This is an uncorrected version of the article. The published version can be found in *ZDM: International Journal of Mathematics education* (2010) DOI: 10.1007/s11858-010-0265-6

Computing Solutions to Algebraic Problems Using a Symbolic Versus a Schematic Strategy

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Article Note

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The study reported in this paper was supported by a grant from the Centre for Research in Pedagogy and Practice, CRP22/03KL. Views expressed in this article do not necessarily reflect those of the National Institute of Education.

Abstract

To improve access to algebraic word problems, primary aged students in Singapore are taught to utilise schematic models. Symbolic algebra is not taught until the secondary school years. To examine whether the two methods drew on different cognitive processes and imposed different cognitive demands, we used functional magnetic resonance imaging to examine patterns of brain activation while problem solvers were using the two methods. To improve our ability to detect differences attributable to the two methods -- rather than participant's abilities to use the two methods --- we used adult problem solvers who had high levels of competency on both methods. In a previous study, we focused on the initial stages of problem solving: translating word problems into either schematic or symbolic representations (Lee et al. 2007, Brain Research, 1155:163-171). In this study, we focused on the later stages of problem solving: in computing numeric solutions from presented schematic or symbolic representations. Participants were asked to solve simple algebraic questions presented in either format. Greater activation in the symbolic method was found in the middle and medial frontal gyri, anterior cingulate, caudate, precuneus, and intraparietal sulcus. Greater activation in the model condition was found largely in the occipital areas. These findings suggest that generating and computing solutions from symbolic representations require greater general cognitive and numeric processing resources than do processes involving model representations. Differences between the two methods appear to be of both a quantitative and qualitative nature.

Keywords: algebra, word problems, pedagogy, working memory, neuroimaging

1 Introduction

The animal problem: A cow weighs 150 kg more than a dog. A goat weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the weight of the cow?

Children, and many adults, find algebraic word problems difficult (e.g., Carpenter, Moser, and Bebout 1988; Kintsch and Greeno 1985; Mayer 1992; Riley and Greeno 1988; Stacey and MacGregor 1999). Amongst the many stumbling blocks are difficulties in transforming questions into equations and constructing appropriate systems of equivalent equations that will result in the solution of the unknown. In Singapore schools, algebraic problems, usually in the form of word problems, are first introduced when children are in Primary 4 (10 year olds). In the animal problem given above, the total weight of the three animals is known, but the weight of each animal is not stated. Children are expected to consider the comparative weight of each animal and utilise a combination of the four arithmetic operations to solve this problem.

1.2 The Model Method

To improve children's access to these problems, children are taught what is locally referred to as "the model method", in which schematics are used to represent the quantitative and qualitative relationships presented in word problems. Unknowns are solved by analysing these diagrams and applying a series of arithmetic procedures (Ng and Lee 2009; Khng and Lee 2009). For the animal problem, for example, a student has to first select a base for comparison. Any of the three animals can be selected as the base from which to construct the model. If the weight of the dog is selected as a base for comparison, the model drawing on the left of Figure 1 represents the information provided in the question. In this drawing, a rectangle or combination of rectangles represents the weight of each animal. The value of the rectangle representing the weight of the dog can be found after the total weight of three identical rectangles has been ascertained. The weight of the cow is found by summing the values of the two rectangles, one representing the weight of the dog and the other representing the difference in weight between the cow and the dog.

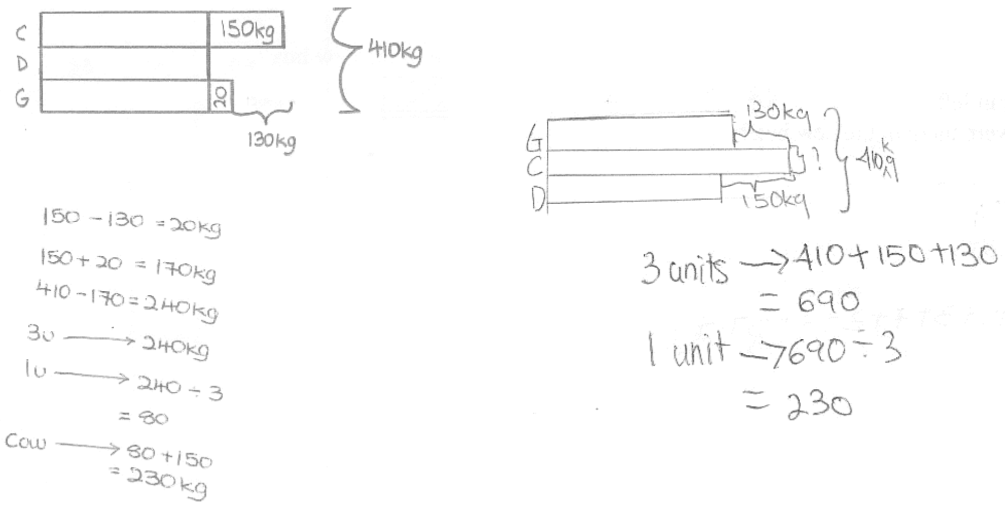


Figure 1. Two students' solutions to the animal problem. Solution on the left uses the weight of the dog as a base for comparison. The solution on the right uses the weight of the cow as a base for comparison.

If the weight of the cow is chosen as the base (see the right hand side of Figure 1), the rectangles are used to represent the relationships between the dog with the cow, and the goat with the cow. Horizontal braces are used to represent the differences in weight between the cow and the dog, and the cow and the goat. The values of the rectangles representing the weight of the dog and the cow can then be equalised to the weight of the cow by adding the respective differences in weight to the total weight indicated to the right of the vertical brace. Using the weight of the cow as the base for comparison leads directly to the solution of the problem.

Although a single equation such as 3 units + 20 + 150 = 410 can be used to capture the information in the model on the right hand side of Figure 1, this is not the norm. Instead the value of the unknown units are solved by an unwinding process in which problem solvers construct a series of arithmetic equations that lead to the value represented by the unknown unit.

In secondary school, children are taught to use the symbolic method to solve algebraic word problems. Some students are able to apply the new method, others are less successful with using it in its entirety. Analysis of students' solutions has shown that they are likely to use either the model method or a mix of the model and the symbolic method (Khng and Lee 2009).

There are a number of reasons for this phenomenon. For some students, their familiarity with the model method may render the method less effortful to deploy. An analysis of the scripts from Khng and Lee (2009) showed that students who used a mix of the two methods may have the motivation to use the symbolic method, but they were not competent in its use. Analysis of their solutions showed that although they were able to construct the initial algebraic equation for a given problem, they were thwarted in their effort to use the symbolic method in its entirety because they were unable to construct the next equivalent equation in a system of equivalent equations. Instead, they returned to the arithmetic method of unwinding to solve for the unknown value.

Some secondary mathematics teachers are troubled by student's reliance on the model method. Although the model method can be used for a variety of problems, the solutions of such problems normally require only the construction of algebraic equations of the first degree. The model method cannot be used to solve problems that require the construction of equations of the second degree and above. Given the importance of such algebraic problems in the sciences, it is important that students are competent users of symbolic algebra.

1.2 The Present Study

The model method was developed and introduced to help children, who have no knowledge of symbolic algebra, solve algebraic word problems (Kho, 1987, 2006; Ministry of Education, 2009). However, there is a diversity of views regarding the place of the model method in the curriculum. Teachers of primary schools often find the model method a powerful tool. However, some teachers of secondary schools view students' knowledge of the model method a hindrance to the learning of symbolic algebra (Ng, Lee, Ang, and Khng 2006). From time to time, there are also suggestions that the symbolic method should perhaps be introduced to children in upper primary schools, perhaps in place of the model method.

