Title	Types of mathematics learners
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TYPES OF MATHEMATICAL LEARNERS

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Abstract

A survey was conducted among 216 Brisbane Year 11 students to explore their self-reported study behaviours in mathematics. Factor analysis of the data produced 9 meaningful factors which were called Diligence, Diversity, Plan, Notes, Memory, Hint, Conformity, Check and Interaction. The pupils were clustered on the basis of these factors into 5 groups known as Conscientious, Independent, Cue-seeking, Rote, and Minimal pupils. The Independent and the Cue-seeking pupils were more successful, while the Rote pupils least successful in a school algebra test and a common calculus test. Pupils generally did not explore and link mathematical ideas together in different ways. The effects of these study behaviours were rather stable. Hence, efforts to help students learn mathematics in an effective way should begin as early as possible.

Introduction

Learning is a highly individual experience. Although pupils may be highly motivated and are willing to put in the necessary effort, this does not always guarantee success, unless they also possess the proper skills in utilising appropriate prerequisite knowledge to master the new learning task. To acquire understanding may require a different approach when compared with rote learning of facts. Pupils' study skills represent their input into the complex interaction between learner, teacher, and instructional materials that make up the major components of the school learning process (Keefe, 1979).

In classroom interaction, the teacher may organise a variety of activities and materials according to some optimal sequence under a conducive atmosphere for learning to take place. However, once the lesson is over, the pupils are expected to do some studying on their own; for instance, to complete assignments, to revise work constantly, to do extra readings, and so on. These activities call into play a set of learning behaviours that are probably under the conscious control of the pupils. These behaviours have been investigated under many different names: learning strategies (Dansereau, 1978), learning styles (Keefe, 1979), study behaviours (Biggs, 1978,), study habits (Holtzman & Brown, 1968), study methods (Entwistle & Wilson, 1977), and others. According to Dansereau (1978), the usually inconclusive effects reported in research on teaching methods may be the result of inefficient learning strategies adopted by the pupils. Furthermore, emphasis on a particular teaching approach, especially rote memorisation, may inadvertently reinforce inefficient study techniques, which limit a pupil's cognitive awareness. Thus, "Educators and researchers should be redirecting at least some of their efforts to the development and training of

appropriate learning strategy skills" (*ibid*, p.2). However, before such training can be undertaken, more information about how pupils actually learn and the effectiveness of different strategies in learning various school subjects should be sought.

This study was conducted for two reasons. First, it is important to know more about study behaviours that can lead to understanding rather than rote learning. Second, in view of the explosion of knowledge, the ability of "learning to learn" becomes an important survival skill.

Literature Review

Research on study behaviours in school mathematics is practically nonexistent. Begle (1979), after reviewing empirical research in mathematics education from 1964 to 1976, found only three studies relating to this issue. Unfortunately, he did not provide details of these studies. An ERIC search was equally fruitless. Walkden and Scott (1980) observed how two weak first year university students learned mathematics and found that they (a) used the "number juggling" technique to mimic worked examples in notes or books in order to solve exercise problems, (b) depended on rote learning, and (c) made no attempt to learn and memorise general mathematical principles. Such isolated anecdotic evidence does not provide a comprehensive picture of the complexity involved in learning school mathematics.

Research conducted on tertiary students from non - mathematical disciplines provides some guidance to the relevant dimensions to be included in this study. Hence, a brief review of this research would not be amiss here.

Early research on study habits concentrated on activities such as selecting a quiet place and a suitable time to study, keeping a study schedule, underlining, using mnemonics, and so on. It also includes the famous Robinson's SQ3R (1970): *survey* the text for headings and key ideas, ask *questions* about these ideas, *read* the text with the aim of answering these questions, *recite* what has been read in own words with the book closed, and *review* the material. Correlations of scales on these study habits with grade point averages ranged from .32 to .66 (Holtzman & Brown, 1968).

Dansereau (1978) distinguished between primary strategies and support strategies. Primary strategies are techniques a person uses to operate on the materials, while support strategies are techniques for establishing an appropriate learning attitude, for coping with loss of concentration, for monitoring and correcting ongoing primary strategies. The effective learners should be aware of their available learning strategies so that they can select those that are appropriate to the demands of various learning tasks (Ford, 1981).

