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Facilitating Sense-Making in Primary Mathematics Through Word Problems

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This paper is based on the pilot study of an on-going research project that aimed to encourage children to engage in sense-making through mathematics word problems. About 400 Primary Three (Year Three) children in one school were involved in an instructional programme *Think-Things-Through* for six months. The two key characteristics of the programme were (1) children engaging in 'what-if' problem posing to extend textbook problems, and (2) children being exposed to word problems that required contextual knowledge for successful solution. The research questions were (1) Were children more able to make-sense when they solved word problems at the end of the programme? (2) Were children able to transfer this ability to solve problems that were not similar to those used in the programmes? A pre-test and a post-test were administered. The tests comprised items in three categories: *programme items* (essentially the same as those used in the programme), *near-transfer items* (similar to those used in the programme but required children to extend their thinking) and *far-transfer items* (not included in the programme). The problems were paired such a *standard* problem was paired with a *non-standard* one. A *standard* problem could be solved by the direct use of a routine procedure. A *non-standard* one required some consideration of contextual factors for successful solution. The findings of the pilot study indicated that children showed improvement in handling *non-standard* items that had been dealt with in the programme. The children were able to transfer this ability to a limited extend. Children were unable to solve *non-standard* items that were not discussed in the programme. The pilot study also provided clear indication that success in *standard* problems was no indication that children engaged in sense-making during problem solving.

Introduction

Word problems have long constituted a major part of school mathematics. A variety of reasons have been put forth to justify this privileged position. Verschaffel, Greer and De Corte (2000) presented a list of such reasons which includes providing opportunities for students to use mathematical tools, motivating the link between mathematics and real-life context, encouraging thinking and the use of problem solving heuristics, and providing a platform to develop new concepts and skills. Several critics (Lave, 1992; Gerofsky, 1996) have questioned such justifications. Gravemeijer (1997) has even suggested that word problems are often mere disguise for practice in the four basic operations. Studies from around the world (cited in Verschaffel, Greer & De Corte, 2000) have indicated that children often do not attempt to make sense of contextual factors when they solve word problems. This phenomenon has been described as the 'suspension of sense-making'. Another line of investigations on everyday cognition demonstrated that people who are successful in using mathematics in their everyday professional and social activities are not so successful in the same mathematics when given in a school context (Scribner, 1984; Lave, 1988; Carraher, Carraher & Schliemann, 1985).

Verschaffel, Greer and De Corte (2000) have called for the re-conceptualisation of the role of word problems in school mathematics. They have suggested the use of word problems to engage students in mathematical modelling. In other words, students solving word problems should engage more in making sense of the semantics of the problem and less in doing tedious computations. A new international assessment framework (OECD, 1999) has defined 'mathematical literacy' as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen." (p.41). Charles and Lobato (1998) has described a 'numerically powerful' child as one who is able to develop meaning for numbers and operations, able to look for relationships among numbers and operations, able to understand computational strategies and uses them appropriately and efficiently, and able to make sense of numerical and quantitative situations.

In Singapore schools, there has been a call for an increased emphasis on thinking and sense-making. Thinking skills are explicitly taught to students and infused into content areas. The increased use of technology in the mathematics classroom also means a decreased emphasis on tedious computations. This paper describes a pilot project that used word problems as a tool to engage children in thinking and sense-making. The first part of the paper describes the key elements and features of the programme. The second part of the paper describes results of the pre-test and post-test taken by a sample of the children who were involved in the project.

Instructional Programme

This section describes the key features of the instructional programme *Think-Things-Through* and provides some examples of word problems used during the programme. The two key features of the word problems used were (1) use of 'what-if' technique to extend textbook problems, and (2) the need for consideration of contextual factors. The application of a procedure or an algorithm was necessary but not sufficient to yield successful responses. Such a consideration of contextual factors during word problem solving is referred to as 'sense-making'. The main purpose of the lessons was to shift the children's attention from a preoccupation with carrying out algorithms to choosing suitable techniques. Three of the problems used during the programme are used to illustrate some features of the programme. The three problems are referred to as *Squares*, *Wires* and *Cities*.

In *Squares,* children were asked to make unit squares using a given number of toothpicks. The main problem was to make four unit squares using 16, 13 and 12 toothpicks, each having a unit length. The worksheet for this lesson and the accompanying teacher's notes is included in the appendix. These problems aimed to include situations that could not be resolved by the mere application of a routine operation. In this case, the application of division was necessary but not sufficient. Children had to make sense of what the remainders mean. The children were encouraged to use problem solving heuristics such as act-it-out and draw-a-diagram. Also, they were encouraged to solve the problem in more than one way. The concept of perimeter was learnt through a series of actions, employing psychomotor intelligence. This problem illustrates how thinking skills (problem solving heuristics) and a thinking disposition (solving problem in different ways) were important components of the programme.