Although many children are able to use the model method to access algebraic problems, whether they can or should do so with symbolic algebra is an issue of concern. A full evaluation of this issue will require a consideration of cognitive, motivational, pedagogical, and policy issues; one issue that interested us was whether the model and symbolic methods drew on similar cognitive processes and imposed similar cognitive demands. If the symbolic method was cognitively more demanding, one needs to evaluate closely whether its introduction to younger children is both practicable and desirable.

When first introduced, the model method was chosen over the symbolic method because the former uses rectangles rather than letters to represent unknowns. Ng and Lee (2009) compared and contrasted the two methods and discussed in detail how the model method helped children overcome some of the difficulties faced by novice learners of algebra. Such difficulties include having to pay attention to the meaning of letter as variables, learning to distinguish between conventions acceptable in arithmetic but not in algebra, and the structural rules underpinning the construction of algebraic objects such as equations and expressions (see Kieran, 1989). Ng and Lee (2009) further argued that although the model method provides a concrete and more visual representation of word problems, the rectangles used in the model method serve a similar structural role as do the x and y in symbolic algebra. Kho (2006) theorised that if children were first taught to use rectangles as a symbol to represent the unknown, they will be better able to appreciate the role of symbols in representing quantities when they are taught symbolic algebra.

Although the structural roles of rectangles and letter-variables may be comparable, there are differences in the surface characteristics between the two methods. The model method makes use of diagrams and alphanumeric labels to depict information contained in word problems. In contrast, the symbolic approach relies on the construction and manipulation of alphanumeric equations. From the students' perspective, Ng (2003) found that about 79% of 114 secondary two students (14+) who participated in that study reported that they found the model method easier because it allowed them to visualise the problem. However 19% of the participants reported that although the model drawings were meaningful, the model rather than the symbolic method required them to pay attention to details such as constructing rectangles of comparable lengths and how rectangles were to be partitioned. With the symbolic method such attention to details was not needed. Though the symbolic approach may be more abstract, these observations suggest that, for some students at least, the model method requires more attentive resources and rely more heavily on visually based processing. Importantly, the students' reports also suggest that there are significant individual differences in how students perceive and utilise the two methods.

Apart from the observational study cited above, there have been few empirical investigations of the cognitive underpinnings of the model versus the symbolic methods. In this study, we used functional magnetic resonance imaging (fMRI) to examine whether the two methods are subserved by similar cognitive processes and whether they impose similar demands. Functional MRI measures hemodynamic changes resulting from the performance of cognitive tasks. Although the physiological mechanisms of such changes have not been fully explained, it is believed to be related to higher glucose utilization by parts of the brain involved in the performance of those tasks. This, together with increased blood flow to affected regions, produces localised changes in the amount of oxygenated versus deoxygenated blood. Functional MRI takes advantage of differences in the magnetic properties of oxygenated versus deoxygenated blood to provide a measure of the extent to which particular brain regions are involved in the performance of a task.

Because many parts of the brain are typically involved in the performance of a higher cognitive task, it is important that study designs are sufficiently sensitive to isolate regions that are specifically involved in the processes of interest. Typically, this is effected by using the subtraction method (first introduced by Donders in 1868). In this paradigm, activation resulting from the task of interest is subtracted from activation from either a resting state or a control condition in which the process of interest is absent.

When combined with suitable experimental designs, fMRI can identify brain regions that are jointly or preferentially activated by specified cognitive tasks (for an introduction to fMRI, see Hernandex-García, Wager, and Jonides 2004; Huettel, Song, and McCarthy 2004). For our study, one of the main attractions of fMRI is that it can provide useful data even when behavioural (e.g., reaction time or accuracy) differences are small. This is particularly important in our study because in comparing and contrasting the two methods, a fundamental prerequisite is that participants can perform both methods. Having participants with similar competencies on the two methods further enhances our ability to detect differences intrinsic to the two methods.

Yet, similar competency will also likely produce similar behavioural outcomes when the two tasks are contrasted. An advantage afforded by fMRI is that it provides data on how each part of the brain reacts as participants solve problems using the two methods. Although the two methods may take a similar amount of time to get to the same solution, the possibility exists that the solutions are generated using different underlying processes. A careful examination of the pattern of activation in different brain areas allows us to makes inferences on the involvement of such processes.

The desire to examine usage of the two methods amongst competent problem solvers also motivated our use of adults. Although using adolescents may have offered greater ecological validity, most would have been recent learners of symbolic algebra and may not have achieved the desired level of fluency. Although a rigorous screening process was an option, it might have led to sample representativeness problems. For these reasons, we opted to use an adult sample.

Although fMRI offers many advantages, it also imposes certain restrictions. First, to minimise movement inside the scanner, the modality of response is highly constrained. In a typical fMRI study, participants respond by key-press only. Second, to isolate and identify the brain activity associated with specific cognitive processes, tasks are usually kept short, typically in the order of seconds. Both features impose constraints on the investigations of processes involved in solving algebraic problems which, under naturalistic conditions, are often assisted by access to pen-and-paper and more liberal time constraints. To overcome these problems, we devised very simple algebraic problems and used a solution validation procedure that can be accommodated to key-press.

1.3 Computing Solutions from Model versus Symbolic Representations

Solving algebraic word problems involve multiple processes. Most information processing models specify at least two stages: problem representation and problem solution (Bobrow 1968; Briars and Larkin 1984; Lewis 1989; Riley and Greeno 1988). Information such as the quantitative relationships between protagonists is first extracted from the word problems. Pre-existing knowledge relevant to the problem is then activated and is integrated with the extracted information. The procedure needed to compute the solution is then planned, followed by computation of solution (Mayer and Hegarty 1996). In our previous study (Lee et al. 2007), we used fMRI to examine differences between the model and symbolic methods in the initial stages of problem solving: problem representation. Participants were asked to transform information from text to either a model or algebraic representation. This was followed by a validation task in which participants were asked to compare the presented representation with the one they had in mind. We found both methods to be associated with activation in the left frontal gyri and the bilateral intraparietal sulci, which have previously been associated with working memory and quantitative processing, respectively. In addition, the symbolic method resulted in greater activity in the posterior superior parietal lobules and the precuneus, which suggested that the symbolic method had greater attentional requirement compared to the model method.

In the present study, we focused on the remaining stages in solving algebra word problems: computing numeric solutions from model or symbolic representations. A number of studies have examined differences in computation or magnitude comparison using symbolic versus non-symbolic stimuli (Ansari 2007; Ansari, Lyons, van Eimeren, and Xu 2007; Piazza, Pinel, Le Bihan, and Dehaene 2007; Venkatraman, Ansari, and Chee 2005; Zago et al. 2008). Ansari et al. (2007), for example, asked participants to make magnitude judgements on quantities depicted by dots versus Arabic numerals. They found bilateral activation in the intra-parietal sulcus for both the symbolic and non-symbolic conditions. Of particular relevance is a study by Venkatraman, Ansari, and Chee (2005). They examined differences in arithmetic computation when addends were presented using either Arabic numerals or arrays of dots. They found the left posterior intraparietal sulcus and bilateral anterior intraparietal sulcus activated in both conditions. In the non-symbolic condition, there was greater activation in the right parietal and frontal cortex. The authors attributed these differences to the novelty and the greater processing demands of the non-symbolic arithmetic task.

