On in-service Mathematics teachers' content knowledge on kinematics

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Received 6 October, 2005
Revised 10 December, 2005

Contents

Abstract

This paper reports a preliminary study on in-service Additional mathematics teachers' knowledge of kinematics concepts. A survey consisting of TRUE / FALSE questions was issued to the participating teachers. The questions were collations of the common misconceptions identified by some local Physics teachers among the local Physics students. The participants were asked to supply the answers to the questionnaire with

their answers substantiated with reasons. In this paper, we discuss the results of the survey done on a group of twenty six in-service Additional mathematics teachers and classify the teachers' misconceptions of kinematics concepts. The finding of this initial survey could be useful to spur further research on Mathematics teachers' subject content knowledge on kinematics. Pragmatically it would also be useful for any agency which is planning for any content upgrading workshops for in-service teachers.

Introduction

Anecdotal evidence during my school attachment of the past few years in the various schools has shown that many Mathematics teachers in the secondary schools, especially those teachers who do not teach Physics or Applied Mathematics as their second or third teaching subject, might not have a sound concrete understanding of kinematics concepts. This can also be clearly seen from my personal interaction with some secondary school students. Many of the students tend to have the opinion that the *physics* taught in Mathematics (which the students refer to the practical graphs section of Mathematics syllabus) is somewhat conceptually different from the *physics* that they learnt in their science subjects (which they refer to the kinematics section of the Newtonian Mechanics of their Physics syllabus) in their curriculum.

In the Mathematics O-Level syllabus before the year 2001, all kinematics problems were classified under the section on Practical Graphs in the chapter of Graphs. Here the students were required only to extract information and perform computation based on either the displacement-time graph or the velocity-time graph. The students must be able to obtain numerical values of the distance traveled, speed, velocity or acceleration without much concern or interpretation needed of the moving particle. In short, students were required merely to perform simple computations to obtain their answers without much qualitative knowledge of the motion of the particle. Practically in all the related examination problems, the particles being considered are always moving in one dimension along a straight line without changing direction. This is a good illustration that mathematics teachers are basically teaching the practice of routine skills rather than mathematical problem-solving (see, for example, Andrews P., 2002), mainly driven by the format and structure of the examinations.

To make matters worst, under such setting, the knowledge of kinematics of one-dimensional motion could even be wrongly generalized to two-dimensional motion. For example, a particle moving with constant speed along a straight line without reversing its direction of travel experiences zero acceleration. One might wrongly generalize to two-dimension that a particle moving with constant speed experiences zero acceleration, even though the direction may not be kept constant (for example, in the case of circular motion with constant speed).

I surveyed a collection of the Practical Graph questions found in the Mathematics examination questions set by the local secondary school teachers in the year end examinations of the past few years for the Elementary Mathematics syllabus. It is evident that many teachers did not even indicate that the moving particles whose motion graphs are shown are moving along a straight line (which they must assume in solving the question)! This shows that the teachers might have treated the statement "the particle is traveling along a straight line" without realizing the related kinematics concepts in motion in higher dimensions.

In the new Additional Mathematics syllabus recently introduced in 2001, more intensive knowledge of kinematics concepts is required of both the teachers and students. This is especially true with the introduction of the topic Relative Velocity into the Additional mathematics syllabus. Mathematics teachers' vague or inaccurate knowledge of kinematics is no longer sufficient to teach the content of this chapter.

As many local in-service teachers did not have sufficient subject content knowledge of Relative Velocity in specific and kinematics in general, I was tasked to design a series of mathematics content workshops for the teachers to "top-up" their subject content knowledge. While it is important that teachers are able to take into account pedagogical considerations in their teaching and how to enhance students' understanding through the use of Instructional Technology (for example, Toh, 2003) or to motivate their students, it is of immediate importance and top priority that teachers themselves have a sound subject content knowledge of this topic and the foundational knowledge of kinematics. In the process of planning suitable materials for the workshops, I felt it necessary to have an understanding of their subject content knowledge of kinematics. This motivated me to conduct a survey on their subject content knowledge of kinematics.

This paper reports the result of the survey conducted on a group of twenty six in-service teachers from different schools in Singapore. This paper begins with a background, discussion of the method of survey, followed by results and discussion. Hopefully the finding of this study could be useful for any future planning of in-service courses to help Mathematics teachers to be better equipped in teaching kinematics related topics in the Mathematics syllabus.

Background

The topic Relative Velocity was incorporated into the Pure Mathematics component of the GCE O-Level Additional Mathematics syllabus in 2001. This topic is compulsory for all candidates studying the subject. Hence, no students are allowed to skip this topic.

