Problem finding in school mathematics: Issues and implications

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Introduction

Compared to problem solving in school mathematics, problem finding has received little attention. Students are almost always presented with problems to solve in their mathematics lessons. Any exposure to mathematical problem finding and posing appears to be an incidental event. But mathematicians, as well as scientists, have for a long time recognised the value of finding good problems. Prominent mathematics educators (Polya, 1957; Freudenthal, 1980) as well as reform initiatives (Cockcroft Report, 1982; NCTM Standards, 1989, 1992) have emphasised problem finding as an important aspect of mathematical education. Evidence from creativity research (Getzels & Csikszentmihalyi, 1976; Jay & Perkins, 1997) suggests that problem finding is a crucial step in the creative process. It is therefore intriguing that problem finding has not received as much attention as problem solving. Are they similar?

Locally, research in problem finding is sparse - the key researchers being Yeap and Kaur (e.g., Yeap & Kaur, 1997; Yeap, 1996). Their pioneering works are making inroads into mathematical problem posing, an activity closely related if not synonymous to problem finding. There is also a concern among practising teachers that the non-routine mathematics problems used in their problem solving classes are fast becoming "routine". How can one generate non-routine problems? Are there strategies or heuristics? Answers to these questions should also benefit teachers of gifted children and the gifted children themselves, for such children need to be challenged with interesting and significant problems as well as learn how to find and formulate problems. There is anecdotal evidence to suggest that the gifted students and their teachers alike find the task of problem finding a challenge. These observations hint at a need, and possibly a timely one, for more empirical or systematic research in mathematical problem finding and posing.

This paper explores the nature of problem finding and examines its role in mathematics education. What is problem finding? Is problem finding different from problem solving? How do people problem find? What cognitive factors are associated with problem finding? How may problem finding be introduced in school mathematics? In what ways can it help inculcate creativity in schools?

Where do mathematics problems come from?

Where do mathematics problems come from? The responses from a group of newly enrolled pre-service primary school teachers are "mathematics textbooks", "mathematics teachers", "past exams papers", "mathematicians" and, half in jest, "people with nothing better to do". The idea of pupils being a source of mathematics problems was met with considerable disbelief. Because teachers' conception of mathematics influence their instructional practice (e.g., Thompson, 1984; Cooney, 1988; Lampert, 1991) and students' beliefs about mathematics determine how they learn mathematics (Schoenfeld, 1992, Borasi, 1990), it important that there is systematic study into role and impact of problem finding in the mathematics classroom. Will problem finding and posing help create an enlightened view of teaching and learning mathematics?

What is problem finding and posing?

What is problem finding? How has it been conceptualised? Reviews of literature on problem finding or posing by Dillon (1982), Silver (1994) and Jay & Perkins (1997) agree that there is relatively little research on problem finding compared to problem solving, but noted that increasingly more are being undertaken. Wakefield (1992) reports a referral to the notion in a statement by Paul Souriau in 1881: "There is something mechanical, as it were, in the art of finding solutions. The truly original mind is that which finds problems" (Wakefield, 1992, p.9).

Since then, many terms closely related to the problem finding have been used, e.g., problem posing, problem formulation, problem construction, problem generation, problem discovery, problem identification, and problem definition (e.g., Jay & Perkins, 1997). Dillon (1982) pointed out that there is no theory of problem finding but only in-depth discussions in his review.

What is problem finding? Csikszentmihalyi (1988) offers a broad definition of problem finding as discovering, formulating and posing a problem. Jay and Perkins (1997) view problem finding as fundamentally the "behavior, attitudes, and thought processes directed toward the envisionment, posing, formulation, and creation of problems, as opposed to the processes involved in solving them." (p. 259)

The literature reports that many notable persons have stated the importance of problem finding. Two such persons are John Dewey and Albert Einstein. Dewey remarked that "A problem well stated is a problem half solved." (in Isaksen, Dorval, & Treffinger, 1994, p. 211), and Einstein stated that "The formulation of a problem is often more essential than its solution, which may be a matter of mathematical or experimental skill. To raise new questions, new possibilities, to regard old questions from a new angle, requires creative imagination and marks real advance in science." (Einstein & Infeld, 1938, p. 92)

It may be inferred from this brief survey that problem finding, problem posing or problem formulation seems to differ from each other only in subtle ways. The existence of these closely related terms may be indicative of complexity in the nature of problem finding. In this paper, the term problem finding will be used in the generic sense and interchangeably with the other terms such as problem posing, unless a distinction is necessary. The next few sections present perspectives from cognitive science, creativity, and mathematics education on problem finding.