Other researchers (Biggs, 1978; Entwistle & Wilson, 1977) assume that the use of a particular learning strategy is strongly related to pupils' psychological traits. Some general conclusions from these studies include:

1. Not all pupils follow the same path to academic success.
2. Pupils may adopt different learning approaches depending on the discipline, the kinds of examination questions expected, the mode evaluation and so on.
3. Learning approaches may be related to motivation and anxiety.
4. There are sex differences in approaches to study.
5. Systematic organisation seems to be the most consistent correlate of academic achievement.

The Present Study

A survey was conducted in 1982 among 216 Brisbane Year 11 students to explore their self-reported study behaviours in mathematics.

The questionnaire on study behaviours consists of 40 Likert type items that attempt to measure the various aspects of study behaviours identified in the review. On the basis of factor analysis, 9 scales were produced to delineate the main components of these study behaviours. (For details, see Wong, 1984). These scales (with Cronbach alpha given in parentheses) can be described as follows:

DILIGENCE (.83): pay attention in class, learn from mistakes, master all work, hand in work on time
PLAN (.75): adopt study plan, revise constantly, set goals for study period
HINT (.45): Pay attention to teacher's hints about test, classify problems according to types and memorise worked examples
INTERACTION (.33): Compare notes with friends, get help with difficulty
NOTES (.58): use notes for homework and revision, do not rely on memory
DIVERSITY (.81): different ways of solving mathematics problems, read more about mathematics, do mathematics puzzles and games, link mathematics ideas together
CHECK (.37): Convince oneself that rule is correct, judge correctness of answers
MEMORY (.62): Memorise definitions, nervous before test because of many rules to remember, confused by different methods
CONFORMITY (.55): Concentrate on how to use rules rather than to derive them, accept everything as correct, copy everything exactly, learn only one method

These scales can be related to the three approaches described by Biggs (1978). The Diligence, Plan, Hint, Interaction and Notes scales describe an "organised, achievement-oriented" approach. The Diversity and Check scales form part of a "deep" or "internalising" approach. The Memory and Conformity scales describe a "surface" or "reproducing" approach.

Cluster Analysis

Cluster analysis is a technique used to group together subjects whose scores on a given set of variables show a high degree of similarity. There are many types of cluster analysis using different similarity coefficients (Anderberg, 1973). This

technique has been used to identify meaningful clusters of students, classes, or teaching styles (Bennett, 1976; Entwistle & Wilson, 1977; Solomon & Kendall, 1979).

In this study, the HGROUP program (Veldman, 1967) was used because of its simplicity. The HGROUP program begins with each object (pupil in this case) as a "group" and then uses a stepwise process to group the objects two at a time on the basis of a minimal increase in a "value-reflecting" function based on the squared deviations from group means. At each step in the combination process, the minimal increase in this function is reported as an "error". The procedure stops when all the objects have been combined into two groups. The number of groups worthy of study is determined at the stage where there is a large increase in "errors". Once the number of groups is selected, the properties of the groups can be identified by examining their profile scores on the original set of variables. This technique was applied to the 9 study behaviours scales in order to obtain types of mathematics learners.

Types of Mathematics Learners

Five types of pupils emerged from the analysis. See Table 1 for the profiles of pupil types.

Table 1: Means of Study Behaviour Scales for Pupil Types

	Type						
	1	2	3	4	5	Whole	F-ratio
(n =)	73	50	36	28	29	216	
DIVERSITY	2.4	2.5	1.8	1.6	1.5	2.1	36.66
PLAN	3.2	2.7	2.3	2.1	1.9	2.6	28.46
INTERACTION	3.0	2.5	2.6	3.1	2.1	2.7	10.28
MEMORY	3.0	2.3	2.7	3.7	3.1	2.9	27.00
CHECK	3.6	3.1	2.4	3.2	2.4	3.1	33.02
CONFORMITY	3.5	2.9	3.8	4.1	3.9	3.5	28.60
DILIGENCE	3.9	3.7	3.7	2.9	2.9	3.6	37.07
HINT	4.0	3.2	4.2	3.6	2.6	3.6	34.57
NOTES	4.4	3.4	3.9	4.1	3.3	3.9	25.60

Note: Scales range from 1 to 5. All F-ratios are significant at the .0001 level. Type 1: Conscientious, 2: Independent, 3: Cue-seeking, 4: Rote, 5: Minimal

From the Total group means, it can be seen that on the average, a typical mathematics pupil was diligent but he or she tended to accept the mathematics given as true. He or she relied on teacher's notes and paid attention to what was important for learning. There was little attempt to study mathematics beyond what was required or to explore mathematical ideas in diverse manners.