Although the role of the intraparietal sulci in magnitude comparison has been widely replicated, more recent studies have emphasised hemispheric asymmetry during the processing of symbolic versus non-symbolic quantities (Cohen Kadosh, Cohen Kadosh, Kaas, Henik, and Goebel 2007; Piazza et al. 2007; see Ansari 2007, for a review). Citing Dehaene’s (1997) and Verguts and Fias’s (2004) models, Piazza et al. (2007) argued that, as a result of schooling and experience, the left intraparietal sulcus may be predominant in symbolic numerosity coding. In contrast, the right intraparietal sulcus remains undifferentiated and is involved in both symbolic and non-symbolic processing.

Although rooted in the literature on symbolic versus non-symbolic numerical processing, comparisons between the model versus symbolic methods were expected to be less pronounced than those found in the symbolic versus non-symbolic literature. Because our study was driven by a naturalistic curricular concern, we gave a lot of emphasis to the ecological validity of our stimuli. The stimuli used in the model condition were similar to those found in schools and consisted of both schematic drawings and alphanumeric labels. As such, stimuli used in the model method were not purely non-symbolic.

To summarise, we were interested in both similarities and differences between the model and symbolic methods. This study expanded on our previous study (Lee et al. 2007) and focused on the later stages of algebraic problem solving: from representation to solution. Similar to our previous findings, we expected both methods to activate extensive areas in the frontal cortex and in the intraparietal sulci. Given previous findings on hemispheric asymmetry (Venkatraman et al. 2005; Piazza et al. 2007), we also examined whether the model and symbolic methods exhibited similar asymmetry. Our previous findings showed that in translating word problems into diagrammatic or symbolic representations, the latter placed more demands in working memory or attentional resources. We examined whether similar differences persisted to the solution production stage when participants were asked to compute a solution from a given model or symbolic representation.

2 Methods

2.1 Participants

Seventeen right-handed volunteers (10 males, 22 to 29 years of age) participated in the study. Participants were chosen based on their grades (Grade A or B) in Mathematics at their Grade 10 and Grade 12 examinations. To improve sensitivity to differences between the model versus the symbolic methods, we reduced individual differences in proficiency on the two methods by screening participants to ensure that they could attain more than 90% accuracy on problems similar to those used in the experiment, and had less than 5% difference in accuracy when using the two methods. All participants gave informed consent and were treated in accordance to applicable ethical guidelines.

2.2 Design

The study was based on a 2 (Method: Model vs. Symbolic) x 2 (Type: Experimental vs. Control) within-subject design. The model experimental condition was designed to replicate the later stages of the algebraic problem solving process. During which, having already created a model representation of information presented in a word problem, participants are faced with the task of generating a strategy for arriving at and computing the solution.

2.3 Materials

In each trial, participants were presented with a model schematic containing two rectangles, labelled J and M. The operands and the relationships between J and M varied across questions. For model experimental condition in Figure 2, for example, J and M are specified as having 31 units together; with J having 9 more than M. In this and all other trials, participants were asked to find the number of units belonging to J. The problems used in the symbolic experimental condition were structurally identical. However, the information was presented as algebraic equations. The example in Figure 2 (labelled SE) specified that J and M owned 38 units together, with J having 12 more units than M.

Eger, Sterzer, Russ, Giraud, and Kleinschmidt (2003) showed that mere exposure to numbers activated cortical areas associated with quantitative comparison. To control for processes not specific to algebraic problem solving, but which are related to the perception of schematics versus alphanumeric information, we designed two modality specific control conditions. In the model control condition, participants were shown schematics similar to those used in the model experimental condition. Participants were shown two numbers, two rectangles, as well as the letters M and J. However, the numbers and rectangles were configured in a manner that was not mathematically meaningful (see Figure 2, condition MC). Participants were asked to look out for and remember the number that was in the same row as the letter J.

The symbolic control condition was constructed with a similar logic. We designed a condition that allowed us to replicate the result of being exposed to alphanumeric and symbolic stimuli, but without participants having a reason or a systematic method to engage in mathematical operations. For the symbolic control condition, we used the same stimuli as the experimental condition, but arranged them in a manner that was not mathematically meaningful. Participants were asked to look out for and remember the number that was on the same row as "J =" or "= J". For the example in Figure 2 (Condition SC), the correct answer is 35. The alternative, 19, is incorrect because there are no "= J" or "J =" in the upper row.

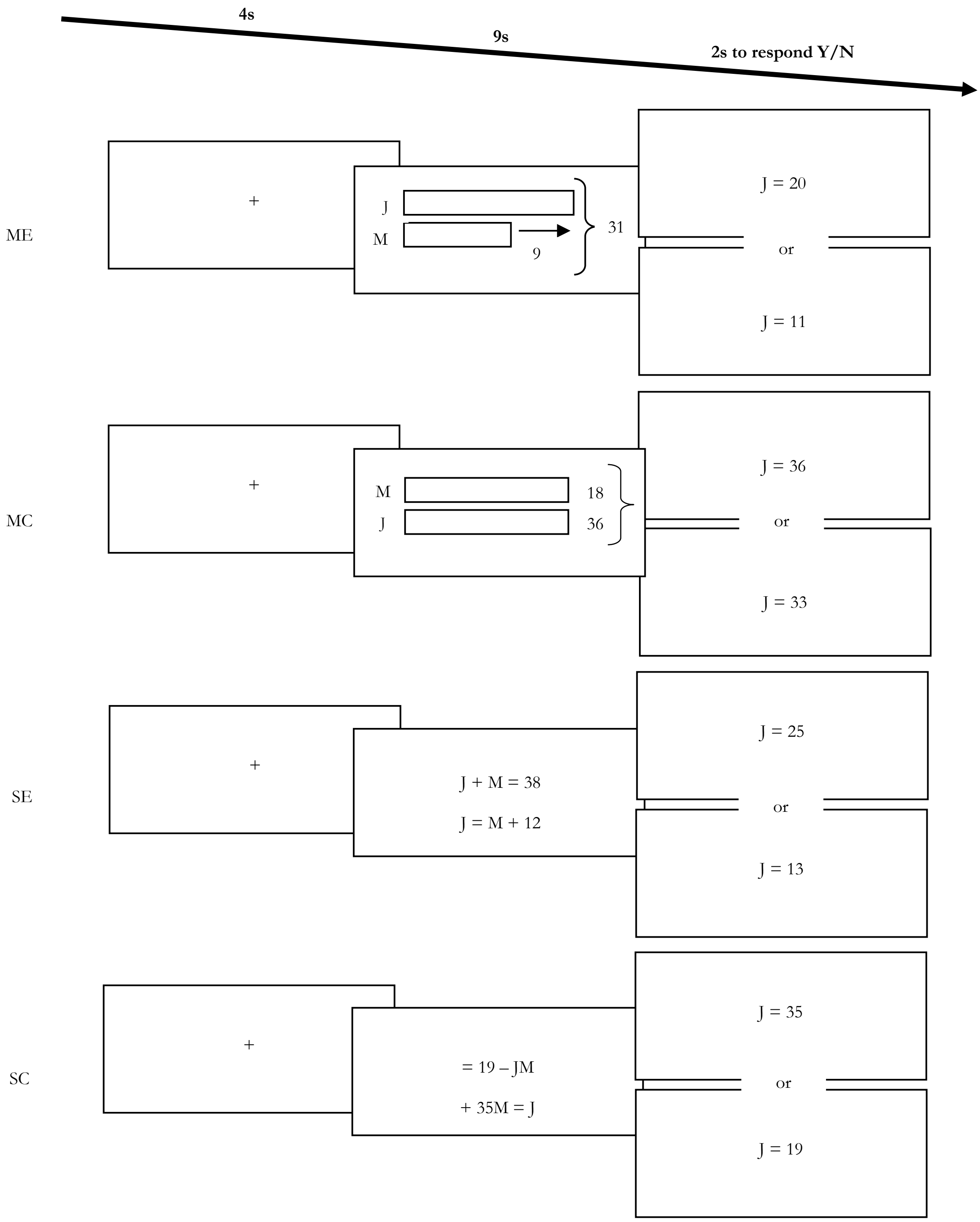


Figure 2. Sequence of stimuli for the four conditions: Model Experimental (ME), Model Control (MC), Symbolic Experimental (SE), and Symbolic Control (SC). Participants first saw a fixation point for 4 s, followed by the problem, which was shown for 9 s. This was followed by the response screen. Participants were given 2 s to validate it against their own answers. Here, the response screen is illustrated with two alternatives (the top slide contains the correct answer). In the experiment, participants were provided with only one response alternative.