Before the implementation of the new Additional Mathematics syllabus in 2001, this topic of Relative Velocity was subsumed under the Particle Mechanics section in the Additional Mathematics syllabus. As the entire Particle Mechanics section was optional, practically every school in Singapore chose to ignore this entire option in order to focus exclusively on Pure Mathematics.

A closer examination of some of the past year questions in the new Additional Mathematics syllabus (starting from the year 2001) reveals that, not only must the candidates have a thorough knowledge of Relative Velocity, the related concepts on kinematics are crucial even for examination purpose to answer some of the questions. For example, one must have a clear perception that when a particle travels with constant velocity, there is no change in direction of motion; hence it must be traveling along a straight line path.

Teachers' Pedagogical Content Knowledge

Teachers need to have a sufficiently good subject content knowledge of their teaching subject in order to be aware of the problems of pupils' misconceptions and "to use a number of different strategies and how to coordinate between strategies depending on the teaching context"(Abd Rahman N, 2004). This is one among the most important attributes of a good classroom teacher.

Usiskin (2001) called this knowledge that a practicing teacher needs to have as the *pedagogical content knowledge*. He classified the pedagogical content knowledge that a teacher needs to have under three broad categories: (1) generalized knowledge from what is required in the syllabus, (2) concept analysis and (3) problem analysis. These three categories of knowledge are of course above the direct content knowledge of the examination syllabus.

To put it in Usiskin's perspective: until the advent of the new Additional Mathematics syllabus, the knowledge of kinematics concepts of two-dimensional motion for teachers teaching 'O' Level Mathematics came under category (1): the generalized knowledge from what is required in the syllabus (that is, one-dimension kinematics problems) while knowledge of kinematics concepts in one-dimensional motion pertains to the direct subject content knowledge of the examination syllabus. This is because in the typical Mathematics examination questions, students were only tested questions of particles moving in one-dimension (in fact, only computational skills were required).

After the introduction of the topic Relative Velocity into the syllabus, the kinematics knowledge of two-dimensional motion can no longer be perceived as *generalized knowledge* from what is required in the syllabus; with the introduction of this topic; this knowledge becomes the direct subject content knowledge of the examination syllabus, that is, directly needed to prepare the students to answer the examination questions. Furthermore, Mathematics teachers, being professional educationists, are expected to be able to minimize any cognitive conflicts among students when the same concept is covered across both the Science and Mathematics subjects. In other words, they must be able to see the relationship between the differences and similarities of the same concepts that are presented across different disciplines.

Method

A group of twenty six in-service Mathematics teachers teaching Additional Mathematics were asked to do a short questionnaire on some kinematics questions before the commencement of the in-service workshop conducted for the teachers. Information regarding the second (and third, if applicable) content subject that they were currently teaching in school (besides Mathematics) were also collected.

The breakdown of the teacher's second and third teaching subjects is tabulated in Table 1 below. It was found that all the participating teachers only taught two subjects in their schools, hence no third subject is recorded below.

Mathematics Only	Computer Applications	English Language	Lower Secondary Science
22	4	3	1

Table 1: *Breakdown of the Second teaching subject of the participating teachers*

The teacher participants were required to identify whether they thought that each statement was TRUE or FALSE. For each of the statement which they believed the answer was FALSE, they were required to write down in the space provided why they thought each statement was FALSE by either giving a special numerical example demonstrating why it was not true, or giving a qualitative explanation of where the concept was incorrect. For each of the statement which they thought was TRUE, they were asked to write a brief statement to give a short explanation.

To ensure greater reliability of the participants' response, the participants were assured of confidentiality of the data they had provided for this initial survey. To reduce the teachers' anxiety level, they were assured that the survey results was not tied to their performance in the workshop but was meant as a means to help them with their kinematics concepts. They were not required to write down their names or the schools they were teaching on the survey question papers.

The questions on the survey forms are appended in Table 2 below.

1.	An object traveling with constant speed does not experience any acceleration.
2.	An object traveling with constant non-zero acceleration definitely experience a change in speed.
3.	The magnitude of displacement of a particle is the distance traveled by the particle.

