Creating Creative Mathematical Resources for Class Use

Fong Ho Kheong
Nanyang Technological University, Singapore

Abstract

This paper discusses a study using constructivism concepts in teaching mathematical problem solving. It is based on a model which comprises 4 stages involving learners to construct their own word problems through recalling, relating and reflection of mathematical contents they have learned. The stages of this model are: (1) *Presentation* of *Related Knowledge* (2) *Construction* of *Provoking Words or Pictures (3) Construction of Mathematical Questions (4) Construction* of *Mathematical Knowledge by Self.* The four stages of the model were tried out with a group of 26 in-service teachers who attended an upgrading programme for teaching lower primary mathematics. In this pilot study, the results showed that the model provides an effective guide for helping teachers to grip with mathematical concepts they have learned through the course. The results also showed that the constructivism activities enable the teachers to create their own mathematical problems for class use.

Introduction

Recent research on mathematics education that focuses on constructivism, has been able to provide some underpinning principles to explain creativity exhibited by some children. According to Hatano (1996), knowledge is acquired by construction, not by transmission alone. This can be confirmed from the observation that for the most part, students' problem-solving procedures are not the result of direct classroom instruction but are the products of their own construction of knowledge or individual problem-solving processes. Fong (1994) made an analysis of children's errors in solving word problems and found that they generally encountered procedural problems in completing a solution. This shows that children created their own problem-solving processes and, as a result, some children had not been able to construct appropriate mathematical principles for solving the problems presented to them and some children came out with some forms of solution procedures different from what have been given to them.

Theoretical Background

Research focusing on contructivism has generally discussed the acquisition of knowledge and concepts (Glaserfeld, 1989; Lehman, 1989), and social constructivism (Cobb, 1991) as a means for the transmission of knowledge and concepts. From a pedagogical viewpoint, Von Glasserfeld (1991) and Lehman (1989) have postulated that knowledge is actively constructed by the learner, not passively received from the environment, i.e. it cannot be simply transferred ready-made from teacher to student but has to be actively built up by each learner in his or her mind. Cobb et al. (1991) and Orton (1994) have discussed the implications of constructivism in classroom teaching. Cobb was concerned with social constructivism, which involves social interaction and communication which help individuals to construct their own meanings. Orton discussed two types of activities for acquiring knowledge: concrete activity and mental activity. The former focused on physical activity with concrete objects, which was seen as the most effective means of promoting the learning of specific ideas. Hart (1989), however, asserted that students do not always see or make the connection between the apparatus and the mathematical concepts. These views, from the constructivism perspective, have provided the author the theoretical base for the construction of a teaching-learning procedure that is discussed in below.

The Activity-based Model for Creating Mathematical Resources

When knowledge is transmitted to a learner and he/she attempts to construct his/her own meaning on the subject matter, mental activities such as recalling, relating and reflecting are activated. A person is said to have accommodated the imparted knowledge and integrated it to his/her existing knowledge when some kind of meaning has been formed in the person's mind. This is the constructivism stage at which the person is attempting to find some form of meaning, from which the acquired knowledge is established.

The model developed for this study views the construction of knowledge as central to the learning and formation of mathematical concept or creating some form of knowledge as a result of rigorous construction of knowledge.

The model consists of the following steps:

(1) *Presentation* of *Related Knowledge*
(2) *Construction* of *Provoking Words or Pictures* for construction of mathematical problems
(3) *Construction of Mathematical Questions* based on the Provoking words and pictures
(4) *Construction* of *mathematical knowledge by self* (This stage involves self-construction and reconstruction of mathematical knowledge through recalling, relating and reflecting.)

The *Presentation of Related Knowledge* stage involves teacher to take initiative to explain the concept and the process/es & procedures for solving a specific categorical problem. Thorough explanation of the essential facts and concepts is given at this stage. The teacher explains how the problem can be solved and a clear solution is given. (In this regard, the learners are expected to have already mastered the pre-requisites and basic skills for solving the problem.)

Next, during the *Construction* of *Provoking Words or Pictures* stage, pupils would be asked to create some words and pictures which are essential for construction of some mathematical problems which have been discussed at the earlier stage (i.e. The Presentation of Related Knowledge stage). At this stage, children are expected to work in groups of four or five. They need to discuss among themselves (a) the categorical question they need to ask, (b) the essential words they need to use so that they can provoke reader to form word problems based on the words and pictures given to them.

