom settings and to improve teaching and learning outcomes in mathematics.

The present study adapted the methods of the TIMSS Videotape Classroom Study to study the teaching practices of a mathematics teacher in a Secondary One class. Five consecutive lessons carried out by the teacher were videotaped. The analysis of the videotaped lessons surfaces information pertaining to the organization of instruction, the nature of discourse for instruction, and the nature of work students are engaged in their mathematics classes.

Background

When Singapore topped the list of mathematics achievement at both the fourth grade and eighth grade level in the Third International Mathematics and Science Study, there was much interest from the local educational community and the general public in identifying the factors that contributed to this spectacular performance (Soh, 1998, Kaur, 2003). Combined with the fact that students in other Asian countries such as Japan and Hong Kong have also regularly performed better than their Western counterparts in other comparative studies, educators world-wide are generally keen to identify characteristics of the Asian classrooms that might have contributed to these outcomes (Hiebert et al, 2003, Leung, 1995 & 2002).

While many factors, both inside and outside the classroom, can affect students' levels of achievement, it has been well documented that teaching makes a difference in student's learning (National Research Council 1999) and it is essential that research address the processes leading to learning in classroom settings (Clarke, 1998). The TIMSS Videotape Classroom Study compared the teaching of eighth-grade mathematics, by videotaping a complete lesson in two hundred and eighty-one eighth-grade classrooms across Germany, Japan, and the United States (Stigler and Hiebert, 1999). A second expanded study was carried out in 1999, involving eighth-grade classrooms in seven countries, to consider whether distinctive patterns can be detected in each country (Hiebert et al, 2003). The projects provided rich and contextual information about what goes on inside the classes, as well as compare actual mathematical teaching methods across the countries (Shimizu, 2002).

This study seeks to adapt the practices of the TIMSS Videotape Classroom Study to examine the teaching practices of a mathematics teacher teaching a Secondary One class in Singapore. Through videotaping the teacher in her classroom over a sequence of five lessons, one seeks to surface information pertaining to the organization of instruction, the language use by the teacher, and the nature of work students were engaged in their mathematics classes.

The Study

The Sample

A female mathematics teacher, referred to as Teacher A in the study, teaching a secondary one class mathematics is the participant of the study. When Teacher A participated in the study, she had been teaching the class for four months. It was observed that she enjoyed a good rapport with her students, and did not seem to have any critical classroom management issues. According to Teacher A, the students in the class observed, were of average ability compared to students of the same cohort in the neighbourhood school. They were generally hardworking, paid attention in class and were motivated to learn.

Methodology

Teacher A was videotaped while teaching mathematics to her secondary one class. The class was not informed in advance of the videotaping. While students were initially surprised to see the researcher and the video camera, they soon accepted the camera and researcher in the class as the lesson progressed.

Classroom observation and videotaping was conducted over a period of two weeks, and a total of five consecutive lessons were documented. Consecutive lessons were observed, so as to capture information, pertaining to how Teacher A introduced the topic Basic Algebra and developed related algebraic concepts over the lessons. Originally the researcher intended to videotape the teacher over five one-hour periods. However Teacher A's mathematics lessons with the class comprised two double period one-hour lessons, and two single thirty-minute periods weekly. Hence the five lessons captured included four one-hour lessons and one thirty-minute lesson.

The researcher videotaped the lesson by setting up a digital video camera at the back of the classroom. Generally the camera focused on Teacher A, capturing everything she was doing to instruct the class. A zoom microphone attached to the camera captured the voice of the teacher when she addressed the whole class clearly. However the microphone was not able to capture the interaction between Teacher A and her pupils when she moved round the classroom attending to individual or small groups of students.

