Mathematical Modelling for Singapore Primary Classrooms: From a Teacher's Lens

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Limited Singapore research indicated a lack of exposure of modelling tasks at primary levels. Teacher reflection is used as a tool in design research cycles exploring the potentials of modelling tasks in a Singapore primary five classroom. Findings reveal that the teacher identified three potentials of a modelling task on children's mathematisation process: the task provided a platform for children to (a) identify variables and form relationships between them, (b) relate school-based math learning to real-world experiences, and (c) justify their mathematical models. Implications on the promotion of modelling tasks at primary schools as well as teacher education are drawn.

Introduction

The Singapore Mathematics Curriculum Framework (MOE, 2007, p.12) identifies problem solving as central in mathematics teaching and learning. One of the goals of mathematics education is the "acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems" (p.12). Traditionally, problem solving skills in primary schools are taught through teacher demonstration and student practice of various word problems which are mainly prescriptive in nature. Open-ended tasks such as investigations and modelling activities are encouraged in recent years to provide students with opportunities for exploration, experimentation, and discussion of mathematical ideas.

Mathematical modelling refers to a process of representing real world problems mathematically so as to understand and find solutions to the problems (Ang, 2010). Modelling activities are espoused to foster mathematical reasoning processes (English & Watters, 2005) and are hence perceived to be strong platforms for problem solving in the Singapore curriculum. Nonetheless, limited local research indicated a lack of exposure of modelling tasks in Singapore primary mathematics classrooms. Several reasons account for this. Firstly, anecdotal evidence suggests that many teachers have taken mathematical modelling to be synonymous with "the model method" (i.e., the use of bar models to represent the information given in a word problem), a common heuristics applied in problem solving at primary levels in Singapore schools (Chan, 2011). Secondly, Singapore teachers are familiar with application tasks and saw little difference between these and modelling tasks as both types of tasks involve the use of real-world contexts (Ng, 2011). Lastly, mathematics teaching in Singapore classrooms is predominantly prescriptive and teacher-centred. Teachers often hold fixed views of mathematics application as the use of formulae, standard algorithms, and predictable working. Modelling tasks which encourages

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multiple mathematical representations of real-world problems pose some challenges to teachers in their comfort zones (Ng, 2011).

This paper is an outcome from a larger study on building teachers' capacity in primary mathematics school-based assessment using *model-eliciting tasks* (see English, 2010) among others. It reports findings on an initial exploration of implementing modelling tasks in a Singapore primary school. The findings are drawn from the teacher's reflections on the modelling task and its potential to promote in-depth mathematical reasoning process in an intact class of 33 students (aged 11) in a primary five mathematics classroom in a Singapore school. Classroom discussions during task implementation are centred on the mathematical processes (e.g., interpreting, reasoning, justifying) that students demonstrated as they developed models representing the real-world problem. Findings are presented from the lens of the regular mathematics teacher of the class who typically delivers teacher-centred prescriptive mathematics lessons. Implications from these findings serve to propose areas of focuses in teacher education as well as the possibility of more varied engagement involving the use of mathematical modelling in primary classrooms.

Theoretical Underpinning

Model development is an important part of the Models-and-Modelling Perspective (MMP) (Lesh and Lehrer, 2003) which this study adopted. MMP emphasizes 'thinking mathematically' and using mathematical interpretations on real-world situations. MMP suggests that beyond content mastery and abilities in solving traditional textbook word problems, it is important that students acquire the ability to represent real-world problems in mathematical terms and to construct models as a solution to the problems. When students are engaged in modelling, their ways of thinking do not manifest as a single, one-dimensional sequence but instead as a series of cycles, in which their mental models representing the given situation are expressed, tested, and revised. In other words, knowledge develops along multiple dimensions from comparing to contrasting, from concretizing to abstracting, from specific to general, from simple to complex or from a collection of un-coordinated, immature ideas to more co-ordinated, mature, and effective knowledge (Lesh & Zawojewski, 2007). The express-test-revise process results in students making better representations or models of the problem situation through purposeful description, explanation, or conceptualization. In a sense, students become involved in mathematizing (quantifying, dimensioning, coordinating, categorizing, algebratizing, and systematizing relevant objects, relationships, actions, patterns, and regularities) (Mousoulides, Sriraman, & Christou, 2007), which is a key aspect of the modelling process.