In *Wires* (Figure 1), a pair of superficially similar problems aimed to include a situation that could not be resolved by the mere application of a routine operation. In this case, the children not only had to make sense of what the answers meant but they also had to use a different procedure to solve the *non-standard* problem. In *Wires*, although division could be used to find the number of pieces, the same operation was not suitable to determine the number of cuts. Other strategies such as drawing-a-diagram or acting-it-out were more suitable.

David cut a wire 918 m long into pieces. Each piece was 9 m long. How many pieces did he get?	David cut a wire 918 m long into pieces. Each piece was 9 m long. How many cuts did he make?

Figure 1 An example of a pair of problems used in the programme

In *Cities*, children were asked to determine possible distance between two cities. Figure 2 shows the problems used in the worksheet. This problem aimed to include a situation that extended the use of mathematics in a one-dimensional world to a two-dimensional one. The situation demonstrated the insufficiency of arithmetic operations in a two-dimensional world. The children were encouraged to use a problem solving heuristic (draw-a-diagram) and different mathematical tools (computing and measuring). Again, thinking skills and disposition were given emphasis.

Figure 2 Another example

Once upon a time, there were three cities, Astro, Bistro and Castro. There was a clever, blue bird. His name was, well, Clever Blue Bird. Clever Blue Bird delivered messages between cities. Clever Blue Bird always flew the shortest distance. He always flew in a straight path. Astro was 325 km from Bistro. Bistro was 270 km from Castro. How far was Astro from Castro?

of a problem used in the programme

The instructional programme encourages children to engage in sense-making during problem solving in several ways. In some problems, the results of the computations had to be considered (for example, *Squares*). In other problems, operations had to be selected. Some operations were not suitable (for example, *Wires*). In yet other problems, assumptions had to be considered (for example, *Cities*). In all the problems, contextual factors had to be considered to obtain a successful or complete set of responses. A range of tools to facilitate thinking was used. Strategies were not limited to just abstract arithmetic operations. More concrete heuristics such as acting-it-out and drawing-a-diagram were used frequently. Thinking dispositions such as solving a problem in several ways and questioning assumptions were given emphasis. During the pilot project, the children engaged in these lessons once every three weeks. For the main project, the lessons would be weekly.

Instrument

In the present pilot study, parallel word problem tests, which comprised three types of word problems, were used. There were items that were essentially the same as those in the programme. Any differences were superficial in nature. These are called *programme items.* There were the *near-transfer items* which required concepts similar, but not identical, to those in the programme. There were also *far-transfer items* which required ideas not included in the programme. In each of the three categories, there were *standard* as well as *non-standard* word problems (Verschaffel, Greer & De Corte, 2000). *Standard* word problems could be solved by the direct application of procedures and algorithms. *Non-standard* word problems required children to consider some contextual factors. The use of a routine procedure would be insufficient. Table 1 shows *standard* and *non-standard* word problems in each of the three categories.

Table 1 Test Items

Types of Items	*Standard* Word Problems	*Non-standard* Word Problems	
Programme Items	Nancy uses 3 straws to make a triangle. How many triangles can she make with 18 straws? (S1)	Mei uses 3 straws to make a triangle. How many triangles can she make with 19 straws? (P11)	Lily uses 3 straws to make a triangle. How many triangles can she make with 20 straws? (P12)
Near-Transfer Items	A taxi can take 4 passengers. At least how many taxis are needed to take 24 passengers? (S4)	A taxi can take 4 passengers. At least how many taxis are needed to take 27 passengers? (P41)	A taxi can take 4 passengers. At least how many taxis are needed to take 25 passengers? (P42)
Far-Transfer Items	A printer prints 12 pages in 1 minute. At this rate, how many pages does it print in 5 minutes? (S5)	Ming does 9 sums in 1 minute. How many sums does he do in 5 minutes? (P5)	

The three items classified as *programme items* were structurally the same as those done during the programme. The three triangle problems in the test (S1, P11, P12) were the same as *Squares* described in the previous section. The *near-transfer* items (taxi problems S4, P41, P42) were similar to *Squares* but required thinking that is distinct from that required by the latter. In the taxi problems, children needed to handle the remainder of a division in a different way from what they did in *Squares*. The *far-transfer items* were totally different from problems solved during the programme. Item P5 involved a non-proportionate event. This idea was not included in the programme. There were a total of 14 items in the instrument. In this paper, the findings were based on children's responses to the seven items describes in Table 1.