At this stage, another group of pupils are supposed to apply the words and pictures given to them to form some word problems. This is the stage at which the pupils create their own questions. If this is given to different groups of pupils, each group may come out with different types of questions. At this stage they have the opportunity to construct the concepts which have learned. The process involves recalling, relating and reflecting on what they have learned in the transmission stage. In the process of construction of knowledge, the learners may reorganise their ideas and accommodate some of the new ideas into their existing concepts in solving the problem.

The final stage of the model involves pupils to reconstruct their own word problems without the use of any helping words or pictures. The accuracy of the problems may be checked by teachers or their peers.

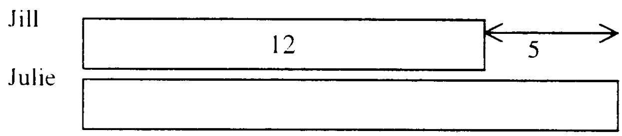
Example

The model above is illustrated below using an actual example which can be applied in a classroom situation. An addition problem involving comparison is used as an example for this purpose.

Jill had 12 wooden beads.
Julie had 5 wooden beads more than Jill.
How many beads had Julie?

<u>*Presentation* of *Related Knowledge*</u>

At the initial stage, a teacher may demonstrate the structure of representing the addition concept using bar diagram as follows. Pupils may be taught to use concrete bars to show comparison between two bars representing the numbers 12 and 5.



From the diagram,

$$\begin{aligned} ? &= 12 + 5 \\ &= 17 \end{aligned}$$

Julie had 7 beads.

The children may be taught to recognise the structure of the model by relating the language structure, the words used and the diagram above. Then using the concept of equal lengths, the unknown bar is equal to the sum of 12 (what Jill had) and 5 (the difference between Jill and Julie had).

At this stage, the teacher attempts to transmit the knowledge for solving addition problem involving the comparison concept. The focus is to transmit essential concept to children to solve word problems. How much children are able to conceptualise the mathematical structure and apply it to a new problem will depend on the way they interpret, relate and construct the knowledge. The solutions resulted from various children support the constructivism theory which states that individual child will construct ideas differently depending on the way they form their ideas

<u>*Construction* of *Provoking Words or Pictures*</u>

At this stage, 4 or 5 children may be formed into a group for peer discussion. They will discuss and decide the categorical question they would like to ask. Once they have decided they will come out with some helping words and diagrams. On the basis of these words and diagrams, one of the members will be asked to write a word problem using the category of question they have decided. Getting one of them to write a question helps them to validate the words and diagrams they have designed. From the given example above, the following words may be suitable to provoke a comparison addition question:

Jill *John* *12* *5*
Beads *more than* *how many*

Pictures of beads, Jill and John may be drawn. Once this is done, this group is ready to hand over to another group of pupils to work out some questions.

<u>*Construction of Mathematical Questions*</u>

The next group will be given the set of words and pictures to work on. Using those words and pictures, they have to create one or more questions on word problems. Probably they may come out with identical or similar problems. They may also create an entire context question which the first group had in mind.

The following are two similar problems to the one which the first had in mind when they produced the above words or numbers.

(a) John had 12 beads and Jill had 5 beads.
How many beads had John more than Jill?
(b) John had 12 beads more than Jill.
If John had 5 beads, find the number of beads John had.

The first question is a subtraction problem. It required you to find the difference between the two numbers. The second question is an addition problem requiring you to add the two numbers. This result shows that different types of questions may be created using the same set of words and pictures.

Construction of Mathematical Knowledge by Self

So far the activity involves pupils into group and one of the purposes for group activity is to allow weaker pupils to interact with better pupils. This enables them to conceptualise and construct the essential knowledge required for this topic. However, opportunity should also be given to each pupil to work individually. In this aspect, each pupil is required to construct their own questions using the different resources (words and pictures worksheets) gathered from the various groups.

Pilot Study

A pilot study was carried out to try out the model of creating mathematical resourses by a class of inservice teachers. A class of 26 teachers participated in this study. They were inservice teachers who had at least some years of teaching experience. They were mainly upper primary level teachers who were expected to teach at lower primary levels of primary schools after they had attended the course.