2.4 Procedure

Participants were given explanations regarding the tasks and were given practice trials prior to entering the scanner. During the experiment, participants were presented with a total of 144 trials, divided into 6 alternating model and symbolic blocks. Within each block, participants were presented with both experimental and control trials, with no more than 3 consecutive trials of each type. The problem stimulus was presented for 9 seconds in each trial. This was followed by a solution screen. Participants were given 2 seconds to validate the presented solution against their own solution using Yes/No response buttons. The solution screen disappeared on key press and was replaced by a blank screen for the remainder of the response period. Half of the presented solutions were correct and the remainder were incorrect. Two types of errors were used to prevent response set and to increase task difficulty. For the experimental conditions, errors were generated based on either arithmetic computational errors or calculations based on finding M rather than J. For the control conditions, errors were based on either remembrance errors (for the example in Figure 2, condition MC, 33 rather than 36) or remembering the number associated with M rather than J (19 rather than 35 in Figure 2, condition SC).

The final block of the experiment contained a memory task consisting of 24 trials. The memory task was included to examine the extent to which activations in the experimental conditions were attributable to mnemonic activities. In each trial, participants were presented with a series of letters, ranging from 3 to 5 letters in length, one at a time. The letters were presented in one second slots arranged randomly within a 9 second presentation window. They were then asked to indicate whether a probe letter, presented after the letters, was part of the original series. Similar to the other conditions, participants responded by Yes/No button press.

2.5 Imaging Protocol

We used a Siemens 3T Allegra system to acquire functional and structural brain images. The functional images provide activation data with high temporal resolution, but poor spatial resolution. To allow us to identify anatomical details, the functional images are supplemented with structural images with high spatial resolution. In each functional acquisition cycle, 36 transverse slices, approximately parallel to a transverse plane formed by the anterior to the posterior commissure (AC-PC) were acquired with an interleaved gradient-echo echo planar imaging sequence. The imaging parameters were as follows: repetition time (TR) = 3000 ms, time to echo (TE) = 30 ms, pixel matrix = 64 × 64; field of view = 192 x 192 mm; slide thickness = 3mm, 0.3mm gap. High resolution co-planar T2 structural images were acquired in the same orientation using a 3 dimensional magnetisation prepared rapid acquisition gradient echo (3D-MPRAGE) sequence.

2.6 Data Analysis

Functional images were pre-processed and analyzed using Brain Voyager QX (Version 1.9). To improve signal-to-noise, the data were cleaned using a Gaussian smoothing kernel of 8mm full width at half maximum (FWHM), applied in the spatial domain. In addition, a high-pass frequency filter was applied following linear trend removal. Because the functional images have lower spatial resolution than the structural images, the functional images were aligned to the co-planar high resolution T2 images to aid identification of anatomical landmarks. The image stacks were then aligned to form high resolution 3D images of the brain. There are significant individual differences in brain anatomy. To allow us to compare activation across individuals, the resulting data were transformed into Talairach space: a standardised 3-dimensional Cartesian co-ordinate system for the brain.

The functional data were analysed with a general linear model. Each trial was modelled using nine finite impulse response predictors, spanning a total of 27s from trial onset, with each predictor or timepoint corresponding to 3s in the experiment. Given the length of the task and the delay in hemodynamic response to cognitive events, we allowed 11s for the actual task -- this corresponds to the actual duration of the task -- and an additional 16s for the hemodynamic response to decay to baseline. A random effects model based on a 2 (Method: Model vs. Symbolic) x 2 (Type: Experimental vs. Control) analysis of variance was used to identify activations across conditions. A visual inspection of the data showed that activation peaked 9 to 12 seconds after event onset. For this reason, and to allow comparison with our previous study (Lee et al. 2007), we focused our analysis on the fourth time point.

Current fMRI methodology treats each voxel in the brain as an independent data point. With over 100,000 voxels in an analysis, inflation to Type 1 error can be expected. In this study, we controlled for multiple comparison with the false discovery rate (FDR) approach (Genovese, Lazar, and Nichols, 2002), using a threshold of *q(FDR)* = .05.

3 Results

3.1 Behavioural Findings

An analysis of variance based on a 2 (Method: symbolic versus model) x 2 (Trial type: Experimental versus control) design revealed a significant interaction effect on the accuracy scores, $F(1,16) = 7.89$, $p = .01$, partial $\eta^2 = .33$. Follow-up *t* tests showed that participants were less accurate in the symbolic experimental condition than in the model experimental condition. There were no differences in accuracy across the two control conditions. We also conducted a similar analysis on the reaction time of trials that were answered correctly. The results showed a cross-over interaction effect, $F(1,16) = 5.40$, $p = .03$, partial $\eta^2 = .25$. However, follow-up *t* tests revealed no simple main effects between the experimental versus control conditions, nor between the symbolic versus model conditions (see Table 1 for means and standard deviations). Only data from the accurate trials were used in the fMRI analyses.

Table 1.

Mean accuracy (proportional score) and reaction times (ms) for all conditions

Conditions	Mean accuracy (*SD*)	Mean RT (*SD*)
Model experimental	.96 (.05)	740.33 (134.20)
Symbolic experimental	.89 (.07)	782.22 (120.35)
Model control	.99 (.02)	748.82 (131.93)
Symbolic control	.99 (.02)	751.98 (120.35)

3.2 fMRI Findings: Similarities between the Model and Symbolic Methods

To locate similarities between the model and the symbolic methods, we conducted a conjunction analysis involving all four conditions: [Model Experimental (ME) > Model Control (MC)] and [Symbolic Experimental (SE) > Symbolic Control (SC)]. To control for differences between the two methods that can be attributed to differences in the modality of the stimuli (e.g., perceiving or reading symbolic versus schematic stimuli), differences between the experimental and control conditions were first estimated for each method using a random effect analysis of variance. We then identified areas that were activated by both methods.

The data revealed extensive activation in the medial frontal gyrus and in the bilateral middle frontal gyri, extending from the precentral sulcus to the anterior section of the middle frontal gyrus. The basal ganglia, extending bilaterally into the anterior sections of the insula, were also activated. We also found bilateral activation along the intraparietal sulci that extended into the precuneus and the posterior superior parietal lobules (see Table 2 for details).