The camera was turned on when Teacher A first greeted the class, and turned off when students thanked the teacher at the end of the class. The researcher was thus able to study the duration of classroom activities by measuring their length on the videotape. Apart from the videotapes of the five lessons, supplementary materials deemed helpful for understanding the lesson were collected. These included copies of textbook pages, worksheets, as well as marked copies of students' work. Teacher A also completed a questionnaire after each lesson, which asked her to clarify her goal

and focus for the lesson, as well as to compare what happened in the videotaped lesson with what would typically transpire in her classroom. An interview was also conducted with Teacher A to clarify her beliefs and ideas on mathematics teaching and learning, and how these ideas were translated in her own teaching practices.

Data Analysis

The videos were transcribed. Data from the transcripts of the videos and relevant documents for every lesson were analysed qualitatively, focusing on the organization of instruction and the processes of instruction, specifically:

- Organization of lessons in terms of whole-class discussion and individual student work

The lessons were first analysed for the distribution of time among whole-class discussion and individual seatwork. These were coded exhaustively, meaning the end of one segment marked the beginning of the next.

Classwork – when Teacher A worked with all of the students in a whole-class situation and teacher talk was predominantly public, where Teacher A intended for all in the class to hear what she says. Activities observed during Classwork include learning a new concept, reviewing a concept, solving mathematical problems together, or sharing solutions to problems the class had been solving. The Classwork segments were further divided into activity segments, where one look into the specific pedagogical functions of the activities occurring during the segments. The following activity segments were coded:

- Teacher Exposition – where Teacher A talked about concepts, ideas, solution strategies or methods. As Teacher A was transmitting lesson-relevant information, her students' roles were limited to listening.
- Teacher Demonstration – where Teacher A illustrated procedures, demonstrated solution steps and communicated information at the same time. Students' roles were to listen as well as to take notes of the procedures and solution steps demonstrated.
- Working at Board – where teacher and students worked collaboratively on a task. Teacher A led the class in working on the task, actively soliciting input from students, through asking questions. This activity segment usually occurred after Teacher Exposition or Teacher Demonstration, as teacher guided students to work on a task, applying the concepts or procedures she had just demonstrated.
- Sharing – where the activity focused on presenting, discussing and reflecting on previously completed tasks. This activity usually occurred right after Seatwork, when Teacher A will spend some time reviewing the tasks assigned during Seatwork. A segment was coded as Sharing only when Teacher A or a student presented the method of solution orally. Whether the Sharing was done by teacher or by a student was also coded.

Seatwork – when students were assigned tasks to work independently, either alone or with their peers at the same cluster of tables. Types of talk observed were predominantly private, where Teacher A addressed an individual student or small groups of students. Most times, Teacher A would also call upon one or more students to come up to the whiteboard to work on a problem, while she moved round the class to check on students' work.

- Distribution of activities of different purposes to develop the content

Next the lessons were examined for the distribution of time assigned for activities of different purposes during the lessons. Information gathered addressed how Teacher A organized her instruction to shape students' learning.

Research has shown that teachers help students to learn through first presenting information, explaining concepts, modeling skills and engaging students in discussion about the content presented. Then teachers must provide opportunities for students to practice or apply what they are learning. Finally students must also receive timely feedback, to help them assess their progress and correct misconceptions (Brophy, 1999).

The lessons were analyzed and divided according to the purpose the segments fulfilled. The following three purposes were defined:

- *Introducing new content*: where students were introduced to content that they had not worked on in an earlier lesson. Teacher A would usually introduce new content through teacher expositions or demonstrations. Occasionally, she would involve students while demonstrating how to solve a new problem. She would elicit their responses to parts of problem, tapping on their prior knowledge.
- *Practicing new content*: where students were practicing or applying content introduced in the current lesson. These segments only occurred in lessons where Teacher A had introduced new content. Students might be given time in class to practice similar problems which Teacher A had just demonstrated or to apply concepts introduced to demonstrate their understanding. Then Teacher A would follow up with a whole-class discussion.
- *Reviewing*: where Teacher A and students were engaged in the review or reinforcement of content presented previously. Teacher A would usually start her lessons by reviewing selected homework problems assigned in the previous lessons. Some students would be assigned problems for which they would have to write out the solution on the board. Then Teacher A would then go through the solutions with the whole class.