Stage 1

Initially the teachers were first presented with the different mathematical structures representing addition operations from different types of addition word problems. At this stage, they were expected to conceptualise and to construct the various structures after the representation on various addition word problems. Generally, there are 4 different types of addition word problems: They are:

(a) *Adding-on Concept*
Ann had 5 apples. Bill gave her 4 more apples.
How many apples had Ann now?

(b) *Putting-together Concept*
Ann had 5 red apples from Mr Li. She had also 7 green apples from Mrs Sim.
How many apples had Ann now?

(c) *Comparison Concept*
Ann had 5 apples. Bill had 3 apples more than Ann.
How many apples had Bill?

(d) *Part-whole Concept*
A class had 24 boys. The rest of the pupils were 15 girls.
How many pupils were there in this class?

Stage 2

The second part of the lesson involved the teachers to form into various groups. Each group was expected to create some words and pictures from which addition problems can be constructed. Appendix 1 shows some of the works of teachers who came out with various types of provoking words and pictures.

Stage 3

When the provoking words and pictures have been constructed, the teachers exchanged their work with another group. Based on the work of the previous group of teachers, the latter group would attempt to come out with addition problems using the given words and pictures. Beside constructing one-step word problems, some teachers even attempted to form two or 3 step word problems. Examples of these problems can be found in Appendix 1.

Discussion

A model for teaching mathematics which comprises the four stages: (1) *Presentation* of *Related Knowledge*, (2) *Construction* of *Provoking Words or Pictures*, *(3) Construction of Mathematical Questions (4) Construction* of *Mathematical Knowledge by Self*, was developed for this study and tested on a group of in-service teachers. The results showed that the teachers were able to conceptualise the addition word problem concepts and they were able to produce some creative materials for class use. The principle under which the model was developed was constructivism which was effective to help teachers developed mathematical questions. The fact that the teachers were able to construct mathematical questions according to the addition concept given to them indicated that constructivism plays an important role in teaching matheamtics. It is not only that they were able to construct mathematical questions they were able to create a new form of mathematical questions by recalling and relating ideas they have acquired before. This approach of teaching allows teachers to optimise the use of constructivism in learning how to solve mathematical problems. This study has shown that integrating constructivism into mathematics pedagogy probably strengthens learners' mental structure so that they can understand and retain the concepts more effectively.

Further investigation needs to be made concerning the following aspects of the research: Would the pupils from schools be able to produce the same work as what the inservice teachers do? Have they performed less competently if the constructivism stage were removed from the model?

References

Brown, J.S. & Burton, R.R. (1978). Diagnosis models for procedural bugs in basic mathematics skills. *Cognitive Science*, 2, 155-192.

Cobb, P.; Wood, T. and Yackel, E. (1991). A constructivist approach to second grade mathematics. In E. von Glassfeld (Ed), *Radical constructivism in mathematical education*. Dordrecht: Kluwer.

Engelhardt, J.M. (1982). Using computational errors in diagnostic teaching. *The Arithmetic Teacher*, April, 16-19.

Fong, H.K. (1994). Information processing taxonomy (IPT): An alternative technique for assessing mathematical problem solving. *Singapore Journal of Education*, 14(1), 31-45.

Hart, K. (1989). *Children's mathematical frameworks 8-13: a study of classroom teaching*. England: NFER-NELSON.

Hatano, G. (1966). A connection of knowledge acquisition and its implications for mathematics education. In Steffe and Nesher (Eds.), *Theories of mathematical learning*. N.Y.: Erlbaum.

Lerman, S. (1989). Constructivism, mathematics and mathematics education. *Education Studies in Mathematics*, 20(2).

Mayer, R. (1983). *Thinking, problem solving, cognition*. New York: Freemann.

Newell, A. and Simon, H. (1972). *Human problem solving*. Englewood, N.J.: Prentice Hall.

Orton, A. (1994). Learning mathematics: implications for teaching. In Anthony Orton and Geoffrey Wain, *Issues in teaching mathematics*. London: Cassell.

Roberts, G.H. (1968). The failure strategies of third grade arithmetic pupils. *The Arithmetic Teacher*, May, 442-446.

Von Glasserfeld, E. (Ed) (1991). Learning mathematics: implications for teaching. In Anthony Orton and Geoffrey Wain, *Issues in teaching mathematics*. London: Cassell.