Table 2

Talairach coordinates of activation maxima common to both the symbolic and model methods: (SE > SC) and (ME > MC)

Brain Regions	Talairach Coordinates							
	Left Hemisphere				Right Hemisphere			
	X	*Y*	*Z*	*t*	*X*	*Y*	*Z*	*t*
Medial frontal gyrus (BA 6)	-2	12	44	7.03				
Middle frontal gyrus (BA46)					43	29	24	5.60
Middle frontal gyri (BA6)	-31	-2	47	6.46	27	1	47	6.81
Middle frontal gyrus (BA10)	-37	41	7	6.30				
Inferior frontal gyri (BA9)	-48	12	25	6.68	45	6	23	5.88
Superior parietal lobules (BA 7)	-31	-55	43	7.11	27	-57	43	6.82
Basal ganglia	-22	4	8	6.81	22	13	4	6.54
					11	-12	7	6.19
Cerebellum	-32	-59	-24	5.95	21	-66	-19	6.18
Inferior temporal gyrus (BA 20)	-50	-52	-10	5.86				

Note. Because the extent of activation at $q(FDR) < .05$ does not allow for clear demarcation of regions of interest, activation maxima and *t* values were harvested at a more stringent level, $q(FDR) < .005$. ME = model experimental, MC = model control, SE = symbolic experimental, SC = symbolic control. Talairach coordinates refer to the distance of activation maxima from the anterior commissure: in x, y, and z axes, measured in mm.

3.3 fMRI Findings: Differences between the Model and Symbolic Methods

To identify differences between the model and symbolic methods, we conducted a random effect analysis of variance based on a 2 (Method: Model vs. Symbolic) x 2 (Type:

Experimental vs. Control) design. We focused on voxels affected by a main effect associated with method and voxels affected by a signification interaction effect.

3.3.1 Main effects

The analyses revealed bilateral areas in the lingual gyri where the model method activated more than symbolic. Using a cluster threshold of $50mm^3$, we identified 13 other clusters that exhibited main effects (see Table 3). These included bilateral activation in the superior parietal lobules extending to the intraparietal sulcus on the left hemisphere, bilateral middle frontal gyri, bilateral precentral, left medial frontal gyrus, and left caudate. The clusters form a subset of areas identified in the conjunction analysis.

Table 3

Talairach coordinates and averaged F values of clusters exhibiting a main effect difference between the symbolic and model conditions

Brain Regions	Talairach Coordinates							
	Left Hemisphere				Right Hemisphere			
	X	*Y*	*Z*	*F*	*X*	*Y*	*Z*	*F*
Inferior Frontal Gyrus (BA 9)	-48	6	25	19.74				
Middle Frontal Gyrus (BA6)					27	-3	45	18.21
Medial Frontal Gyrus (BA 6)	-8	9	48	21.24				
Precentral gyri (BA6)	-27	-2	47	8.21	44	4	28	1.72
Superior parietal lobules (BA 7)	-33	-55	43	21.03	26	-60	43	21.06
Basal ganglia	-10	4	0	19.59				
	-17	11	5	19.60				
Inferior occipital gyri (BA 18)	-26	-94	-2	20.00	28	-89	-1	29.26

	-44	-75	0	18.62				
Cuneus(BA 18 bilateral)	-16	-84	20	19.75	4	-75	7	24.66

Data from these clusters were extracted to the Statistical Package for the Social Sciences (SPSS), in which we examined whether the main effect was due to activation in the model or symbolic condition. With the exception of clusters in the cuneus, all others revealed stronger activation in the symbolic than in the model condition. It should be noted that areas identified in this analysis are those in which both symbolic conditions -- experimental and control -- activated more strongly than the model condition (see Figure 3). Thus, they reflect a general modality based effect: differences were the results of both non-mathematically related processing and processes resulting from having to compute from the two representations. To identify areas activated uniquely by mathematically related processing in the two formats, we examined the interaction effects.

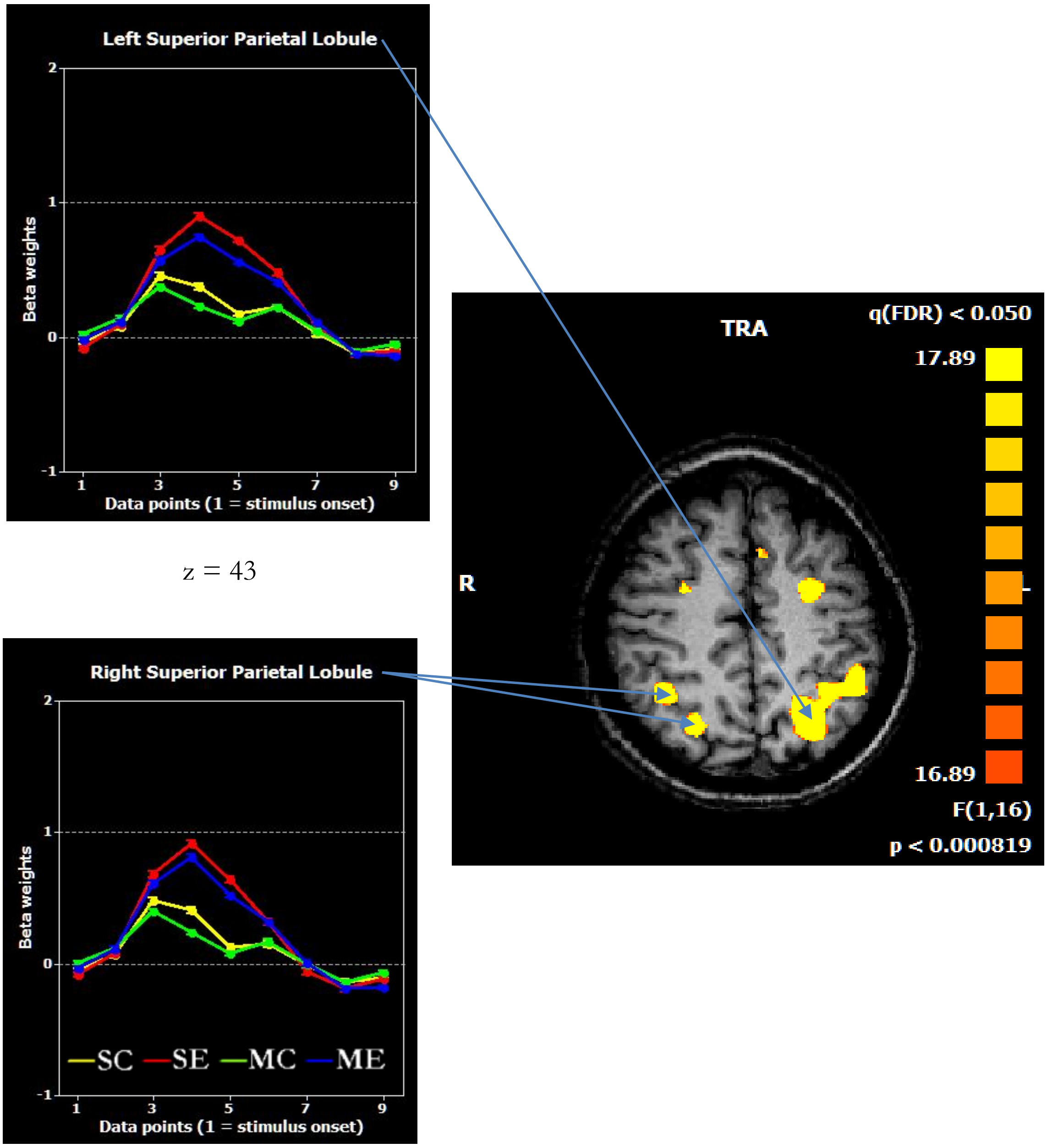


Figure 3. A transverse (TRA) slice showing several areas affected by the main effects involving the [Symbolic Experimental (SE) + Symbolic Control (SC)] > [Model Experimental (ME) + Model Control (MC)]. The slice is taken at 43 mm dorsal to the AC-PC plane (z = 43). A threshold of *q(FDR)* < .05 was used to determine whether a voxel was activated. The left side of the slice represents the right (R) side of the brain.