There were some segments of non-mathematical activity, such as greeting students and giving instructions for them to bring out the relevant books and papers before the lesson proper starts and assigning homework towards the end of the lesson, which were not coded.

- Nature of teacher discourse during the lessons

Types of teacher talk

The teacher plays a crucial role in shaping the nature of interaction in the classroom through the way she presents information and poses question, sending signals about the knowledge and ways of thinking and knowing that are valued (Ball, 1991).

In analyzing teacher discourse over the five lessons, the following seven mutually exclusive categories were coded:

- Elicitation - where Teacher A intended to elicit an immediate communicative response from student(s), including both verbal and non-verbal responses. These were usually questions posed by Teacher A to an individual or to a whole class.
- Information - where Teacher A provided information to the students, not expecting any communicative or physical response from students.

- Direction - where Teacher A intended to cause students to perform some physical or mental activity.
- Uptake - where Teacher A responded to student verbal or physical response. It might be evaluative comments such as "Correct", "Good", or "No", repetition of student response, or reformulation of student response.
- Response - where Teacher A responded to a student elicitation
- Provide Answer - where Teacher A provided the answer to her own elicitation
- Other - when the utterance did not fit into any of the above categories or that was not intelligible.

Types of Questions

Besides presenting information and modeling application of skills, teachers should use questions to stimulate students to process and reflect on content, recognize relationships among key ideas and think about it, applying it in problem solving (NCTM, 1991). While closed-end and factual questions are appropriate for assessing prior knowledge and reviewing new learning, open-ended questions allow students to apply, analyse, synthesize and evaluate what they are learning (Brophy, 1999).

The questions posed by Teacher A in class were coded as follows:

- Name/State type of questions - questions about the content covered that requires a relatively short response. These included questions which request a student to provide information, either from the textbook or from the problem statement.
- Yes/No type of questions - questions about the content covered that requires a simple yes or no response from students. Questions that require students to choose between two alternatives were also categorized as a Yes/No questions.
- Describe/Explain type of questions - open-ended questions that requires students to describe a mathematical object, not just labeling it, or to explain a generated solution method, not just providing an answer, or to provide reasons why something is true or not true.

- Nature of work students were engaged in

Students' opportunity to learn mathematics is shaped, in part, by the content of the mathematics presented (National Research Council, 1999). The types of mathematical problems that students were engaged in over the five lessons were examined.

Hilbert et. al. (2003) defined problems as events that contained a statement asking for some unknown information that could be determined by applying a mathematical operation. Problems could vary greatly in length and complexity, ranging from routine exercises to challenging problems. Problem statements were analysed for the following three types of mathematical processes implied:

- Using procedures - where the problem statements suggested that the problem was typically solved by applying a procedure or set of procedures.
- Stating concepts – where the problem statements called for a mathematical convention or an example of a mathematical concept.
- Making connections – where problem statements implied the problem would focus on constructing relationships among mathematical ideas, facts, or procedures

Results & Discussion

- Organization of lessons in terms of whole-class discussion and individual student work

Classwork

Figure 1 shows that Classwork took up most of the lesson time. Over the five lessons observed, Teacher A spent an average of 64% of the total Lesson time, engaging students in a whole-class situation. The average length of a Classwork segment was ten minutes, comprising mainly teacher-directed activities.