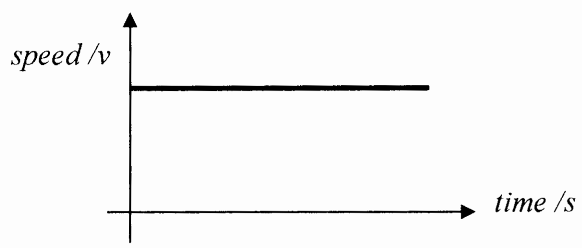
4.	An object traveling with minimum velocity is traveling with minimum speed.
5.	When an object travels with positive acceleration, it travels with increasing speed.
6.	Acceleration is defined as the rate of increase of velocity with respect to time.
7.	When an object travels with positive velocity, its speed is increasing.
8.	When an object has a positive displacement, its speed is increasing.
9.	The velocity of an object is defined as the rate of change of its displacement with respect to time.
10.	The speed of an object is defined as the rate of change of distance traveled with respect to time.
11.	The speed of an object equals the magnitude of its velocity.
12.	Refer to the speed-time graph below. 
13.	There is nothing wrong with the statement "The velocity of a car is 2 m/s".

Table 2: *Survey questions*

The participating teachers were given about thirty minutes to complete the questionnaire and submit their answers at the end.

The questions were classified into three main categories as follow:

Category 1: *Knowledge of Kinematics in Two-Dimensions:* Questions 1, 2 and 12 (see Table 2 above) involve knowledge of objects moving in more than one dimension. In general, objects moving with constant speed may not have zero acceleration. Teachers are at least required to relate the concepts to examples of motion in two dimensions.

Category 2: *Elementary Conceptual Difference between Vector Quantity and Scalar Quantity:* Question 13 involves knowledge of scalar and vector quantities, and that there is difference between the terms speed and velocity. Teachers are required to know that speed is a scalar quantity while velocity is the vector counterpart of speed.

Category 3: *Knowledge of Kinematics in One-Dimension:* Questions 3, 4, 5, 6, 7, 8, 9 and 10 involve knowledge of the terms displacement, velocity and acceleration and their scalar counterparts (namely, distance traveled, speed and the magnitude of acceleration). The questions also involve the signs of the respective vector quantities and the significance of the signs of these vector quantities.

The data was then recorded according to the correctness of the answers and the reasons substantiating their answers in Table 3 below. The column "Wrong Response" in Table 3 is a collation of those entries when either (1) the answer is wrong or (2) the answer is correct but *substantiated by wrong reasoning.* Data in which no reason was given to substantiate the response would be ignored (In this study, it turned out that the all the teachers' responses were substantiated with reasons, hence the sum of wrong response and correct response for each question equals 26, the total number of participants, for each question recorded below).

Finally after the survey, the correct answer and reason to each of the questions in Table 2 were given to the participants. The participating teachers were also encouraged to give their verbal feedback on the questions.

Results

The response of the in-service teachers were collected and are summarized in the fourth and the fifth columns of the Table 3 below. The second column gives the correct answer to the corresponding question in Table 2 and the third column states the correct reason for the answer of each of the questions.

Question Number	Correct Answer	Correct Reason	Wrong Response	Correct Response
1.	FALSE	Not necessarily true when the object does not travel in the same direction throughout	25 (96%)	1 (4%)
2.	FALSE	An object traveling with non-zero acceleration may not be traveling with the same direction, hence it may have an acceleration	22 (85%)	4 (15%)
3.	FALSE	Displacement of a particle indicates the position of the particle, not the distance it travels.	12 (46%)	14 (54%)
4.	FALSE	It is possible that an object traveling with minimum velocity may be traveling the fastest.	11 (42%)	15 (58%)
5.	FALSE	It is not true if the object is not traveling in the same direction.	23 (88%)	3 (12%)
6.	FALSE	Acceleration should be defined as the rate of change of velocity with respect to time, not "increase"	17 (65%)	9 (35%)
7.	FALSE	The sign of velocity indicates the direction it is traveling, not the increase in speed.	6 (23%)	20 (77%)
8.	FALSE	Displacement indicates the position, not the rate of change of speed	3 (12%)	23 (88%)
9.	TRUE	The statement follows from definition	2 (8%)	24 (92%)

10.	TRUE	The statement follows from definition	0 (0%)	26 (100%)
11.	TRUE	It can be proved that the speed equals the magnitude of its velocity.	1 (4%)	25 (96%)
12.	FALSE	There is no statement that the particle travels in the same direction throughout	23 (88%)	3 (12%)
13.	FALSE	Velocity is a vector quantity, hence the direction needs to be stated	1 (4%)	25 (96%)

Table 3: *Summary Results of the Teachers' Response*

There were four participants who responded with FALSE for Question 1 but substantiated their answers with wrong reasons, hence as mentioned in the preceding paragraphs these responses were classified under "Wrong Response". Since air resistance is not known, the teachers felt that it was not possible to conclude that the particles travel with any acceleration.