However, this general pattern becomes more complicated when the five types are examined for possible meaningful clustering.

The *Conscientious* pupils employed most of the strategies in their learning. They frequently copied everything exactly, but at the same time, tried to check that these notes made sense.

The *Independent* pupils were also diligent, checked that the work made sense, and paid attention to teacher hints. However, they seldom planned their study, discussed mathematics with classmates, or memorised things exactly. They projected an image of learning mathematics in their own, probably active, way.

The *Cue-seeking* pupils were diligent, paid attention to hints and copied everything exactly. However, they seldom checked whether the work made sense or not, planned their study, discussed mathematics, or did mathematics in different ways. These pupils probably relied on external cues to determine what they should learn.

The *Rote* pupils, though not particularly diligent, tried to memorise everything as given. They sometimes compared notes probably to ensure that they had copied these notes and teacher hints correctly.

The *Minimal* pupils, with comparatively low means on most scales, expended as little effort as possible in their study of mathematics. Like the Rote pupils, they accepted the mathematics given as true.

Some Characterisation of Pupil Types

There was no significant sex difference.

Pupils' first semester mathematics scores on Preparatory Mathematics were used as a measure of their prior achievement. Their subsequent scores on Algebra were used as the dependent variable. Since these scores were school-based and hence not directly comparable, it was decided to "standardise" individual scores within each class to maintain their relative class positions and at the same time to reduce the effect of differential standards among the classes. These scores are reported in Table 2.

Table 2: Sex and Achievement Scores for Pupil Types

Type	Sex		Prior	Algebra	Adjusted
	Male	Female	Score	Score	Mean
CONSCIENTIOUS	36	37	0.00	0.03	0.04
INDEPENDENT	34	16	0.47	0.49	0.09
CUE-SEEKING	19	17	0.30	0.20	-0.03
ROTE	18	10	-0.71	-0.53	0.02
MINIMAL	17	12	-0.50	-0.68	-0.27
Whole Sample	124	92	0.00	0.00	

Significant differences were obtained for the prior score and the subsequent algebra score. The Independent and the Cue-seeking pupils were more successful while the Rote and the Minimal pupils were the weaker ones. The Conscientious pupils,

forming the largest group, generally had average achievement.

The differential effects of various patterns of study behaviours on subsequent achievement were confounded by a similar pattern for prior achievement. An analysis of covariance was used to control indirectly group differences in prior achievement so that more precise analysis of subsequent achievement could be studied. The ANCOVA showed that the prior score accounted for a substantial 67% of variance in subsequent algebra score. It appeared from the adjusted means in Table 2 that no significant differences were obtained. This indicated that study behaviours contributed little more to the prediction of subsequent achievement when prior achievement was included. The similar pattern of differences in prior and subsequent achievements, however, suggests that study strategies had stable effects on achievement. This suggestion needs to be further investigated using a longitudinal approach.

81% of the students also took a common Calculus test near the end of their Calculus course. This test has two components that provide an Instrumental score that covers skills of differentiation and a Relational score that examines concepts of differentiation and limits. The means of these scores are given in Table 3.

Table 3: Means of Calculus Scores for Pupil Types

Type	n	Total	Instrumental	Relational
CONSCIENTIOUS	63	24.6	7.9	7.6
INDEPENDENT	46	27.5	9.5	8.8
CUE-SEEKING	27	26.0	8.1	8.9
ROTE	17	19.1	4.6	7.9
MINIMAL	22	20.4	5.5	6.8
Whole Sample	175	24.5	7.7	8.0

Note: Maximum marks are 69 (Total), 23 (Instrumental) and 22 (Relational).