Figure 1: Percentage of time during lessons spent on Classwork

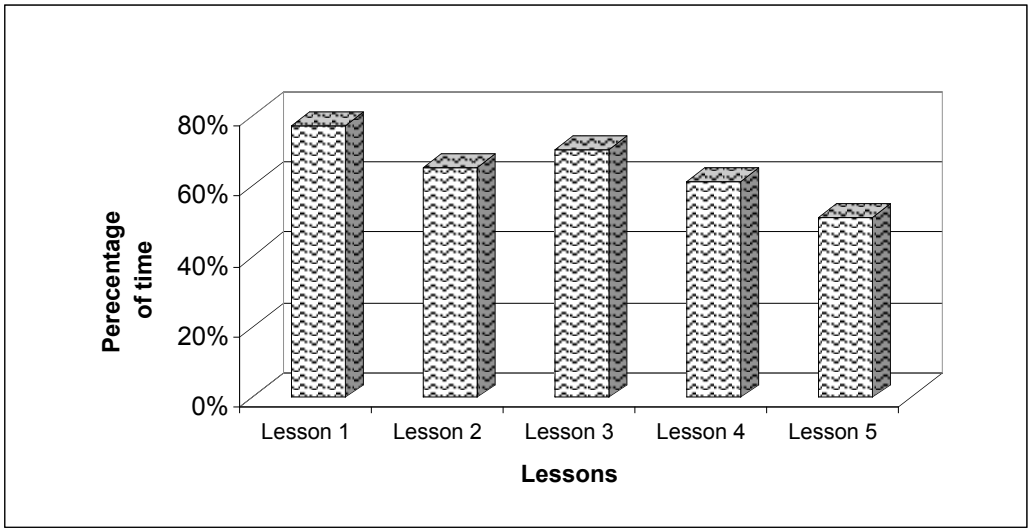


Table 1 shows the activity segments that were observed over the five lessons. It was observed that the nature of activities during Classwork changed slightly over the five lessons. In the first 2 lessons, where Teacher A was introducing basic algebraic concepts, more of the activity segments were Teacher Exposition and Teacher Demonstration. Activity segments also included Working at Board, where Teacher A led students to apply concepts and practice procedures, such as simplifying algebraic expressions on the board. In Lesson 3 and 4, most of the activity segments were Sharing by Teacher and Students, as the class focused on reviewing related homework and sharing solutions to problems.

Table 1: Activity Segments observed during the Lessons

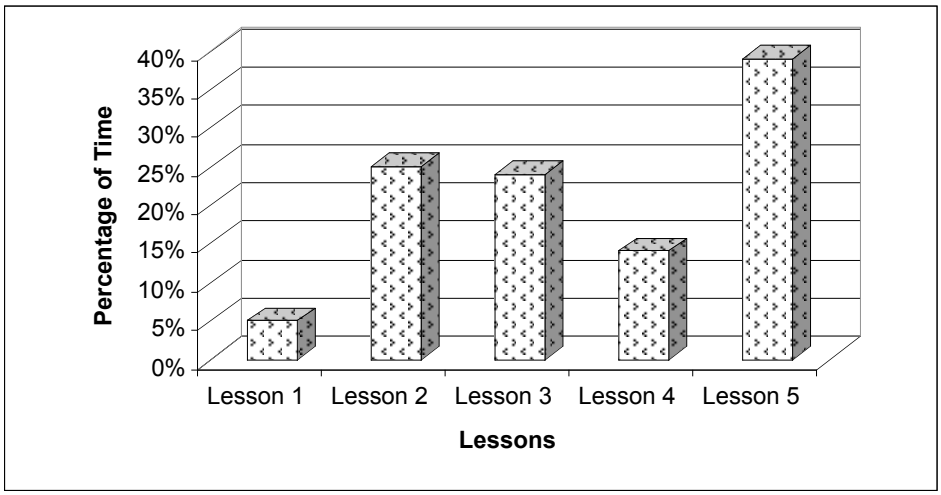
	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Teacher Exposition	X	X	X		X
Teacher Demonstration	X	X			X
Working at Board	X	X			X
Sharing by Teacher	X	X	X	X	X
Sharing by Students			X		

Generally, the nature of activities during Classwork was mainly teacher-directed, and teacher talk dominated. Student participation was limited to answering questions when teacher A tried to elicit responses from students while working on problems collaboratively. Sometimes Teacher A would direct her questions at a particular student, requesting that the student answer the question. More frequently, however, the questions were directed to the class in general, and anyone in the class might attempt to answer.