Problem finding in cognitive science

Cognitive scientists (Newell & Simon, 1972; Langley, Simon, Bradshaw, & Zytkow, 1987) argue that problem finding is but a special form of problem solving. According to Newell and Simon (1972) a person faces a problem "when he wants something but does not know immediately what series of actions he can perform to get it." (p. 72). To solve the problem, the person searches through a problem space, starting with an initial (given) state and using tools such as means-ends analysis, to reach a goal (desired) state. Problem finding is construed as solving the problem in which the goal is a "problem". Or, as Getzels (1982) puts it, in problem finding, the problem solver is confronted with "the problem of the problem".

Through computer simulations, Langley et al (1987) were able to reproduce many discoveries in science (e.g., Kepler's Third Law of planetary motion, Balmer's series in atomic spectra) using induction and heuristic strategies on raw data available to the scientist who made the discovery. It is a process which requires the formulation and re-formulation of problems.

If problem finding is a special form of problem solving, efforts to introduce problem finding in schools may be able to draw on the resources accumulated for problem solving.

Problem finding in creativity

Problem finding has long been recognised as a crucial step in the creative process (Wallas, 1927; Hadamard, 1954; Runco & Chand, 1992). According to Csikszentmihalyi (1988), creative thinking lies in the ability to discover new problems never before formulated. Howard Gardner (1994) defines a creative person is one who "solves problems, fashions products, or *poses new questions* within a domain in such a way that is initially considered to be unusual but is eventually accepted within at lease one cultural group." (italics mine, p. 71)

Getzels and Csikszentmihalyi (1976) undertook one of the first empirical studies of problem finding and creativity. They defined problem finding as "the way problems are envisaged, posed, formulated, created" (p. 5). In their study, student artists were observed as they selected and arranged materials to paint a still life. The quality of problem finding was judged on the basis of breadth, depth, and uniqueness of exploration of objects prior to actual drawing. The behavioural patterns of the students whose work was rated by a panel of judges as original were compared to those of students whose work were rated as lacking originality. Within its limits, the study found strong empirical relationship between problem finding approach and creativity in the resulting painting. It was also linked to the students' success as professional creative artists seven and eighteen years later (Csikszentmihalyi, 1990).

To characterise the problem finding approach, Getzels and Csikszentmihalyi (1976) introduced the notion of a *presented - discovered - created* problem continuum. A presented problem is one that is clearly formulated, has a acceptable method of solution, and a solution is known to the presenter. The problem is presented to a solver who must figure out the solution. Much of school mathematics consists of presenting students with problems to solve. In a discovered problem situation, the problem is vague and yet to be defined and so no agreed-upon method and solution. Investigational work (Cockcroft, 1982), which is a relatively new feature in school mathematics, presents opportunities for problem finding of this nature. In a created problem situation, the problem does not exist and has to be invented. Can we expect school children to create mathematics problems?

Problem finding appears as an essential step in numerous models of creative problem solving (e.g., Wallas, 1927; Guildford, 1950; Osborne, 1963; Parnes, 1967; Isaksen and Treffinger, 1987; Isaksen, Dorval, & Treffinger, 1994). Using ideas from the works of Osborne and Parnes on creativity, Isaken, Dorval and Treffinger (1994) developed a Creative Problem Solving (CPS) model. Briefly, the CPS consists of three components. The *Understanding the problem* component consists of mess-finding, data-finding and problem-finding. The *Generating Ideas* component is concerned with finding idea to solve the problem. The last component, *Planning for action*, contains the solution-finding and acceptance-finding stages. In this model, the *problem-finding* step is both a divergent and a convergent process that uses the results from the *messing-finding* and *data-finding* steps to formulate a problem statement.

It can be seen from this quick glance into the literature on creativity that problem finding is associated with the creative person and creative process. But, what is the exact connection between problem finding and creativity? How may problem finding or the notion of problem continuum be used to infuse creativity into the mathematics class? To what extent along the problem continuum can students be expected to engage in problem finding?