Again, the Independent and the Cue-seeking pupils scored significantly higher than the other pupil types on the whole test and the Instrumental component. But there was no significant difference for the Relational score. ANCOVA with prior achievement as the covariate did not produce significant result for the Total and Instrumental scores. There was significant difference at 5% level for the Relational score. The regression lines for various Pupil types on the Relational score are plotted.

A striking interaction was found for the Independent pupils. Among the weaker pupils, those who were classified as Independent tended to have the worst Relational score. A possible reason is that their 'independence' may indicate a failure to attend to the important discussions that went on in the lessons. The situation was reversed for the more able pupils. In this case, the Independent pupils had the highest Relational score. Hence, their 'independence' may indicate that they had understood the lessons so well that much out of class studying was not very necessary.

The Relational scores of the other 4 types clustered around the mean, showing a generally low performance on the Relational items, irrespective of the types of study strategies reported.

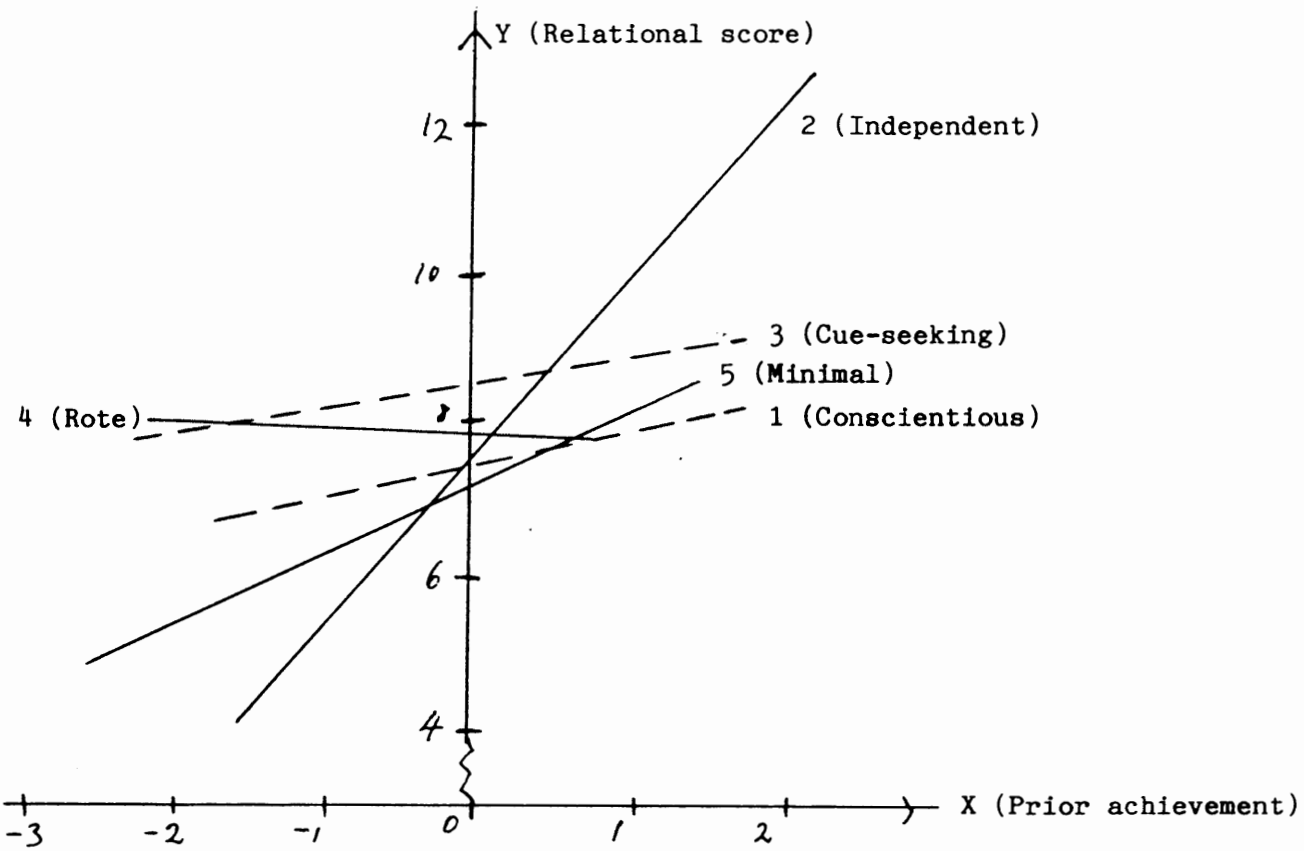

Table 4 shows results of pupil types with respect to attitudes toward mathematics, self-concept in mathematics, and motivation towards learning mathematics.