Research Methodology

This study adopted design research methodology. The researchers and teachers worked closely together in two iterative cycles comprising three phases over a four-month period: design, teaching experiment, and retrospective analysis (Research Advisory Committee, 1996). Each implementation cycle became an opportunity to collect data to inform subsequent design. Design researchers, upon a parallel and retrospective process of reflection of the design and its outcomes, refined their initial hypothesis and principles towards a coherent theory concerning their understanding of their design experience (Edelson, 2002).

Specifically, the first cycle of this study familiarized the teacher with features of mathematical modelling tasks and some facilitation skills to promote children's mathematisation process. In the design phase of this cycle, the teacher completed a researcher-designed model-eliciting "Bus Route Task" (see Appendix) while working with her colleagues in small group discussions facilitated by the researchers. The task encouraged a diversity of mathematical approaches. It provided a platform for students to abstract mathematical relationships among key variables (i.e., distance, time, and costs) interpreted within the real-world experiences of the children. The teacher subsequently implemented the same task with her primary 5 students during three consecutive one-hour sessions over three days in the teaching experiment phase whilst the researchers took the role of non-participant observers in the class (Cohen, Manion, & Morrison, 2011). Following each implementation session, the teacher engaged in a 20-minute interview to reflect on her facilitation strategies, outcomes, and students' engagement with the task. In the retrospective analysis phase of the first cycle, the teacher engaged in an overall post-task reflection with the researchers. Here, the teacher's reflection is a critical instrument to inform subsequent design of the teaching experiment during the second cycle (Dolk, Widjaja, Zonneveld, & Fauzan, 2010). The intent is to build the teacher's capacity to design, facilitate, and evaluate learning from her own mathematical modelling task.

The following section presents findings from teacher reflection in the first cycle during the teaching experiment and retrospective analysis phases involving the Bus Route Task. Transcripts of the video-recording of the teacher's discourse with students working in groups of four served as supplementary data to complement the teacher's reflection accounts. The focal point of analysis is the teacher's views on the potentials of the Bus Route Task on children's mathematisation processes.

Findings and Discussion

The potentials of the Bus Route Task on children's mathematisation process as identified by the teacher in her reflection centred around three predominant areas.

(a) Identification of Variables and Formulation of Relationships between them

The excerpt below extracted from teacher interviews during the teaching experiment phase revealed the teacher having detected that all eight groups of students had successfully identified the mathematical topics from their prior learning which were relevant for mathematising the given problem situation (line 5).

Researcher	...So in terms of mathematical thinking and presentation, outcomes and all that, are there things that you're particularly happy about?
Teacher	…generally, I was quite happy with the progress they made because initially I thought they would not be able to bring the maths out, but they were actually able to do that. They were able to identify the topics that were relevant for this task.
Researcher	....are there particular questions that you're very happy that you've asked in order to steer them towards to what you were intending to do?
Teacher	…they were able to actually see that these (the variables) are not just isolated data that they just going to present, that they could actually find the relationship (between the variables)...So they could tell me things like you know when it's (the bus route is) longer, it's more expensive and (a) shorter (bus journey) is cheaper.

In addition, the students were also able to articulate crucial relationships between key variables (i.e., distance, cost) in the problem during her facilitation (line 9 in the excerpt below). Some of the groups even went on to identify other variables affecting the choice of the most efficient bus route such as the time taken for the entire journey, the number of bus stops, and speed of the buses (a topic they have yet to learn at the point of implementation of the task).