Problem finding in mathematics education

The term *problem posing* instead of *problem finding* is used in mathematics education. Pimm (1989) uses the term *asking mathematical questions*. Research on mathematical problem posing, although little compared to problem solving, is attracting the attention of mathematics educators (e.g., Silver, 1994; English, 1997; Silver, Mamona-Downs, Leung, & Kenny, 1996). According to Silver (1994), "Problem posing refers to both the generation of new problems and the re-formulation, of given problems...posing can occur before, during, or after the solution of a solution. (p. 19)

A typical research approach in mathematical problem posing by students consists of giving the students some information and requesting them to generate questions or problems using the information. Figure 1 shows a couple of examples.

Figure 1. Examples of problem posing tasks

1. Example from (Silver & Cai, 1996)
   Write three different questions that can be answered from the information below.

   Jerome, Elliot, and Arturo took turns driving home from a trip. Autro drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles.

Example from Yeap and Kaur (1997)

What is the question?
... and the answer is x = 3. What could the question be?

What if?
Find dy/dx if y = sin3x.

What if (i) y = 3 sin3x (ii) y = sin(x/3), etc
What's the problem?
Create a calculus problem that involves maximum/minimum values. Solve the problem.

In Silver and Cai (1996), the problems posed were examined for solvability, linguistic and mathematical complexity, and relationships within the set of posed problems. Yeap and Kaur (1997) attempt to apply Marzano's (1992) *Dimensions of Thnking and Learning* framework to analyse the problems posed. The structures of the problems posed by students in these exercises are similar to those they have encountered in their mathematics lessons.

Placed on the presented-discovered-created problem continuum, the problem posing situations described in Silver and Cai, and Yeap and Kaur lie near the discovered-problem situation. It should be interesting to examine the quality of problem posed and mathematical ability, at different points along the continuum.

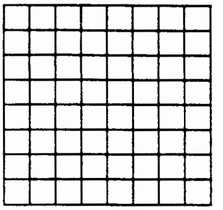
Problem finding strategies

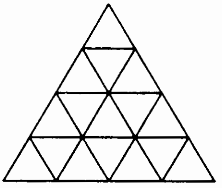
What are the strategies for problem finding? Cognitive science views problem finding as problem solving, and so the many strategies for solving problems apply in finding problems (Langley et al., 1987).

Stephen Brown and Marion Walter (1990, 1993) introduce an approach for posing mathematics problems from any given problem situation. Their strategy, called the *What-If-Not?*, consists of five steps:

1. Choosing a Starting Point
2. Listing Attributes
3. Do a What-If-Not-ing on or two attributes
4. Question Asking or Problem Posing
5. Analysing the Problem

To briefly illustrate, consider the problem of counting the number of squares in an 8x8 grid. Taking this as the starting situation, we list as two attributes of the situation, squares and dimensions of grid (8 x 8). Next we carry out "What-If-Not-ing". What if not squares or 8x8? This suggests triangles, rectangles, hexagons, etc., for the first attribute, and 16x16, nxn, 8x4, mxn, etc., for the second attribute. From these, problem possibilities can easily be generated. Fixing "triangle"as an attribute and asking similar questions about the new attribute, may be lead to the problem: How many equilateral triangles are there in the equilateral triangle shown on the right?





This discussion raises a related point: What is the role of divergent thinking in listing the attributes and generating problems?

Problem finding strategies: Revisiting Polya

Polya's influence on mathematics education and problem solving is legendary. Problem posing is much alive in his famous four-stage model of problem solving. In *Devising a plan*, Polya suggests restating the problem, restating it still differently if needed, imagining a more general problem or a more special problem, or an analogous problem. When he advises solvers to keep only a part of the condition, drop the other part, or change the unknown or the data, he recommending problem re-formulation. In the *Looking back* phase, he advocates applying the result, or the method of solution to some other problem. By specialising or generalising from the result we can derive other problems. Indeed, in urging the solver to consider the unknown, the data and the condition in the *Understanding the problem* phase, Polya has provide a general approach to posing mathematics problem – new problems may be generated by altering unknowns, the data, or the conditions of a given problem. In view of the extensive use of Polya problem solving model in school mathematics world wide, it may be worth considering how the model can be made explicit for posing problems as well.

In revisiting Polya, one finds that he recommends the use of analogy as a heuristic in problem solving. Polya's (1954) illustrates how analogy works in his book *Mathematics and Plausible Reasoning: Induction and Analogy in Mathematics*, which is a continuation of *How to solve it*. What is analogy and what is its role in problem finding?

