<table>
<thead>
<tr>
<th>Title</th>
<th>Linear algebraic equations: Are they the same?</th>
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<tbody>
<tr>
<td>Author(s)</td>
<td>Yu Yoong Kheong and Ng Swee Fong</td>
</tr>
<tr>
<td>Source</td>
<td>4th Redesigning Pedagogy International Conference, Singapore, 30 May to 1 June 2011</td>
</tr>
</tbody>
</table>

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Linear Algebraic Equations: Are they are the same?

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Importance of the study

• To ascertain what difficulties secondary two students have solving linear algebraic equations.
Research Questions

1. Which were the more challenging linear equations encountered by the Secondary Two Normal (Academic) students?

2. What were the common errors made by Secondary Two Normal (Academic) students in solving algebraic equations?
Methods

• Participants
  An intact class:
  Male: 21
  Female: 17

• Participants were given one hour to complete nine pairs of items.
## Instrument and the rate of success

<table>
<thead>
<tr>
<th>Item</th>
<th>Linear equation</th>
<th>Numbers of incorrect solution</th>
<th>Percentage of incorrect solution</th>
<th>Numbers of incorrect solution</th>
<th>Percentage of incorrect solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1a</td>
<td>$x + 2.6 = 12.4$</td>
<td>4</td>
<td>10.5%</td>
<td>4</td>
<td>10.5%</td>
</tr>
<tr>
<td>Q1b</td>
<td>$14.3 = x + 3.5$</td>
<td>9</td>
<td>23.7%</td>
<td>8</td>
<td>21.1%</td>
</tr>
<tr>
<td>Q2a</td>
<td>$x - 2.5 = 11.4$</td>
<td>3</td>
<td>7.9%</td>
<td>3</td>
<td>7.9%</td>
</tr>
<tr>
<td>Q2b</td>
<td>$15.34 = 4.2 - x$</td>
<td>17</td>
<td>44.7%</td>
<td>17</td>
<td>44.7%</td>
</tr>
<tr>
<td>Q3a</td>
<td>$3x = 7.5$</td>
<td>0</td>
<td>0.0%</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>Q3b</td>
<td>$1.56 = -7x$</td>
<td>17</td>
<td>44.7%</td>
<td>17</td>
<td>44.7%</td>
</tr>
<tr>
<td>Q4a</td>
<td>$3x + 5 = 7.5$</td>
<td>8</td>
<td>21.1%</td>
<td>7</td>
<td>18.4%</td>
</tr>
<tr>
<td>Q4b</td>
<td>$25 = 2x + 7$</td>
<td>12</td>
<td>31.6%</td>
<td>9</td>
<td>23.7%</td>
</tr>
<tr>
<td>Q5a</td>
<td>$4x - 6 = 14.4$</td>
<td>9</td>
<td>23.7%</td>
<td>8</td>
<td>21.1%</td>
</tr>
<tr>
<td>Q5b</td>
<td>$28 = 14.4 - 5x$</td>
<td>19</td>
<td>50.0%</td>
<td>19</td>
<td>50.0%</td>
</tr>
<tr>
<td>Q6a</td>
<td>$6x - 6 = 9 - 4x$</td>
<td>3</td>
<td>7.9%</td>
<td>2</td>
<td>5.3%</td>
</tr>
<tr>
<td>Q6b</td>
<td>$8 - 3x = 7 + 5x$</td>
<td>20</td>
<td>52.6%</td>
<td>18</td>
<td>47.4%</td>
</tr>
<tr>
<td>Q7a</td>
<td>$\frac{2x + 5}{3} = 3.1$</td>
<td>14</td>
<td>36.8%</td>
<td>11</td>
<td>28.9%</td>
</tr>
<tr>
<td>Q7b</td>
<td>$\frac{1}{2} = \frac{2x + 5}{3}$</td>
<td>20</td>
<td>52.6%</td>
<td>15</td>
<td>39.5%</td>
</tr>
<tr>
<td>Q8a</td>
<td>$\frac{6 - 3x}{2} = 2$</td>
<td>28</td>
<td>73.7%</td>
<td>21</td>
<td>55.3%</td>
</tr>
<tr>
<td>Q8b</td>
<td>$\frac{3}{4} = \frac{2x - 5}{3}$</td>
<td>25</td>
<td>65.8%</td>
<td>19</td>
<td>50.0%</td>
</tr>
<tr>
<td>Q9a</td>
<td>$\frac{2x - 4}{3} = \frac{x + 3}{2}$</td>
<td>20</td>
<td>52.6%</td>
<td>16</td>
<td>42.1%</td>
</tr>
<tr>
<td>Q9b</td>
<td>$\frac{3x + 2}{3} = \frac{x - 2}{5}$</td>
<td>18</td>
<td>47.4%</td>
<td>12</td>
<td>31.6%</td>
</tr>
</tbody>
</table>
Findings

• Students were most challenged by
  (i) *Rational equations where the unknown had negative coefficients*,
  (ii) *Unknown is on the right of the equal sign*, and
  (iii) *Equation with unknown with negative coefficients*
Pedagogical implications 1

• How do we improve students’ ability in solving linear algebraic equations?

• Provide varied examples where the variable is on the right-hand side of the equal sign as well as on the left-hand side of the equal sign.

• Do not solve these equations:
  • $3x + 2 = 17$ as well as $17 = 3x + 2$

• Compare the solutions of these equations. These equations have the same solution because they are equivalent equations.
  (notion of $a = b$, $b = a$)
Pedagogical implications

- Do not solve these equations.
  - \(3x + 2 = 17\) and \(17 = 2 - 3x\)
- Compare and contrast these two equations.
- Would they have the same solutions. Explain your answer.