Teacher	So what is your, what is your recommendation?
Student	Our recommendation route is the pink route.
Teacher	So what are you basing this on?
Student	From the bus fare and the, the…the length of the…
Teacher	The distance?
Student	Ya.
Teacher	Okay, you are going to base on bus fare and distance. Is there any relationship you found? Between the two.
Student	The shorter the distance the cheaper the bus fare.
Teacher	The cheaper the bus fare.

The students were observed to have demonstrated the ability to draw upon their school-based mathematical knowledge in their model development. They made sense of the data in the map, selected the relevant quantities for comparisons, and defined the operations to create a meaningful representation of the real-world problem situation. Such findings were contrary to the teacher's initial assumptions about her students' ability in mathematical modelling (line 4 in the first excerpt). To this end, it appeared that her students have demonstrated that they were able to represent real-world problems mathematically in their first attempt at a modelling task.

(b) Relating School-Based Mathematics to Real-World Experiences

In mathematising the Bus Route Task, the students drew on their real-life experiences to make sense of the situation to create a model. In particular, speed was identified to be an important variable determining the efficiency of travelling at the various given bus routes. However, the students also realised that there was insufficient information about the speeds of the buses in the task. Hence they decided to make assumptions about the speeds of the buses in their mathematical approach. This was noticed by the teacher. During the retrospective phase of the second implementation session, the teacher articulated that '*there was even one group that talked about speed. They suddenly brought in speed. Then they realised that the speed actually is also relative. I mean you know it's … they were going to assume that all the buses were going to move at the same speed.*' The following excerpt records the conversation between the teacher and a student group illustrating how a group member reasoned that the speed of the bus would have an impact on whether the bus route was efficient (lines 18-19).

John	We used a string and a ruler to measure how long the route is. First we used the string and place it on the map according to the highlighted route then after that we make a marking on the string. We used the ruler to measure how, how long the distance is.
Teacher	Okay. That is for the…shortest (route).
John	Shortest (route).
Teacher	How about fastest? You said fastest also. (Being the most) Efficient (route) means fastest (journey). How do you…how do you show that? Uh you are saying that the pink route is fastest? Okay. How do you show that?
John	Because uh…it has the shortest distance and you can reach uh…the destination the fastest.
Teacher	Okay. Fastest and shortest. Do you agree that it's actually linked?
John	No.
Teacher	Shortest means fastest. How many of you uh agree with them that shortest is the fastest? [Nods from some students in the group.] So you are in agreement with them. How about those…is there anyone who disagrees? Shortest may

	not be fastest. Yes Tom, why do you feel that way?
Tom	Because the bus may go very slow. Do not mean it's (the bus journey of a specific route will be) the fastest (most efficient).
Teacher	Okay, so how did you cover (investigate) that? He says that…you are talking about the speed of the bus right? Did you cover that part of it? What did you say about that?
Tom	Uh…it has…(we assume that) all the buses are (travel) at the same speed.

Besides speed, the students also went beyond their textbook knowledge to articulate other variables that are based on their real-world experiences. For instance, some students tried factoring in population density as part of the decision making process on route efficiency. They believed population density has an effect on the popularity of the bus stop which in turn could have affected the boarding time of each bus service (lines 12 to 15 in the excerpt below from the same student group).

Teacher	So if they (the other students) assumed that the buses are travelling at the same speed, then will you accept the argument that the shortest (route) is the fastest (most efficient bus route)? You do? Is there anyone has any other comments on this? John?
John	What if there's….one of the, one of the shortest route has a traffic jam?
Teacher	[Directing at Tom] Okay so did you make any assumptions then? [Repeating John's comments to Tom] He says shortest route may have more traffic jam. What do you have to say to justify what you have said against what he said?
Tom	(Maybe we can assume that) there are no traffic lights...
Teacher	Any other, any other factors you think that need to be taken into account here for fastest (the most efficient bus route)? Yes?
Harry	You know sometimes you go stop at bus stop, there are a lot of people queuing up. You know like…just like morning one we go to school you know.
Tom	Maybe and probably that the bus stop got a lot of people queuing up to go up the bus then will delay uh.
Teacher	Okay do you agree with Harry?
Harry	Ya.