3.3.2 Interaction effect

Using a cluster threshold of 50mm^3, we identified 13 clusters that exhibited interaction effects. These included bilateral activation in the middle frontal gyri, inferior frontal gyri, precuneus, and the basal ganglia (see Table 4). Data from these clusters were extracted to SPSS, in which we conducted pairwise *t* tests to identify sources of interaction.

Table 4

Talairach coordinates, averaged F values of all activated voxels, and t values from follow-up tests on clusters affected by an interaction effect involving the experimental and control trials of the symbolic versus model conditions

Brain Regions	Talairach Coordinates				Follow-up *t* tests			
	X	*Y*	*Z*	*F*	ME>MC	SE>SC	SE>ME	SC>MC
Right middle frontal gyrus (BA 9)	36	20	29	17.48	1.50	4.70***	3.15**	-.53
Left middle frontal gyrus (BA 10)	-34	52	7	17.59	4.19**	8.19***	6.33***	.55
Right superior frontal gyrus (BA 10)	29	50	14	19.16	2.60*	5.80***	2.35*	-1.63
Right inferior frontal gyrus (BA 11)	25	33	-17	21.30	1.02	5.32***	3.45**	-1.26
Left inferior frontal gyrus (BA44)	-49	10	17	20.75	6.20***	8.52***	5.49***	.48
Right postcentral gyrus (BA 2)	56	-21	45	17.89	3.19**	-.31	-1.17	1.88

Bilateral anterior cingulate (BA 24)	4	26	22	22.42	1.53	5.55***	3.02**	-.98
Bilateral precuneus (BA 7)	-1	-60	38	20.36	2.35*	4.45***	4.42***	-.40
Right supramarginal gyrus (BA 40)	46	-47	35	21.41	.08	2.66*	1.59	-1.50
Basal ganglia	1	-1	10	20.46	4.91***	7.31***	4.98***	-.31
Left lingual gyrus (BA 18)	-15	-63	8	17.49	1.91	4.33**	-2.24*[1]	-4.83***[2]
Medial and Right cerebellum	6	-64	-19	19.83	5.95***	7.89***	3.75**	-.15
Left cerebellum	-30	-67	-22	16.36	4.86***	6.14***	3.14**	-.24

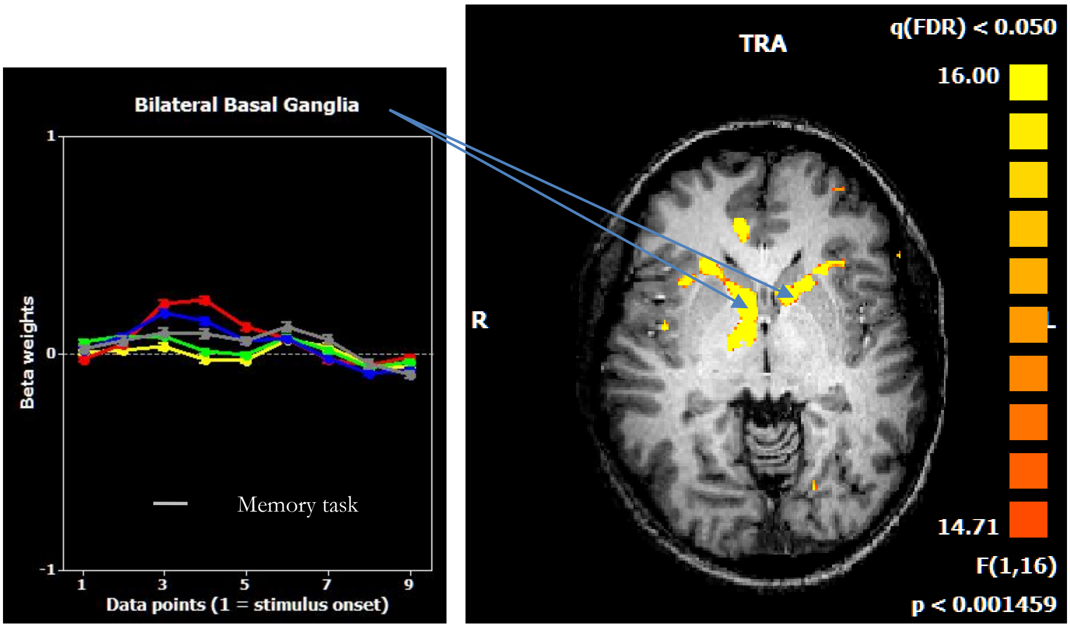
Note. ME = model experimental, MC = model control, SE = symbolic experimental, SC = symbolic control

[1]Significant but in the opposite direction, i.e., ME > SE

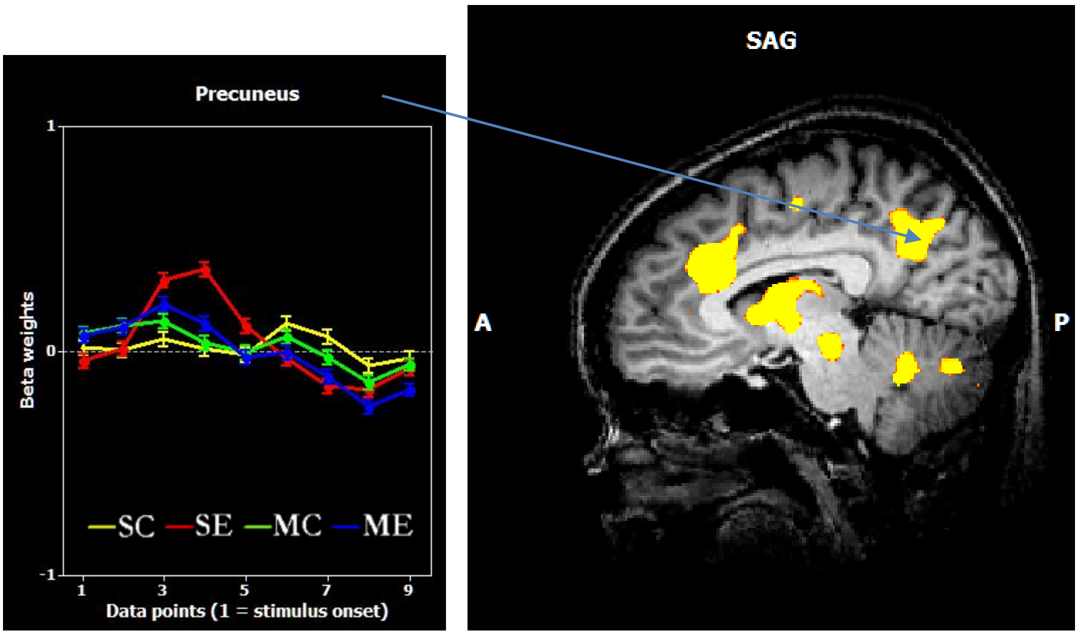
[2]Significant but in the opposite direction, i.e., MC > SC

* < .05, ** < .01, *** < .001

In most areas, the symbolic experimental condition activated more strongly than did the model experimental condition. The symbolic experimental condition was also associated with stronger activation than its control (see Figure 4). With the exception of the lingual gyrus, there was no significant difference between the two control conditions. These findings show that in most areas, the interaction effects resulted from larger differences between the symbolic conditions (SE > SC) than in the model conditions (ME >MC).



z = 1



x = 6

Figure 4. A transverse (TRA) and a sagittal (SAG) slice showing several areas affected by an interaction effect involving the Model Experimental (ME), Model Control (MC), Symbolic Experimental (SE), and Symbolic Control (SC) conditions. The transverse slice is taken at 1 mm dorsal to the AC-PC plane (z = 1). The sagittal slice is taken 6 mm to the right of the medial line (x = 6). A threshold of *q(FDR)* < .05 was used to determine whether a voxel was activated. The left side of the transverse slice represents the right side of the brain. The left side of the sagittal slice represents the anterior of the brain. A = Anterior, P = posterior.