Seatwork

Figure 2 shows that, in general, less time was allocated to Seatwork, where students get to work on assigned tasks on their own. It was observed that over the five lessons, Seatwork accounted for an average of 20% of total Lesson time. In the first lesson, which was an introductory lesson on Algebra, only one segment was coded as Seatwork. Teacher A spent most of her time defining terms and explaining concepts to the whole class. In Lesson 5, more time was allocated for Seatwork where students get to apply concepts and practice procedures taught.

Figure 2: Percentage of time during lessons spent on Seatwork

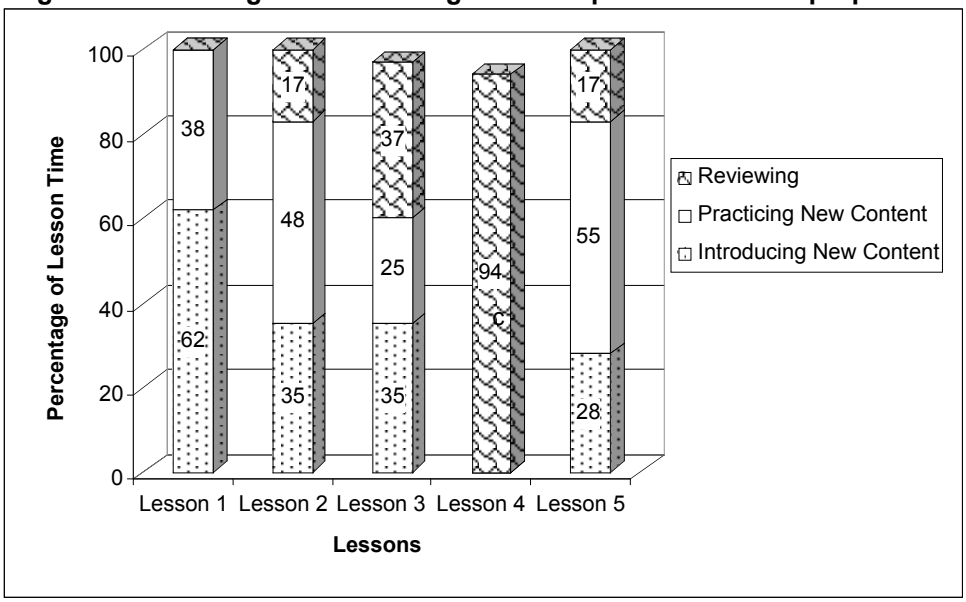


Seatwork segments were comparatively fewer and shorter with average Seatwork segment lasting five minutes, inclusive of time taken for Teacher A to prepare students for the tasks. In most of the Seatwork segments, one or more students were assigned by Teacher A to work out the problems at the board, while the rest of the class work on the problems at their desks. Teacher A would then moved around the class to check students' work, occasionally interrupting the class to highlight common misconceptions or errors among students as they work on their tasks.

During Seatwork segments, students generally did their work individually, turning to each other or to Teacher A to verify the final answers to the tasks assigned. Occasionally, Teacher A did suggest that students exchange work for checking, and to help each other identify errors, but this practice was not observed widely in the lessons.