A large part of the ability of the students in the sample in making connections between their real-world experiences and the demands of the task could have resulted in the teacher's focused questioning during facilitation. For example, the teacher prompted her students to look more closely at the information given in the map so as to select other useful information or '*more realistic factors'* affecting bus route efficiency. She wanted them to '*see from the real life point of view.*' As a result, some groups were able to use variables such as the number of bus-stops and traffic conditions as part of their arguments for or against bus route efficiency. In developing their model, students' knowledge of pertinent mathematics (e.g., length, time, distance, average) and their flexibility in drawing upon their real-world experiences on identifying these variables were visible in the mathematising process. They were observed to link the mathematics they learnt in the classroom to what they understand in the real world.

(c) Mathematical Justification of Models

In addition, the students also attempted to justify their models. For instance, in the excerpt below, the group tried to explain their model comprising of sets of distance measurements of each bus route provided in the problem. They not only measured the routes with a string and ruler but also took an average value of the multiple measurements of each route for accuracy. Their model led to a conclusion that the most efficient bus route would be the route with the shortest distance measured. They later proposed verifying this conclusion by calculating the bus fare for the journey. This is interesting because bus fares in the Singapore transport system are already pro-rated according to the distance travelled.

Teacher	1	This (the distance recorded here) is based on?
Student	2	Ruler length (we used the ruler to measure the length of the string used to
	3	trace the bus route on the map)
Teacher	4	The length.
Student	5	The average also.
Teacher	6	Okay, the distance. What else are you going to do? Is that enough you
	7	think? No? How else would you want to get evidence (to determine the
	8	most efficient bus route)?
Student	9	The bus fare. Find the bus fare.

Moreover, the teacher detected that the justification process of model development task allows for in-depth communication of mathematical reasoning during group discussions as compared to her regular mathematics lessons. Indeed, the teacher reflected on how some group members "argued" for their choice of the most efficient route based on their mathematical reasoning and subsequently reached a conclusion within the group:

> "When they were talking about (which was) the shortest route, then the other one (student) said no. Then they will argue away. Of course they have to prove why is the shortest (the most efficient route) and things like that…But they have reached a conclusion (later). They are quite sure that they can justify. It should be okay. They have their arguments."

It is evident that from the teacher's interview, she could see that the students refined their models that suggested their growth in refining their mathematical ideas by way of their arguments and their reasoning.

Implications and Conclusion

This paper reports findings from teacher reflection on the potentials of mathematical modelling tasks at primary five. The opportunities afforded by the task were appreciated by the teacher who was surprised at the quality and sophistication of the mathematisation processes during the task. The teacher detected three areas that the Bus Route Task provided a platform for: (a) the identification of key variables and their relationships, (b) relating school-based mathematical knowledge and skills to the real-world experience, and (c) justification of mathematical models developed. These findings echo the arguments made by the proponents of model-eliciting tasks in mathematics classrooms (e.g., English, 2010, Lesh and Lehrer, 2003). In this regard, mathematical modelling has not only opened the eyes of the teacher towards a more student-centred approach mathematics lesson but also allow for richer communication of ideas by the students. In justifying their models, students communicate their thinking more explicitly than they would in their regular mathematics class.

Although the possibility of including mathematical modelling tasks in Singapore primary mathematics classrooms may not be as remote as one may think, there has to be concerted efforts at guiding teachers in facilitating and evaluation of learning from such tasks in order for the children to reap benefits from these tasks. In addition, as Stillman (2010) emphasised, a positive climate for active, purposeful mathematical discussions would be necessary to set the stage for learning during modelling tasks.

References

Ang, K. C. (2010). Mathematical modelling in the Singapore curriculum: Opportunities and challenges. In A. Araújo, A. Fernandes, A. Azevedo & J. F. Rodrigues (Eds.), *Educational interfaces between mathematics and industry: Proceedings of the EIMI 2010 Conference* (pp. 53-62). New York: Springer.