In the lingual gyrus, both model conditions activated more than the symbolic conditions. Data from the postcentral gyrus exhibited a crossover interaction effect with only differences between the model experimental and model control attaining significance. The right supramarginal cortex also exhibited a crossover effect, with only the symbolic experimental condition activating more than the symbolic control condition. However, in neither of these areas were differences between the two experimental conditions significant.

3.4 fMRI Findings: Relationship with Mnemonic Processes

In a previous study, the basal ganglia were found activated in a symbolic condition similar to that used here (Lee et al. 2007). Here, we also found activation in the basal ganglia. It has been argued that the basal ganglia are sensitive to both mnemonic activities and the retrieval of particular types of proceduralised information (Anderson 2005). To examine whether the present finding can be attributed to mnemonic activities, we conducted a conjunction analysis in which we examined areas activated in both the memory tasks and the experimental conditions, that is, (a) Memory & (ME+MC) > (SE+SC) and (b) Memory & (SE+SC) > (ME+MC). Review of time courses obtained from the basal ganglia showed that the activation peak for the memory task was later than those found in the other conditions. The conjunction was conducted using data from the sixth timepoint of the memory task and the fourth timepoint of other conditions. These analyses showed no basal ganglia involvement in our memory task.

4 Discussion

This study builds on our previous study (Lee et al. 2007) in which we found transforming word problems into a symbolic representation activated areas associated with attentional or working memory processes more than did transforming the same problems into a model representation. Our present findings bear on the latter stages in the problem solving process: computing numeric solutions from either symbolic or model representations. Participants were given structurally identical questions expressed in either a symbolic or a model format. Similar to our previous study, we selected only participants who were competent in solving algebra problems and who exhibited similar behavioural competencies at screening. As expected, there were both similarities and differences. Both methods activated extensive areas in the middle and medial frontal gyri. Also activated were large areas in the parietal lobes: along the intraparietal sulci extending to the posterior superior regions. Both the basal ganglia and cerebellum were also activated. The symbolic method resulted in greater activation in the right superior frontal gyrus, bilateral anterior cingulate, and the precuneus. We discuss these findings in greater details below.

Consistent with previous findings, the intraparietal sulcus was activated bilaterally in both the model and symbolic experimental conditions. Dehaene and his colleagues argued that one of the main functional characteristics of the intraparietal sulcus is to serve as a mental number line (Dehaene, Piazza, Pinel, and Cohen 2003). In previous studies, it was found to be active in both symbolic and non-symbolic numeric processing (Fias, Lammertyn, Reynvoet, Dupont, and Orban 2003; Piazza, Izard, Pinel, Le Bihan, and Dehaene 2004). Venkatraman et al. (2005), in particular, found the intraparietal sulcus activated when participants were asked to engage in arithmetic computation. In their study, they found computation carried out on non-symbolic stimuli activated the right parietal lobe more than did computation with symbolic stimuli. In the present study, intraparietal sulcus activation was found in both the model and symbolic conditions and was likely related to participants' engagement in arithmetic computation: a necessary final step in solving algebraic problems.

Notably, we also found the intraparietal sulcus activated more strongly in the symbolic than in the model condition; suggesting that having to compute solutions from symbolic representations drew on more numeric processing resources than does computing from model representations. There was also some evidence of hemispheric asymmetry (see Figure 3). In the intraparietal sulci, activation in the symbolic condition was more extensive in the left hemisphere (6153 voxels) than in the right (2637 voxels). This finding parallels those found in other studies. Piazza et al. (2007), for example, argued that because of schooling and experience, the left intraparietal sulcus might be predominant in symbolic numerosity coding. In the present study, the areas activated in the left parietal lobe included the angular gyrus. It is possible that participants used different strategies across the two methods. Because symbolic algebra is utilised in different topics in mathematics, participants' experience with symbolic algebra in these different contexts may have led to a greater reliance on processing related to mathematical fact retrieval (e.g., Grabner et al. 2009a, b). Another possibility is that the left dominance stems from the more language-like presentation of algebra equations.

Although both explanations are speculative, the latter, orthographically based explanation seems more likely. The finding of left dominance came from tests for main effect in the analysis of variance. Because the area was not affected by an interaction effect, the finding of hemispheric asymmetry is not specific to the experimental condition. The lack of an interaction effect is not consistent with the mathematical fact retrieval explanation. If added familiarity or experience with symbolic algebra led to greater reliance on fact retrieval, its effect should only be observed in the symbolic experimental, but not the symbolic control condition. The lack of specificity over the experimental and control conditions suggests that a modality based explanation, such as the orthographic explanation, is more likely.

Other areas activated by both the model and symbolic conditions are similar to those found in our previous study (Lee et al. 2007). Both conditions activated extensive areas in the bilateral middle frontal gyri extending into the medial frontal gyri and the frontal poles. These frontal areas are associated with working memory and were characterised by Owen, McMillan, Laird, and Bullmore (2005) as being involved in assimilation or the reorganisation of information into pre-existing knowledge structures. These are likely to be key processes in solving algebra problems. Participants had to draw on what they know about algebra equations or model representations to understand the questions and to extract the appropriate information necessary for computing the solutions. This finding is consistent with related work from the developmental literature, which shows strong associations between measures of working memory capacity and proficiency in solving algebraic word problems (Lee, Ng, Ng, and Lim 2004; Lee, Ng, and Ng 2009).

Although both the symbolic and model conditions activated the middle frontal gyrus and precuneus, there is also an area inferior to the commonly activated area in the precuneus that exhibited an interaction effect. Follow-up tests showed that relative to their respective control conditions, the symbolic condition activated more strongly than did the model condition. The precuneus has been associated with the execution of complex arithmetic tasks (Zago and Tzourio-Mazoyer 2002) and goal directed shifts of attention (Behrmann, Geng, and Shomstein 2004). In Dehaene's tripartite framework, the precuneus was classified as a part of a parietal number processing circuit that fulfilled attentional selection and orientation functions (Dehaene et al. 2003). In our previous study (Lee et al. 2007), the precuneus was also found more strongly activated in the symbolic than the model condition. In addition, there are areas in the bilateral middle frontal gyrus, left medial frontal gyrus, right superior frontal, and anterior cingulate in which the symbolic condition activated more strongly. A recent study on arithmetic calculation showed that the executive demands of numerical manipulation are subserved by a network involving the anterior cingulate, the orbital part of the inferior frontal gyrus, and the caudate (Zago et al. 2008; see also Stocco and Anderson 2008). Taken together, these findings suggest that, compared to the model method, additional attentional and executive resources are required both in translating a word problem into an algebraic equation and in generating a numerical solution from that equation.