Objective: **To encourage students to be more reflective.**
Reflexive Property -> $a = a$

- Do the students perform checking?
- Did they explicitly substitute the solution to the left hand side and right hand side of the initial equation for verification?

<table>
<thead>
<tr>
<th>Figure 1a: Incorrect solution of item set 1a.</th>
<th>Figure 1b: Correct solution of item set 1a.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 2.6 = 12.4$</td>
<td>$x + 2.6 = 12.4$</td>
</tr>
<tr>
<td>$x = 9.8$</td>
<td>$x + 2.6 - 2.6 = 12.4 - 2.6$</td>
</tr>
<tr>
<td>$x = 9.8$</td>
<td>$x = 9.8$</td>
</tr>
</tbody>
</table>
Reflexive Property -> $a = a$

• From 1a, the student obtained an incorrect solution of 9.6 for $x$.
  \[ x + 2.6 = 12.4 \]

• If the student performed checking, the student would realise that
  \[ 9.6 + 2.6 = 12.2 \]
  and
  \[ 12.2 \neq 12.4 \]
Symmetric Property $\Rightarrow$ if $a = b$, $b = a$

$$14.3 = x + 3.5$$

$$x + 3.5 = 14.3$$

$$x + 3.5 - 3.5 = 14.3 - 3.5$$

$$x = 10.8$$

*Figure 2a:* Incorrect solution (E1) of item set 1b. Unknown at the right hand side.

*Figure 2:* Correct solution with the use of symmetric property
Transitive Property -> if $a = b$, $b = c$ then $a = c$

**Figure 8a:** Incorrect solution of item set 8b due to E4.

\[
\frac{3}{4} = \frac{2x - 5}{3} \\
3(3) = 4(2x - 5) \\
9 = 8x - 20 \\
9 + 20 = 8x - 20 + 20 \\
29 = 8x \\
\frac{29}{8} = \frac{8x}{8} \\
3.625 = x
\]

**Figure 8b:** Correct solution of item set 8b.

\[
\frac{3}{4} = \frac{2x - 5}{3} \\
9 = 8x - 20 \\
9 + 20 = 8x - 20 + 20 \\
29 = 8x \\
\frac{29}{8} = \frac{8x}{8} \\
3.625 = x
\]
Suggested Teaching Strategies

• Notion of variation
  – Strategies to sensitize students that some equations are the same and others are not.
  – In each set of tasks, two equations are presented to students.
  – Students are NOT to calculate for the roots of the equation. They are to reflect whether Equation A and Equation B have the same solution, i.e. whether equation A and B are equivalent AND to explain why.
Notion of Variation

• Instructions

• Here are some examples of equations.
  – a) In each pair of equations, state whether the equations are equivalent.
  – b) Explain your answer.
  – c) Which equation would be more difficult?
  – d) Why do you say so?
Set 1

These equations are equivalent.
Reasons: *They have the same solutions/roots*

*Equation B is more difficult because of the position of x to the right of the equal sign.*
Set 2

A
\[ x + 13 - 2 = 25 - 2 \]

B
\[ x + 13 = 25 \]

These equations are equivalent.
Reasons: *They have the same solutions/roots*

*Equation A is more difficult because of the complex structure.*
Set 3

These equations are not equivalent. For each pair of equations, they do not have the same solution/root.

*Equation B is more difficult.*

Reasons: Position of $x$ and the coefficient of $x$ is a negative integer.
Rationale for such activities

• Students seem to exhibit a Pavlovian response to equations. When presented with an equation, the instinctive response is to solve.
• The three sets of tasks puts a stop to this instinctive response.
• Set 1 is easier than Set 2. Set 2 is constructed based on Set 1 but it is more complex of the two. Set 2 challenges students to study the equations properly.
• They ask themselves. What remains the same? What changes?
• By asking which two equations are equivalent, students are asked to consider the structure of the equation. The position of the letter x does not matter as long as the equations are structurally the same.
• Set 2 is a variation of Set 1 but the complex structure is meant to challenge students to think further. How are the two equations alike? How are they different?
• How is Set 2 different from Set 1?
• Set 3 is the most difficult of the three because of students have to work with the negative coefficient of x. Because it is more, such questions require students to exercise greater care when solving such problems.
Efficacy of Such Tasks

• Such tasks have been tried with Secondary 2 students.

• Reports provided by teachers suggest that the students appreciate such tasks as they require them to think.
• Thank You
Redesigning Conference 2011

Linear Algebraic Equations: Are they all the same?

Here are some examples of equations.

a) In each pair of equations, state whether the equations are equivalent.
b) Explain your answer
c) Which equation would be more difficult?
d) Why do you say so?

<table>
<thead>
<tr>
<th>Set 1</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x + 13 = 25$</td>
<td>$25 = x + 13$</td>
</tr>
</tbody>
</table>

These equations are equivalent.
Reasons: *They have the same solutions/ roots*

*Equation B is more difficult because of the position of $x$ to the right of the equal sign.*

<table>
<thead>
<tr>
<th>Set 2</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x + 13 - 2 = 25 - 2$</td>
<td>$x + 13 = 25$</td>
</tr>
</tbody>
</table>

These equations are equivalent.
Reasons: *They have the same solutions/ roots*

*Equation A is more difficult because of the complex structure.*

<table>
<thead>
<tr>
<th>Set 3</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$3x + 5 = 47$</td>
<td>$47 = 5 - 3x$</td>
</tr>
</tbody>
</table>

These equations are not equivalent. For each pair of equations, they do not have the same solution/root.
*Equation B is more difficult.*
Reasons: Position of $x$ and the coefficient of $x$ is a negative integer.