Chan, C. M. E. (2011). *Conceptions of mathematical modelling*. Paper presented at the Connecting to practice: 15th International Conference on the Teaching of Mathematical Modelling and Applications.

Cohen, L., Manion, L., & Morrison, K. (2011). Research methods in education (7th ed.). London: Routledge

Dolk, M., Widjaja, W., Zonneveld, E., & Fauzan, A. (2010). Examining teacher's role in relation to their beliefs and expectations about students' thinking in design research In R. K. Sembiring, K. Hoogland & M. Dolk (Eds.), *A decade of PMRI in Indonesia* (pp. 175-187). Bandung, Utrecht: APS International.

Edelson, D. C. (2002). Design research: what we learn when we engage in design. *The Journal of the Learning Sciences, 11*(1), 105-121.

English, L. D. (2010). Young children's early modelling with data. *Mathematics Education Research Journal*, *22*(2), 24-47.

English, L. D., & Watters, J. J. (2005). Mathematical modelling in third-grade classrooms. *Mathematics Education Research Journal*, *16*(3), 59-80.

Lesh, R.,& Lehrer, R. (2003). *Models and Modeling Perspectives on the Development of Students and Teachers.* In Mathematical Thinking and Learning, 5(2&3), (109-129). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 763-803). Charlotte, NC: Image Age Publishing.

Ministry of Education [MOE]. (2007). *Ministry of Education Mathematics Syllabus – Primary*. Singapore: Curriculum Planning and Developmental Division.

Mousoulides, N., Sriraman, B., & Christou, C. (2007). From problem solving to modelling: The emergence of models and modelling perspectives. *Nordic Studies in Mathematics Education*, *12*(1), 23-47.

Ng, K. E. D. (2011). *Facilitation and scaffolding: Symposium on Teacher Professional Development on Mathematical Modelling - Initial perspectives from Singapore*. Paper presented at the Connecting to practice - Teaching practice and the practice of applied mathematicians: The 15th International Conference on the Teaching of Mathematical Modelling and Applications, Australian Catholic University, Melbourne, Australia.

Stillman, G. A. (2010). Implementing applications and modelling in secondary school: Issues for teaching and learning. In B. Kaur & J. Dindyal (Eds.), *Mathematical applications and modelling: Yearbook 2010 of the Association of Mathematics Educators* (pp. 300-322). Singapore: World Scientific.

Research Advisory Committee. (1996). Justification and reform. *Journal for Research in Mathematics Education*, *27*(5), 516-520.

Appendix 1 – The Bus Route Task

Determining the Most Efficient Bus Route

Ms Chang has recently moved to Block 297C Punggol Road. She is going to start teaching at Punggol Primary school next week and needs to know how to travel to the school. However, the MRT is always too crowded for her to take and it also requires her to take a feeder bus which results in inconvenience. Ms Chang realizes that there are three bus services that ply different routes to her school. Help her to find the most efficient route to travel by bus from her home to the school. The location of her home is marked in the map. Currently the three bus services that are available for Ms Chang to choose are Service 124, Service 62 and Service 89. The routes for Service 124, Service 62 and Service 89 are marked as blue, yellow, and pink lines respectively on the map. The bus stops along each bus route are marked with stickers with corresponding colours.

Your task is to give Ms Chang a proposal consisting of the following:

1. How your group determines what is meant by the "*most efficient*" bus route
2. *Assumptions* about the problem your group made in order to help Ms Chang
3. The *mathematics* used to decide which route is the most efficient
4. How your group *justifies* that the selected route is the most efficient
5. The final *recommended* route for Ms Chang

For us to better understand your work, you can attach the following to your proposal:

(a) A map containing the chosen bus route.
(b) The information you found useful for this task

Bus Fares

Distance Range	***Bus Fare (Cash)***
First 1 km	$1.10
Up to every 0.7 km increase	30 cents increase

Note: The student-groups were given a map in which the three different routes for selection were marked using pink, blue, and yellow colours. Bus stops were indicated along the routes using coloured stickers. In Singapore, bus fares are calculated according to the distance travelled.