Table 4: Personal Characteristics for Pupil Types

	Type				
Variable	1	2	3	4	5
Maths is Easy	3.0	3.4	3.0	2.2	2.5
Enjoy Maths	3.7	3.8	3.1	2.5	2.8
Maths is Important	3.7	3.6	2.9	3.0	2.8
Self-concept	3.5	3.7	3.4	2.7	2.8
Motivate to learn Maths	0.9	0.9	0.8	0.5	0.5

Note: The first 4 scales range from 1 to 5; the Motivate scale ranges from 0 to 1.

The Conscientious and the Independent pupils had more positive attitudes towards mathematics, were better motivated and had higher self-confidence than the other types of pupils.

Conclusion

Five pupil types were identified from pupils' self-reported study behaviours. The types differed in mathematics achievements and various affective characteristics. This lends support to the assumption of linking study behaviours to personal characteristics. The more successful pupils adopted an active approach defined by planning, checking for understanding, and working diligently. On the other hand, a passive approach that relies on rote learning, memorisation and conformity was related to poor achievement.

Survey data may not reflect accurately what pupils actually do. Hence, the above characterisation needs to be further explored with case study methodology preferably conducted over a long period of time.

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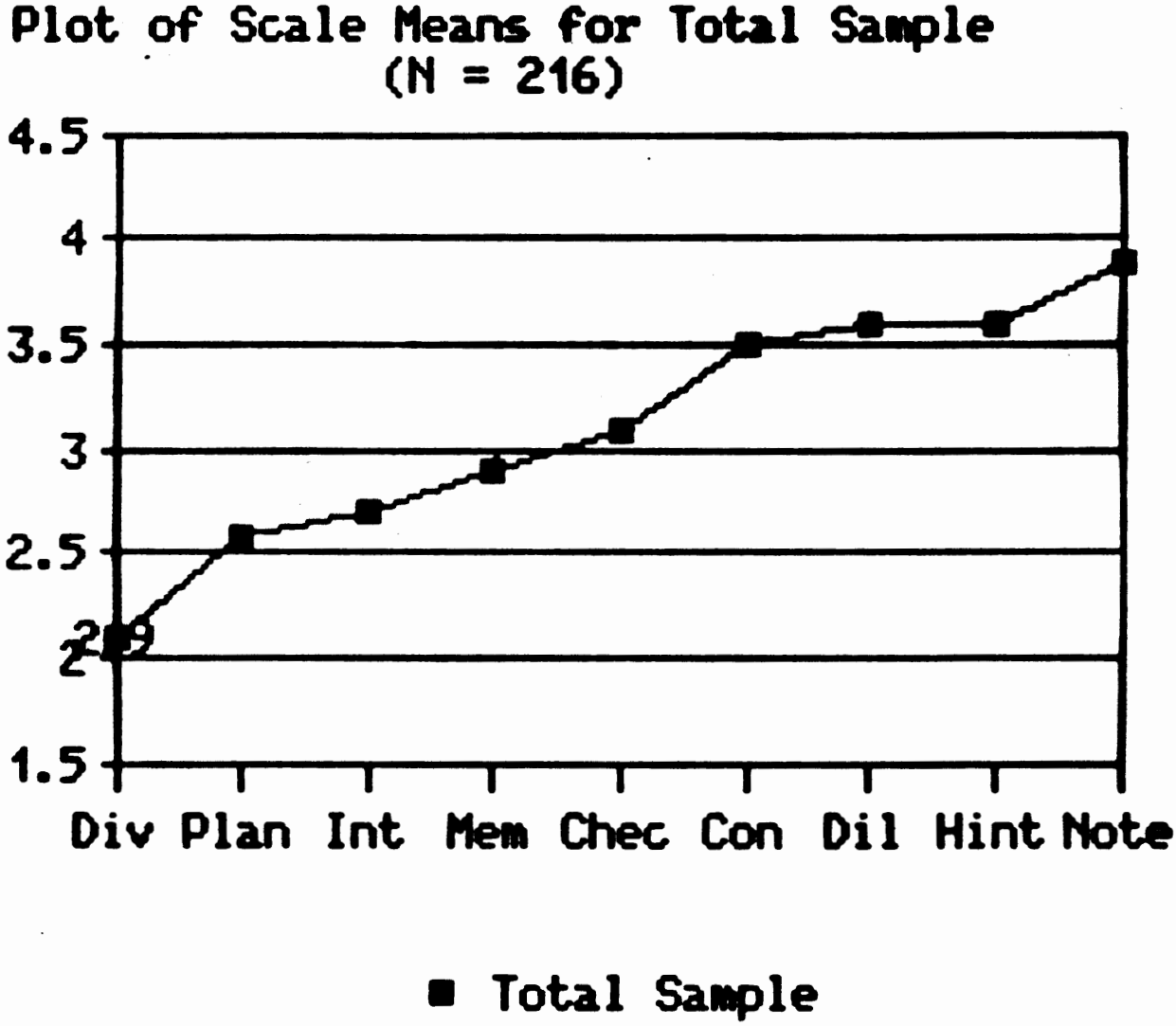
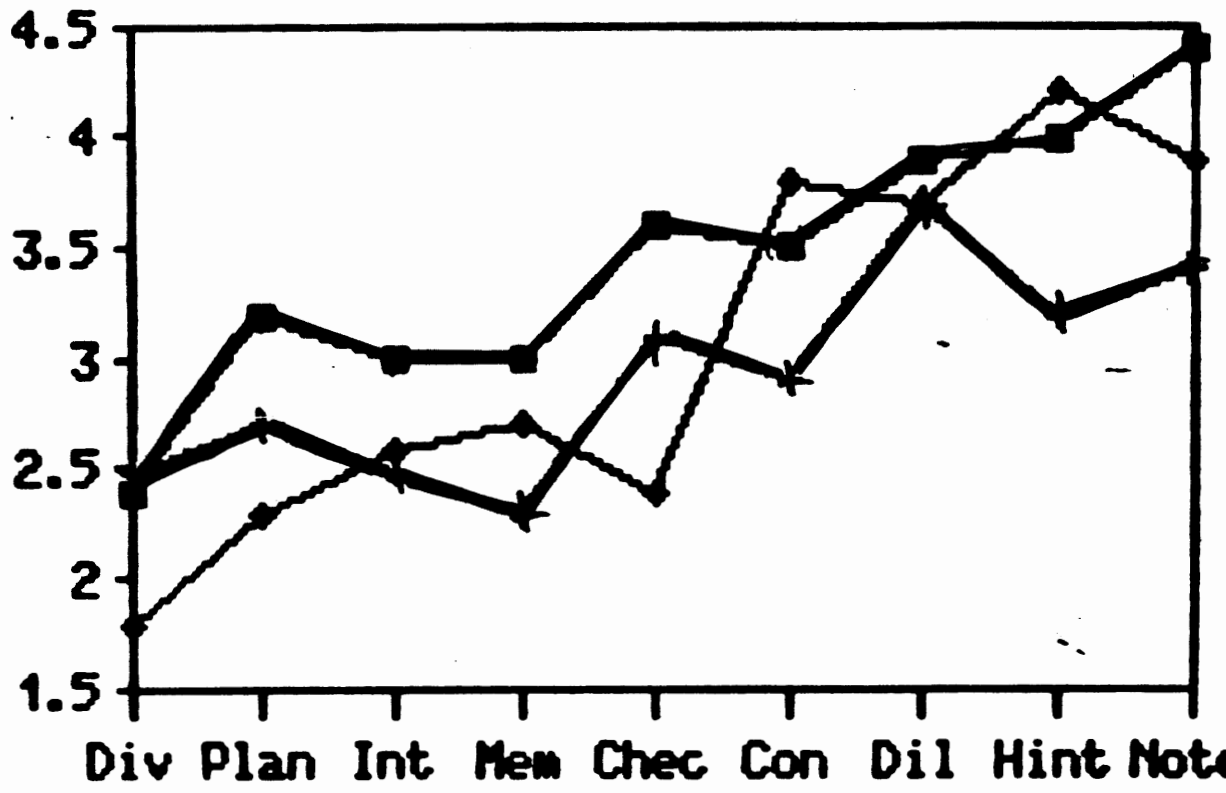
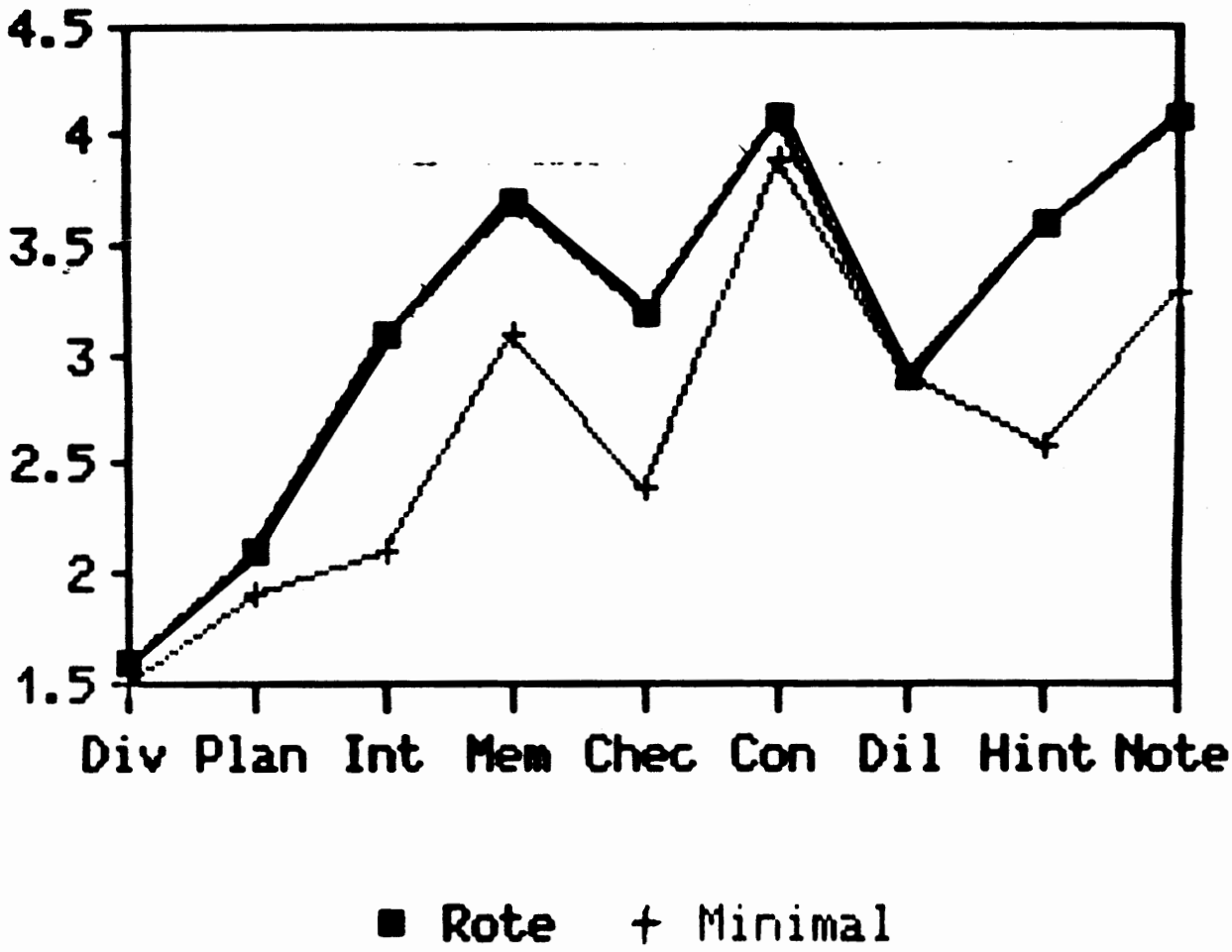