Analogical thinking is pervasive in our everyday life. It is also recognised as closely linked with creative thought. An example is Kekule's discovery of the structure of benzene ring using "whirling snakes" as the analogous situation. Gick and Koh (1987) characterise the process of analogical problem solving as follows:

1. constructing mental representations of the source and the target;
2. selecting the source as a potentially relevant analogue to the target;
3. mapping the components of the source and target; and
4. extending the mapping to generate a solution to the target.

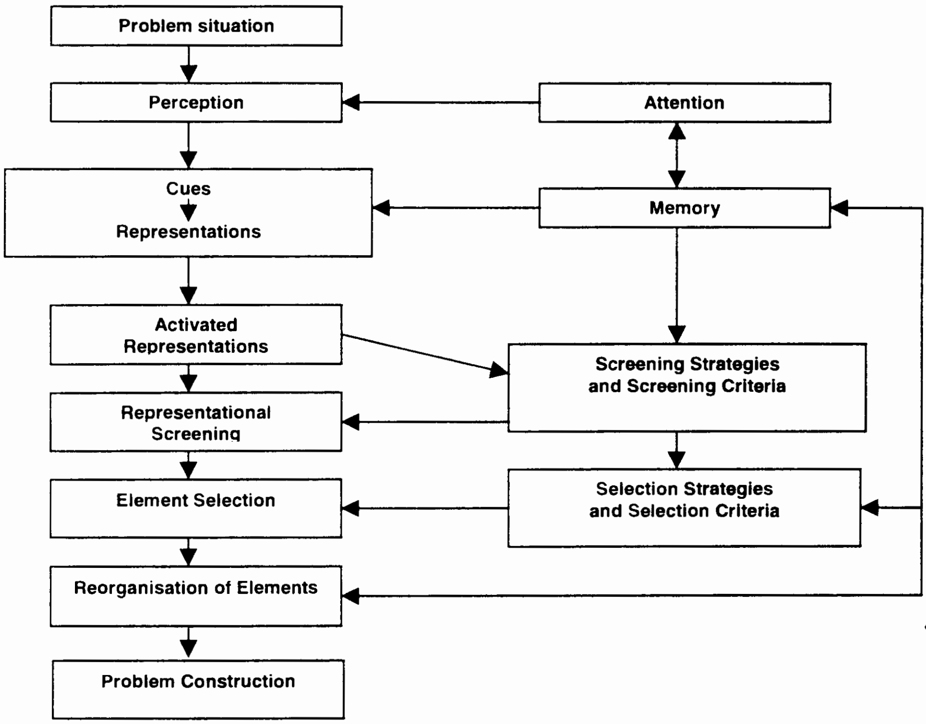
Because analogous objects agree in certain relations of their respective parts (e.g., rectangular parallelogram and rectangular parallelopiped) new problems may be obtained or suggested by varying these analogous parts. The source from which an analogue is selected may come from within the domain (e.g., the sides of a triangle and the faces of a tetrahedron) or from a different domain. It is the ability to drawn analogy with a remote field that is called creative.

It is clear from this discussion that there are some strategies for generating problems. Analogy is one of them. An issue that can be raised is whether school mathematics problem posing be built on Polya's model, given its wide spread used? What would be the relation of the "What-If-Not...? strategy in this scheme?

Cognitive factors in problem finding

What cognitive processes underlie problem finding? The construction of mental representations and the comparison of mental representations to map relational elements in analogical thinking are examples of cognitive processes involved in the finding and posing of problems. Mumford, Reiter-Palmon, and Redmond (1994) propose a model of problem construction that traces the cognitive operations underlying problem finding (Figure 2).

Figure 2. Model of Problem Construction Operations (adapted from Mumford et al, 1994 p. 17)



The problem finder builds representations of problem possibilities based on perceptual cues in the problem situation (event). The representations are screened and the most strongly activated problem representation selected. The result provides a set of problem elements that can be organised into a problem proper. The end stage is where a problem is constructed from the elements selected. Analogy is recognised as part of the problem construction process, which is utilised possibly in the activation, selection and reorganisation of the problem representations.

Re-examination of the cues may activate a different representation. The type (e.g., consistent or conflicting) and diversity of environmental cues are likely to influence the problem constructed. Will greater diversity in cues result in greater variation in problem posed?