Subjects

For the purpose of this paper, the analysis was conducted on 75 Primary Three (Year Three) children. The children were from two intact classes. One of the classes (n = 38) comprised children with high academic achievement while the other (n = 37) comprised children with low academic achievement. Those with high academic achievement were in the top quartile of the cohort. Those with low academic achievement were in the third quartile of the cohort. The academic achievement of the children was based on the children's previous year's achievement test scores. The entire cohort of about 390 children was involved in the programme.

Results

The results will be discussed in terms of items types and children's achievement profiles. The children's performance levels in the *standard* word problems were the same in both the pre-test and the post-test. About 60% of the low achieving children and about 90% of the high achieving children were successful in the *standard* problems in both pre-test and post-test. The only exception was problem S5, where the low achieving children did significantly better in the post-test. The children typically had no difficulty in selecting the correct operation and executing the operation accurately.

In the *non-standard* word problems, where the children were required to engage in sense-making, the children were less successful. The percentage of children who were successful in these problems was significantly lower than that in the corresponding *standard* problems. Successful responses to *standard* problems that were not matched by successful responses in *non-standard* ones were clear indications that standard problems were not suitable items to determine if children could make sense of the computation they carried out. In other word, success in *standard* word problems was not a good indicator of children's ability to apply mathematics.

In the *non-standard* problems, there were different levels of incorrect responses. At the lowest level were those who did not respond and this would include children who could not comprehend the problem. At the next level were those who were willing to compromise and engage in suspension of sense-making. These children were using operations almost randomly. Then, there were children who could select the correct operation, performed them correctly but could not make sense of the remainder. These children would either give an answer that is one more than the correct response or stated that no triangles could be made because there was a remainder to the division.

For the low achieving children, success in *programme items* was significant higher at the end of the programme. About 10% of the low achieving children were successful in these items at the start of the programme and about 40% were successful at the end of it. Although inability to comprehend and inability to make sense persisted, several children from these categories were successful at the end of the programme. By comparison, about 60% of the low achieving children were successful in the *standard* problem. Similarly, the high achieving children showed improvement in the *programme* items. About 55% of the high achieving children were successful in these items at the start of the programme and about 80% were successful at the end of it. By comparison, 95% of them were successful in the *standard* problems.

Table 2a Low Achieving Children's Pre-Test[1] Performance on *Programme Items* (n=35)

Item	Correct Response	Correct Method		Use incorrect operations	No response
		Answer that is 1 more	Unable to make sense		

		than the correct answer	of the remainder		
S1	22 (.63)	0	1 (.03)	7 (.20)	5 (.14)
P21	3 (.09)	0	15 (.43)	11 (.31)	6 (.17)
P22	5 (.14)	0	16 (.46)	9 (.26)	5 (.14)

1Two children were absent.

Table 2b Low Achieving Children's Post-Test[2] Performance on *Programme Items* (n=33)

Item	Correct Response	Correct Method		Use incorrect operations	No response
		Answer that is 1 more than the correct answer	Unable to make sense of the remainder		
S1	22 (.67)	0	2 (.06)	7 (.21)	2 (.06)
P21	13 (.39)	4 (.12)	7 (.21)	7 (.21)	2 (.06)
P22	15 (.45)	4 (.12)	7 (.21)	4 (.12)	3 (.09)

2Four children were absent.

Table 2c High Achieving Children's Pre-Test Performance on *Programme Items* (n=38)

Item	Correct Response	Correct Method		Use incorrect operations	No response
		Answer that is 1 more than the correct answer	Unable to make sense of the remainder		
S1	36 (.95)	0	0	2 (.05)	0
P21	20 (.53)	0	12 (.32)	4 (.11)	2 (.05)
P22	22 (.58)	1 (.03)	9 (.24)	4 (.11)	2 (.05)

Table 2d High Achieving Children's Post-Test[3] Performance on *Programme Items* (n=37)

Item	Correct Response	Correct Method		Use incorrect operations	No response
		Answer that is 1 more than the correct answer	Unable to make sense of the remainder		
S1	35 (.95)	0	1 (.03)	1 (.03)	0
P21	29 (.78)	1 (.03)	6 (.16)	1 (.03)	0
P22	30 (.81)	0	6 (.16)	1 (.03)	0

3One child was absent.

Table 3a Low Achieving Children's Pre-Test[1] Performance on *Near-Transfer Items* (n=35)

Item	Correct Response	Correct Method			Use incorrect operations	No response
		Answer that is 1 more than the correct answer	Unable to make sense of the remainder	Others		
S4	24 (.69)	0	1 (.03)	-	4 (.11)	6 (.17)
P41	0	0	18 (.51)	4 (.11)	8 (.23)	5 (.14)
P42	0	4 (.11)	15 (.43)	2 (.06)	8 (.23)	6 (.17)

1Two children were absent.