- Distribution of activities of different purposes to develop the content

Figure 3: Percentage of time during lessons spent on different purposes



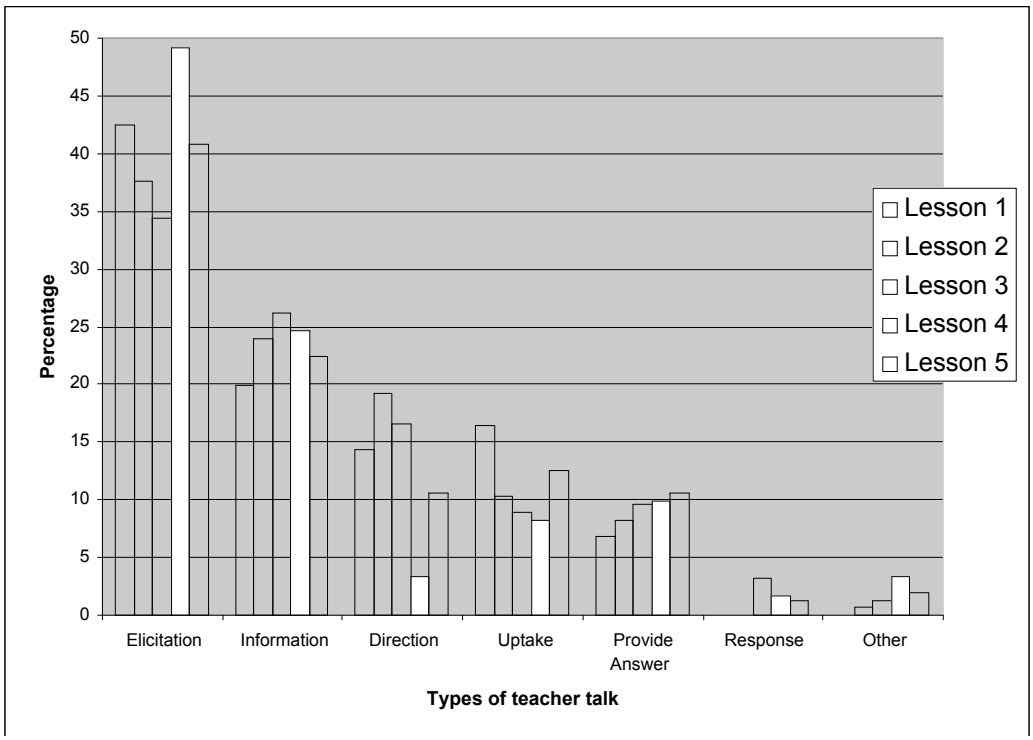
As shown in Figure 3, over the five lessons, Teacher A had allocated approximately equal amounts of time to each of the three purpose types. Students were given time to practice new content in class each time new content was introduced. It was observed that Teacher A also felt it important to review homework assigned and to clarify misconceptions surfaced at the beginning of each lesson.

- Nature of Teacher Discourse during the Lessons

Types of teacher talk

Figure 4 shows the distribution of different types of Teacher Talk over the five lessons. It was observed that the distribution of teacher discourse over the five lessons was similar.

Figure 4: Distribution of Types of Teacher Talk over the five lessons



The most frequent type of teacher utterances was Elicitations, where Teacher A asked questions to elicit response from students. Teacher A attempted to engage her students with the content through questions to check their understanding. The next most frequent type of utterances coded was Information, where Teacher A explained concepts or demonstrate solutions to mathematics problems.

Types of Questions

It was observed that the most frequent type of content elicitations occurring over the five lessons were Name/ State type of questions. The following is an example of such an exchange sequence:

Teacher A: *OK,* $p \times 2qr \times 4$. *A simple one. Let's see if we can simplify it straightaway. <u>How many terms are there</u>?*

Student(s): *3*

Teacher A: *3 terms. 1st term is p, 2nd term is 2qr and 3rd term is 4. They are all linked by multiplication. So we just multiply numerals to numerals, letters to letters. <u>So what is this straightaway</u>?*

Student(s): *8pqr*

The next most frequent type of content elicitations were Yes/No type of questions. Examples of such exchanges are:

Teacher A: [Referring to the expression: $\frac{-17x+18}{20}$]

Some of you ask me this question. The answer in the textbook is different: $\frac{18-17x}{20}$. *<u>Are these 2 answers the same</u>?*

Teacher A: *Look at h.* $-\frac{3}{7}a \times \left(-4\frac{2}{3}\right)$. *<u>Does this a belong to the numerator or the denominator</u>?*

Student(s): [Not sure] *Numerator/ Denominator*

Teacher A: *Numerator. Alright? If a is at the bottom, it should be* $-\frac{3}{7a}$.