From the data collected above, the top four questions with the greatest number of wrong response were Question 1 (96% wrong response), Question 5 (88% wrong response), Question 12 (88% wrong response) and Question 2 (85% wrong response). The four questions with the greatest number of correct response were Question 10 (100% correct response), Question 11 (96% correct response), Question 13 (96% correct response) and Question 9 (92% correct response).

Verbal Response

During a casual talk after the survey with four participants, three of the participants mentioned that they had never encountered mathematical problems involving motion in two dimensions in their teaching career. That was the reason why they had responded to the questions with only one dimensional motion in mind. Most likely they had forgotten the further knowledge of kinematics which they had learnt in their undergraduate days.

One participant mentioned with regard to Question 12 that he was used to computing acceleration as the gradient of the speed-time graph. He had not taken into consideration that the particle might not be traveling in the same direction throughout the motion. He further added that he would not know how to handle the case of non-uniform direction.

Discussion

Elementary Conceptual Difference between Vector Quantity and Scalar Quantity:

From the high percentage of correct response of Question 13, it can be seen that practically all the teachers in the survey know the difference between vector and scalar quantities. This is understandable as in the Mathematics curriculum, students need to be aware of the existence of both the vector and the scalar quantities. In fact, Vectors is one chapter of the Mathematics syllabus.

Knowledge of Kinematics in Two-Dimensions:

It could be seen from Table 3 that Questions 1, 2 and 12 have the highest percentage of wrong response. It is clear from the participating teachers'response that they did not have sound generalized kinematics concepts involving particles moving in more than one dimension. In the case of uniform motion (along a fixed direction), the teachers equated zero acceleration with having constant speed. This was erroneously generalized to motion in two dimensions.

Thus, it can be concluded that most of the participants' concepts of kinematics is strictly restricted to one-dimensional motion which might be wrongly generalized to two dimensional motions. Thus, the Mathematics teachers need to build up on their foundation in the general concepts of kinematics.

Knowledge of Kinematics in One-Dimensional Motion:

Sign of Acceleration for one-dimensional Motion. From the response to Question 5 of Table 3, it can be seen that there is a rather high percentage of about eighty-eight percent of wrong responses of participants, who thought that positive acceleration implies increasing speed, without knowing that the sign of the acceleration is a matter of the convention taken in each situation.

Signs of Displacement and Velocity one-dimensional Motion: From the highest percentage of correct response for Questions 7, 8, 9 and 10, it is clearly suggestive that teachers have fairly clear knowledge of the significance of positive/negative signs of displacement and velocity for particles moving along one-dimension.

Conclusion

The result in this initial survey offers some initial information on the kinematics knowledge of the in-service Additional mathematics teachers. However, the result might not be generalizable to the entire population of all Additional mathematics teachers in Singapore, since the sample size was rather small for inferential statistics to be meaningful. Moreover, the sample involved was only thirty teachers sent by the Ministry for content upgrading based on their professional needs and the number of training hours to be fulfilled. Thus, the sample involved is not random and might not be representative of the entire population. In order to have a more accurate generalization, a larger random sample may be more useful. Moreover, there is a large proportion of Mathematics teachers, the number of which could amount to approximately 20% of the entire Mathematics teachers by a rough estimate, in the local schools whose second teaching subject is Physics. These teachers, expected to have a much stronger foundation in kinematics, were not available to be participants of this study.

Nevertheless, this initial survey gives an initial idea of the aspects of Mathematics that in-service Mathematics teachers might need help in, especially with the more science-related mathematical concepts. This result could provide some inkling for those agents who are planning series of in-service content upgrading workshops for teachers. Perhaps, the finding of this initial survey could be useful to spur further research on Mathematics teachers' subject content knowledge on kinematics.

References

Abd Rahman, N. (2004). Physics Teachers Strategies and Reflective Ability for Addressing Pupils' Misconceptions in the Classrooms. *Paper presented at the British Educational Research Association Annual Conference, University of Manchester,* 16-18 Sep 2004.

Andrews, P. (2002). Secondary Mathematics Teachers' Rationales for the Teaching of the Subject. *Paper presented at the British Educational Research Association Annual Conference, Univeristy of Exeter, England,* 12-14 Sep 2002.

Toh, T.L. (2003). On Using Geometer's Sketchpad to Teach Relative Velocity. *Asia-Pacific Forum on Science Learning and Teaching*, *4*(2), Article 8. Available: http://www.ied.edu.hk/apfslt/v4_issue2/toh/

Usiskin, Z. (2001). Teachers' Mathematics: A Collection of Content Deserving to be a Field. *The Mathematics Educator, 6*(1), 81-99.