Table 3b Low Achieving Children's Post-Test[2] Performance on *Near-Transfer Items* (n=33)

Item	Correct Response	Correct Method			Use incorrect operations	No response
		Answer that is 1 more than the correct answer	Unable to make sense of the remainder	Others		
S4	19	0	1	0	7	6

	(.58)		(.03)		(.21)	(.18)
P41	3 (.09)	12 (.36)	4 (.12)	5 (.15)	5 (.15)	4 (.12)
P42	4 (.12)	14 (.42)	4 (.12)	0	6 (.18)	4 (.12)

2Four children were absent.

Table 3c High Achieving Children's Pre-Test Performance on *Near-Transfer Items* (n=38)

Item	Correct Response	Correct Method			Use incorrect operations	No response
		Answer that is 1 more than the correct answer	Unable to make sense of the remainder	Others		
S4	33 (.87)	0	0	0	3 (.08)	2 (.05)
P41	10 (.26)	13 (.34)	9 (.24)	0	3 (.08)	3 (.08)
P42	11 (.29)	10 (.26)	8 (.21)	1 (.03)	4 (.11)	4 (.11)

Table 3d High Achieving Children's Post-Test[3] Performance on *Near-Transfer Items* (n=37)

Item	Correct Response	Correct Method			Use incorrect operations	No response
		Answer that is 1 more than the correct answer	Unable to make sense of the remainder	Others		
S4	35 (.95)	0	1 (.03)	0	0	1 (.03)
P41	15 (.41)	17 (.46)	4 (.11)	0	0	1 (.03)
P42	15 (.41)	17 (.46)	4 (.11)	0	0	1 (.03)

*1 child was absent.

In *near-transfer* items, the number of children who were successful after the programme was only slightly higher than before. Among the low achieving children, while none of them were successful prior to the programme, about 10% were at the end of the programme. Among the high achieving children, about a quarter were successful at the start of the programme and 40% were successful at the end of it. Most encountered difficulty because they were unable to interpret the remainder in a manner that was different from the *programme* items. In the *near-transfer* items, children had to be able to see that the number of vans required should be one more than the quotient of the division, no matter how small the remainder was. This idea was different from the *programme* items where the number of triangles or squares formed is one less than the quotient of the division.

In the *far-transfer* items, children were required to handle one aspect of sense-making that was not included in the programme. The *non-standard* problem involves an activity that occurs at a non-constant rate. Children who engaged in sense-making would not use the method for a constant rate event to solve the problem. They would hesitate to use multiplication.

Table 4a Low Achieving Children's Pre-Test[1] Performance on *Far-Transfer Items* (n=35)

Item	Correct Response	No response	Lack of Sense-making Response	Other Incorrect Response
S5	21 (.60)	6 (.17)	-	7 (.20)
P5	0	8 (.23)	25 (.71)	1 (.03)

1Two children were absent.

Table 4b Low Achieving Children's Post-Test[2] Performance on *Far-Transfer Items* (n=33)

Item	Correct Response	No response	Lack of Sense-making Response	Other Incorrect Response
S5	30 (.91)	2 (.06)	0	1 (.03)
P5	0	4 (.12)	21 (.64)	8 (.24)

2Four children were absent.

Table 4c High Achieving Children's Pre-Test Performance on *Far-Transfer Items* (n=38)

Item	Correct Response	No response	Lack of Sense-making Response	Other Incorrect Response
S5	38 (1.0)	0	0	0
P5	0	1	37 (.97)	0

Table 4d High Achieving Children's Post-Test[3] Performance on *Far-Transfer Items* (n=37)

Item	Correct Response	No response	Lack of Sense-making Response	Other Incorrect Response

S5	37 (1.0)	0	0	0
P5	0	0	33 (.89)	4 (.11)

Generally, none of the children were successful in the *non-standard far-transfer* item at the start and at the end of the programme. It is, however, encouraging to note there was a decrease in number of children who were willing to use multiplication in a non-constant rate event. Those who were unwilling to use multiplication, however, were not successful either as the chose other equally unsuitable operations.

Discussion and Conclusion

Were children more able to make-sense when they solved word problems after the programme? Were children able to transfer this ability to solve problems that were not similar to those used in the programmes? The results of the pilot study seem to suggest that, given that the frequency of programme lessons was low, both low and high achieving children were more successful in problems that required some sense-making after the programme. This success was however limited to problems which were essentially the same as those in the programme. The more dissimilar the problems were from those used in the programme, the less likely they would be successful. This indicated that the transfer of this ability to engage in sense-making during problem solving, to a large extent, did not occur.