Problem finding and problem solving

The connection between problem finding and problem solving seems to be an issue (Brown & Walter, 1990; Csikszentmihalyi, 1988; Simon, 1988). In what ways are the two processes similar? Silver (1994) states that problem posing can occur before, during and after solution of a problem. In revisiting Polya's four-stage model of problem solving, we saw that problem formulation and re-formulation are inseparable from the solving process. Brown and Walter (1990) argue that the two activities are closely intertwined, initially to reconstruct the task of solving a novel problem by posing new problem(s) and finally to understand the solution to a problem by testing it on other problems. Simon (1988) claims that problem finding is a special case of problem solving, whereas Csikszentmihalyi (1988) points out that problem finding precedes problem solving and calls for creative thinking. In terms of the cognitive processes involved, problem solving is mainly a convergent process in reaching a solution whereas problem finding is divergent as well as convergent in reaching a problem formulated. English (1997) maintains that problem posing takes students beyond the parameters of the solution process, for example, to the origins of ideas in a problem. To Jay and Perkins (1997) problem finding is more often associated with breaking domain boundaries than is problem solving.

Is the ability for problem solving the same as for problem finding? Smilansky (1984) found little connection between competence in posing problems and competence in solving problems in the same domain. 129 tenth and eleventh grade pupils completed the Raven Progressive Matrices Test, and thereafter to pose a new problem that could be included in the test as its most difficult item. Of 57 students at the highest level of problem solving only 12 also rank highest in problem posing, suggesting that ability to solve problems does not include the ability to pose problems.

Problem finding and problem solving seem tightly enmeshed, one leading into the other. The distinction between the two is not clear-cut. Differences may lie in the type of cognitive processes involved in their execution.

Other variables of interest

What is the relationship between problem finding and other variables of interest such as cognitive and thinking styles, context, and metacognition?

It may be expected that cognitive styles such as field-independence or field-dependence (Morgan, 1997) would bear on the type of problems posed. As the terms imply, field-dependence people are more dependent on visual cues in the surrounding to make sense what is perceived than are field-independent people. According to Mumford's et al (1994) model of problem construction, environmental cues act on the problem finding process at its onset. What are the relations among cue diversity, cognitive style, and originality of problem posed?

Context, in terms of cue diversity or in which mathematics is learnt, is also expected to influence problem finding. Research (Hinsley, Hayes and Simon, 1977) reveals that people have a body of information about each problem type which is potentially useful in formulating problems of that type for solution. This information directs their attention to important problem elements, making relevance judgements, and retrieving information concerning relevant equations. Also, people are known to categorise problems without completely formulating them for solution. We may therefore expect students to pose typical school mathematics problems if these are the only types they have been exposed to, or the only context they ever learnt their mathematics in.

Metacognitive skills and strategies are known to determine success in problem solving performance (Schoenfeld, 1992). As can be seen in Mumford's et al (1994) model, metacognition may be involved in determining the focus of attention, the choice of representation activated, and the use of the sceening and selection strategies. Metacognition is likely to play a key role in problem finding and formulating problems because finding and formulating, especially at the created problem end of the presented-discovered-created problem continuum, is a highly self-directed effort. What exactly is the role of metacognition in problem finding?

Implications for problem finding and posing in school mathematics

The concepts of problem finding and problem posing merit further attention by the mathematics education community as they cut across the domains of mathematics and creativity. Both of these have been the focus of recent educational reforms. Problem finding is a notion associated with creativity whereas problem posing is linked to mathematics education.

Problem finding, being a relatively new notion, raises the question of how it may be introduced in schools. How does one get started on problem finding and posing? Should one explicitly teach problem finding strategies such as "What-If-Not...?". Should problem finding evolve from teacher questioning?

Using analogy is not a simple task. Domain specific knowledge is required, sometimes at a sophisticated level. Neglected so far in school mathematics, should analogy now be taught? Students may then start at the presented-problem end of the problem continuum, and move to the created-problem end as they become better at generating problem isomorphs or analogues.

How may a problem posing classroom be organised? Should it be individualised work or collaborative work? On the created-problem end, time for incubation may be needed.

How may problem finding and posing be assessed? Should it be assessed at all? If so, should the assessment criteria be based on the complexity of structure of the problem posed, originality, aesthetics or significance?

Conclusion

This paper has raised more questions than it has answered. This could be due to the multifaceted nature of problem finding, as testified by the number of terms used to describe it and its knowledge, behavioural, attitudinal, cognitive and metacognitive components alluded to in the paper. It has been said that problem solving is the heart of mathematics. Perhaps, problem finding is the soul of mathematics.

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