The symbolic experimental condition was also associated with stronger activation in the basal ganglia. The basal ganglia have been associated with working memory in general and as part of a centralised communication and control relay in the ACT-R model (Anderson 2005). More recent studies suggest that it is part of a network that supports the executive demands of numerical processing (Ischebeck, Zamarian, Egger, Schocke, and Delazer 2007; Stocco and Anderson 2008; Zago et al. 2008). We conducted a conjunction analysis to examine whether the basal ganglia activity was similar to those associated with a simple memory maintenance task. The analysis revealed no overlapping activation in the basal ganglia. Given the nature of our memory task, this finding allows us to conclude that the basal ganglia activation, seen in the symbolic condition, is not likely a result of simple encoding, maintenance, and retrieval. However, more specific hypotheses regarding the retrieval of procedural memory or involvement in executive control remain viable.

Only in the cuneus and superior occipital gyrus did the model method activate more than the symbolic method. In some of these areas -- the superior occipital, the lingual gyrus, and the caudal sections of the inferior occipital lobules, both the experimental and control trials of the model method activated more than the symbolic conditions. In the absence of an interaction effect, these findings suggest that differences are due not to having to carry out computation based on model representations, but to generic processes associated with processing models and model-like schematics. These findings suggest that participants spent more time viewing the stimuli in both model conditions, possibly because it requires more visual perceptual resources to decode stimuli in these conditions.

4.1 Conclusions

Both the model and symbolic methods activated areas in the frontal gyri and intraparietal sulcus. Differences were also found in the precuneus and the basal ganglia. In all four locations, the symbolic method activated more strongly than did the model method. Findings from the frontal gyri and the precuneus are consistent with findings from our previous study (Lee et al. 2007). In that study, participants were asked only to translate algebra word problems into either equations or model representations. Problems were structured so that computation of the solutions was neither necessary nor possible. In contrast, computation is explicitly required in the present study. Despite these differences, the finding that very similar areas in the frontal gyri and precuneus were activated suggests that the symbolic method is more demanding on attentional or working memory resources.

Unlike our previous study (Lee et al. 2007), having to compute from symbolic versus model representations is associated with differential activation in the intraparietal sulcus. The finding suggests that processing from symbolic representations draws on more numeric processing resources than did processing model representations. There was also some evidence of hemispheric asymmetry, suggesting that linguistic processes play a more prominent role when processing symbolic stimuli.

One caveat to our interpretations is that the behavioural findings showed that participants in the symbolic experimental condition were less accurate than were participants in the model condition. Is it possible that the activation differences merely reflect this difference in task difficulty? Several factors mitigate against this interpretation. First, the activation findings were generated from only trials on which participants were able to solve the task successfully. Nonetheless, it is possible that participants can find using a particular method more difficult, but still use it successfully. However, the finding that there were no significant differences in reaction time between the two experiment conditions suggest that the findings are not greatly affected by the ease with which the accurate items were solved. Though of less direct relevance, it should also be borne in mind that the overall accuracy on both tasks were high (.89 in the symbolic experimental condition and .96 in the model experimental condition). As such, the differences were found despite participants not experiencing major difficulty with either method. Future studies may want to consider using an even more stringent inclusion criterion in sample selection.

Another limitation of our study is that, in trying to preserve ecological validity, we used a task that is more complex than is customary in fMRI studies. We are mindful that multiple processes are likely to occur as problem solvers compute solutions from a model or symbolic representation. The design of our tasks does not allow us to say which of these processes are responsible for the observed differences.

4.2 Curricular Implications

We examined whether the model and symbolic methods are supported by different cognitive processes and whether the two methods imposed different cognitive demands. One of the questions that motivated this enquiry is whether the symbolic method can or should be introduced to children in upper primary schools. Given children's ready engagement with the model method, it is understandable for curriculum designers to explore whether they can be introduced directly to the symbolic approach and avoid the confusion that is sometimes associated with having multiple strategies for solving the same kind of problem. In this study, we focused on only one aspect of these concerns and examined whether the two methods imposed similar cognitive demands on problem solvers. In drawing some tentative curricular implications, we are mindful that a resolution will require a fuller evaluation of related motivational, pedagogical, and policy concerns.

The findings suggest that the symbolic approach is more effortful even amongst competent adult problem solvers. Furthermore, there are some suggestions that linguistic processes play a more prominent role when processing symbolic stimuli. Both findings give us pause in considering whether symbolic algebra should be introduced earlier. Nonetheless, such reservations should be tempered by several caveats. First, the present finding provides but one piece of the puzzle. As stated earlier, curricular decisions require the consideration of a multitude of factors. Second, although the tasks we used are complex relative to tasks typically encountered in fMRI studies, they are simple in the context of schools. Previous findings suggest that the intraparietal sulcus is sensitive to the complexity of the computational task (Zago et al. 2001).

Our findings, especially with respect to the hemispheric asymmetry in the intraparietal sulcus may well be moderated by such considerations. Third, our participants were adults competent in both methods. It is tempting to argue that if competent adults have to exert more resources on the symbolic method, it must be more so for children. However, it must be borne in mind that such relationships need not be linear. Even for simpler numerical processing, recent findings show that there are developmental differences in both the pattern and extent of activation between children and adults (Davis et al. 2009; Kaufmann et al. 2008; Kucian, von Aster, Loenneker, Dietrich, and Martin 2008; Rivera, Reiss, Eckert, and Menon 2005). Whether findings will extend to children who are learning school algebra will require further investigations.

With these caveats in mind, our findings provide some tentative pointers that may be of significance for curriculum development. Many primary teachers know of the equivalence between the two methods, namely that a rectangle used in the model method is a placeholder which can be replaced by the letter *x,* however, they do not see that their work with the model method help prepare children for work with symbolic algebra (Ng, Lee, Ang, & Khng, 2006). Sharing our findings with primary teachers highlights to them the significance and the importance of their work with the model method: that the drill and practice questions they provide to children are engaging many of the same processes as symbolic algebra.

The findings from this study also have significant implications for the practice of secondary teachers. In a previous study, a majority of secondary two students found the model method easier because it provided them a means to visualise the problem and it makes "understanding of the problem easier" (Ng, 2003, p. 13). Our present findings also showed the model to be "easier". Even amongst adults with similar competency in the two methods, more attentional or working memory resources were recruited in the execution of the symbolic method. If these findings generalise to learners of algebra, they suggest that learners will benefit from pedagogical strategies that reduce the attentional demands of symbolic algebra. From this perspective, using familiar representations to construct new representations has the potential to promote learning. Examining the effects of different ways to present algebraic problems, Koedinger and Nathan (2004) concluded that the use of more familiar representations can assist in meaning making and promote self-monitoring. Being able to relate new information to old may also reduce working memory requirements by giving access to pre-existing procedures and knowledge structures that are shared between the two methods.

Naturally, such strategies will not solve all the obstacles associated with solving algebraic problems. Given an algebraic equation, students must also be provided with practice on how to construct a system of equivalent equations that will eventually result in the resolution of equations (Steinberg, Sleeman, & Ktorza, 1991). However, in light of our findings, secondary algebra students who have previously been exposed to the model method should perhaps be asked to construct relevant model drawings, from which teachers can highlight the role of rectangles as placeholders for variables, which could be replaced with letters. Students can then be asked or be shown how such pictorial representations can be transformed into algebraic equations, and vice versa.

4.3 Summary

To conclude, there is some scepticism amongst classroom practitioners as to whether fMRI related research can inform practice. In relation to the teaching of algebra, the model method was thought to provide children better access to algebra because it is less abstract and more visual than symbolic algebra. Our findings offer new insights on the reasons why many students find the model method easier. Contrary to expectation, we found no evidence that it relies more extensively on visual processes than does the symbolic method. Instead, we found that it imposes greater demands on attentional resources. Based on our findings, we offered some speculation on how teachers can make use of students' prior knowledge of the model method to serve as a bridge to formal algebra.

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