While the children were engaged in problems that required them to engage in sense-making during the programme, the frequency of the programme lessons was low (once in three weeks for six months). The other lessons were conducted using textbook problems which were mostly *standard* problems. The children's familiarity with *standard* problem was reflected by the constant high success rate in these problems in both the pre and post tests. We would like to suggest that, in order for children to transfer this sense-making disposition to problems other than those included in the programme, there must be a culture of sense-making. This can only happen if children were exposed to *non-standard* word problems at a more frequent interval. In the main study, children will have at least one programme lesson each week.

It was, however, clear that *standard* word problems were unable to capture if children engaged in sense-making when they solved problems. It was consistently found that success in *standard* problems was not matched by success in *non-standard* ones. One aspect of the present research is the development of a set of questions to measure students' ability to engage in sense-making during word problem solving (Yeap & Lee, in preparation).

In the main study, which involves Primary Three and Four (Year Three and Four) children in two schools, the *Think-Things-Through* lesson will be conducted at least once a week during regular mathematics lessons. The quantitative aspect will investigate effects of the programme on achievement test score, ability to engage in sense-making in familiar and unfamiliar situations and ability to solve non-routine problems. The qualitative aspect will investigate how a disposition to engage in sense-making can evolve via classroom instruction.

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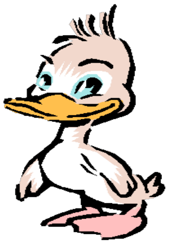
Think-Things-Through Project

Name: [ ]	**Primary 3**
Title: *Square Challenge*	**Date**

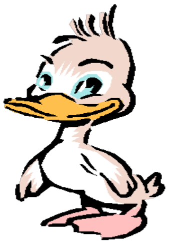
You can make 1 square using 4 toothpicks.

Each side of the square has a length of 1 toothpick.



Draw your shape here. What is the perimeter?

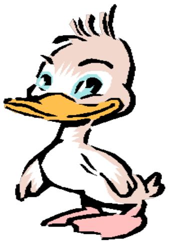
Shape: Perimeter = ………. units



Draw your shape here. What is the perimeter?

Shape: Perimeter = ………. units

Shape: Perimeter = ………. Units



Draw your shape here. What is the perimeter?

Shape: Perimeter = ………. units

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Think-Things-Through Project

Teacher's Notes

Title: Square Challenge

Reference: None

This lesson can be used in the topic Perimeter.

We are using problems where children create possible shapes (squares) using a given number of toothpicks (or substitutes). In each case they are asked to find the perimeter of the shape.

Give each child / pair about 20 toothpicks.

Segment	Procedure	Rationale / Notes
1	Ask children to show a square using the toothpicks. Ask them to name one property of a square. Show them a triangle using 3 toothpicks and ask them if it is a square. Show them a rectangle using 6 toothpicks and ask them if it is a square. Show them a rhombus using 4 toothpicks and ask them if it is a square.	Some children may make squares bigger than unit squares. Clarify if necessary that we will only make squares with sides of 1 toothpick.
2	Ask the children how many toothpicks are needed to make 1 square. Ask them how many toothpicks are needed to make 2 squares. A common answer will be 8. Challenge them to make two squares using only 7 toothpicks.*	

	Get children to draw the shape onto their worksheet and calculate the perimeter.	To think in a different way.
3	Ask the children how many toothpicks are needed to make 1 square. Ask them how many toothpicks are needed to make 4 squares. A common answer will be 16. Challenge them to make 4 squares using only 13 toothpicks.* Tell them there are three different ways to do this. Ask them to find two ways. Get children to draw the shapes onto their worksheet and calculate the perimeters.	
4	Challenge them to make 4 squares using only 12 toothpicks.* Get children to draw the shape onto their worksheet and calculate the perimeter.	
5	Conclude the lesson by telling the children that although 1 square needs 4 toothpicks, 4 squares may need 16, 13, 12 toothpicks. Challenge them to show how 6 squares can be created using 12 toothpicks. Encourage them to work this out with their parents / siblings.*	

Lesson Features:

1. These are problems that show children that simple multiplication does not work all the time, in geometry in this instance.
2. The children use the heuristic: **act it out** and **draw a diagram**.
3. Children are also encouraged to **solve the problem in another way.** This is metacognition, one component of the curriculum. It also requires some creative thinking, another emphasis of the Thinking Schools philosophy.
4. The concept of perimeter is learnt through a series of action (psychomotor intelligence).