It was observed that Teacher A asked fewer of Describe/Explain type of questions. On a number of occasions, Teacher A provided the response to her own Describe/ Explain questions.

Teacher A: [Referring to working on the board that shows $(3y \times 3y) + 2x = 9y^2 + 2x$]

Add 2*x to twice of* 3*y*, *<u>correct or wrong</u>?*

Student(s): *Wrong…*

Teacher A: *Benny, <u>where is your mistake</u>?* [Pause]

Teacher A: *Twice of* 3*y means* 3*y times* 2. [Writes on board $3y \times 2$]

Not $3y$ *square. This one is* [referring to $(3y \times 3y)$] $3y$ *square.*

While Teacher A spent a relatively high proportion of lesson time involving her students through questioning, most of her questions were closed questions which required students to provide short factual responses. Questions to elicit thinking and promote critical thinking and mathematics communication were conspicuously absent. While Teacher A did allow students some time to think about what she was saying, generally she did not lead students to probe deeper into the mathematical concepts.

Questions posed were usually directed at whole class. It was noted that generally, students who were more confident tend to give the answer, which might then be echoed by the rest of the class. As Teacher A did not actively organize students' participation in class, she was not able to check for individual students' understanding of the content taught, through their oral communication in class. Instead she shared that she would keep track of students' progress through their written work, as well as through assigning the class short tests conducted within curriculum time.

- Nature of Work Students were Engaged In

Over the five lessons observed, Teacher A spent much time going through selected problems from the textbook. Her students were also assigned work from the textbook, either to apply and practice concepts taught in class or as homework. It was noted that the problems encountered by students were mainly of the first two types, problems to be solved by applying a set of procedures or problems to be solved by stating concepts.

Table 2: Examples of the three types of problem statements

Using Procedures	(i) Evaluate the expression $3a$-$3(2c$-$b)$, given that $a = 2$, $b = -3$ and $c = 4$. (ii) Simplify the following expressions: $\frac{2}{5}\left[2x + \frac{1}{2}(4x - 12)\right]$.
Stating Concepts	(i) Express the following polynomial: $3x + 7x^2 + 4 - 5x^3$ so that the degrees of the terms are in descending order." (ii) Write an algebraic expression for each of the following: (a) the cost of x litres of petrol at \$1.10 per litre (b) three times the variable x divided by the sum of 3 and k.
Making Connections	Suppose x and y are positive real numbers. Which of the following expressions must be larger than x and y? (a) $(x + y + 1)^2$ (b) $(x+y)^2$ (c) xy (d) $x^2 + y$.

(Examples from New Syllabus Mathematics 1, 2001)

A large proportion of the problems required students to manipulate algebraic symbols to simplify or to evaluate expressions. Teacher A felt that it was important for

students to be familiar with these procedures, as the ability to manipulate algebraic expressions is a basic foundation skill which students will need in order to develop more advanced algebraic concepts and competencies.

In teaching the skills of evaluating and simplifying algebraic expressions, Teacher A implicitly shared her students the concept of algebra as generalized arithmetic. She reminded students that "*whatever that you learn about rational numbers, fractions and whole numbers, this time round, apply the same thing to algebra.*"

While solving problems, Teacher A emphasized the need to present mathematics systematically. She was most particular about the way students present their solutions. Usually, she would demonstrate with a worked example on the board, showing students how to solve the problem step by step very systematically, and then require her students to follow the same format when they solve a similar problem. There were many occasions in class when she talk about the need to present the working properly.

"... you skip your steps, you lose your marks. Clear? And please use your brackets wherever possible, wherever necessary..."

Teacher A would also explain to her class how she would be awarding marks for their assignments. She emphasized that marks will be allocated for the correct answer as well as the working, to motivate students to present their solutions properly.