Plot of Scale Means for Total Sample
(N = 216)
4.5
4
3.5
3
2.5
2
1.5
Div Plan Int Mem Chec Con Dil Hint Note
Total Sample

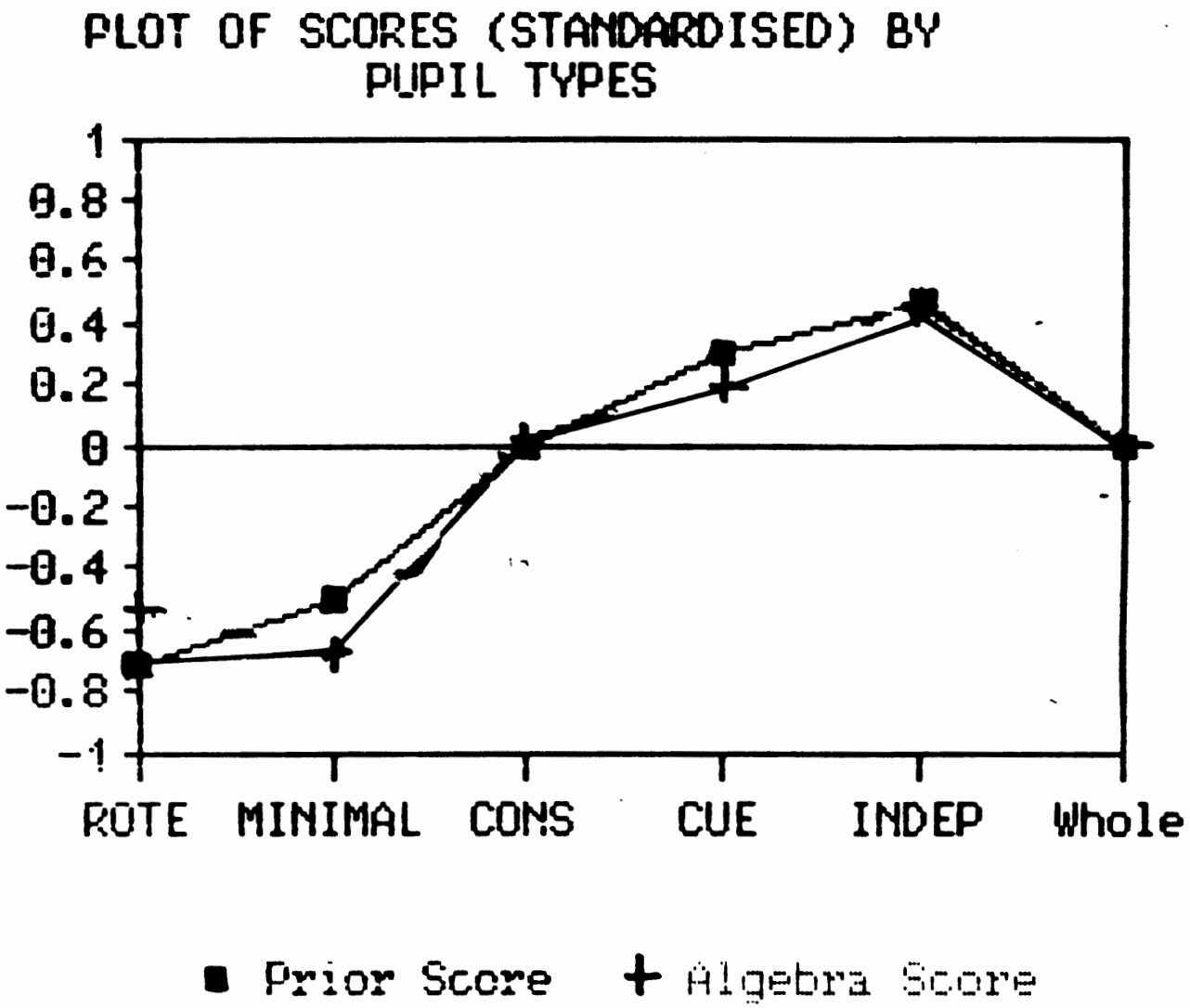
Plot of Scales with Pupil Types



■ Conscientious + Independent ◆ Cue-seeking

Plot of Scales with Pupil Types




PLOT OF SCORES (STANDARDISED) BY
PUPIL TYPES
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
ROTE
MINIMAL
CONS
CUE
INDEP
Whole
Prior Score
Algebra Score

PLOT OF MATHS SCORES BY PUPIL TYPES