"...so please ensure that your substitution is correct. This will entitle you to a method mark..."

It was noted that the nature of work students were engaged in, during the lessons, followed closely with the examination syllabus. Teacher A was conscious of her need to get her students ready for the semestral examinations. She would highlight types of questions which students were likely to encounter in their examinations, as well as what the examiner might be looking out for in the answers. Teacher A selected problems for discussion and practice from the textbook, which in turn, were much influenced by the examination syllabus and included many paper and pencil type of exercises, featured in written examinations.

Conclusion

In summary, the findings from the classroom observations showed that the lessons were mainly teacher-centered, with students spending more time on whole-class discussion of the content covered and hence proceeding at the same pace set by Teacher A. There was an equal emphasis on introducing new content, practicing the new content in class and reviewing content presented in previous lessons, as approximately equal amounts of time was allocated to each of the three purpose types over the five lessons.

The analysis of teacher discourse revealed that Teacher A tended to present knowledge by telling and explaining. Questions were often directed to the class as a whole and most questions check for recall of facts or procedures. The type of mathematical tasks students were engaged in were mainly practices to consolidate their knowledge of concepts and skills. There was little evidence of mathematical tasks that required students to engage in problem solving and develop mathematical

thinking, despite the introduction of new educational initiatives of infusing IT and thinking skills in past five years.

References

Ball, Deborah. (1991). "Implementing the Professional Standards for Teaching Mathematics: What's all this talk about discourse?" *Arithmetic Teacher*, **39** (**3**). 44 – 48.

Brophy, J. (1999). Teaching (Education Practices Series No. 1). Geneva: International Bureau of Education. Retrieved Nov, 3rd, 2003. from http://www.ibe.unesco.org.

Clarke, D. (1998). *Study Design: Learner's Perspective Study*. Australia: University of Melbourne.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K., Hollingsworth, H., Jacobs, J., Chui, A., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P. and Stigler, J. (2003). *Teaching Mathematics in Seven Countries: Results From the TIMSS 1999 Video Study*. Washington, DC: U.S. Department of Education.

Kaur, B. (2003). TIMSS - Students' and Teachers' Perspectives on Mathematics Instruction in Singapore Schools. In B. Kaur, D. Edge & B.H.Yeap (Eds.) *TIMSS and Comparative Studies in Mathematics Education*. Singapore: The Mathematics Educator – Monograph No 1.pp 85 – 96.

Leung, F. (1995). The Mathematics Classroom in Beijing, Hong Kong, and London. *Educational Studies in Mathematics,* **29** (**4**), 297–325.

Leung, F. (2002). Why East Asian Students Excel in Mathematics? – Characteristics of High Achieving Classrooms. In D. Edge & B.H. Yeap (Eds.) *Mathematics Education for a Knowledge-Based Era*. pp. 127 – 131. Singapore: Association of Mathematics.

National Council for Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics*. Reston, Va: NCTM.

National Research Council. (1999). How People Learn: Brain, Mind, Experience, and School. In J.D. Bransford, A.L. Brown, and R.R. Cocking (Eds.). *Committee on Developments in the Science of Learning, Commission on Behavioral and Social Sciences and Education.* Washington, DC: National Academy Press.

Shimizu, Y. (2002). Capturing the structure of Japanese mathematics lessons: Some findings of the international comparative studies. In D. Edge & B.H. Yeap (Eds.) *Mathematics Education for a Knowledge-Based Era*. pp. 168 – 176. Singapore: Association of Mathematics.

Soh, K.C. (1998). How do Japanese, American, and Singaporean Primary School Mathematics Teachers Teach. *The Mathematics Educator*, **3** (**2**), pp 88-99.

Stigler, J.W. and Hiebert, J. (1999). The Teaching Gap: Best Ideas from the World's Best Teachers for Improving Education in the Classroom. NY: Free Press.