# METACOGNITIVE STRATEGIES IN MATHEMATICAL PROBLEM SOLVING

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Problem solving is a complex task involving many types of knowledge and skills. While it is agreed that to be a successful problem solver, one requires specialized knowledge such as linguistic knowledge, factual knowledge, schematic knowledge, strategic knowledge and procedural knowledge (Mayer, 1987), a number of researchers have also included metacognitive knowledge as another important factor that differentiates between the good and the average problem solver (Gagne, 1985; Briars & Larkin, 1984, Lester 1982). Past studies on metacognition have concentrated on tasks involving reading and memory work and little work is done with metacognition in problem solving. Only lately have researchers begun to look at metacognitive skills in problem solving and have started to develop theoretical frameworks (Garofalo & Lester, 1985; Lester, 1985, Schoenfeld, 1985).

## What is metacognition?

Metacognition is generally considered as "knowing about knowing" or what Schmitt and Newby (1986) refers to as "a body of knowledge that reflects knowledge itself". In other words, metacognition involves knowing the cognitive processes associated with an instructional task, and being able to use and monitor appropriate cognitive processes during the task. Although metacognition has been loosely defined, most psychologists (e.g., Brown, 1978; Flavell, 1976) consider metacognition to consist of two separate but related aspects (a) **knowledge about cognition** which includes "one's knowledge concerning one's own cognitive processes and products or anything related to them " (Flavell, 1976, p. 232) and (b) **regulation of cognition** which refers "to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective (p. 232)."

## Knowledge about cognition

Knowledge about cognition implies that a person is knowledgeable about variables that will affect one's instructional performance in a learning situation or during an instructional process. Drawing from their research on metamemory, Flavell and Wellman (1977) suggested three variables that could influence a person's performance. They are **person** variables, **task** variables and **strategy** variables. According to Garofalo & Lester (1985) these variables are applicable to one's performance in problem solving.

Knowledge about person variables involves knowing one's own cognitive resources, strengths, weaknesses and cognitive abilities. Garofalo and Lester (1985) proposed that in mathematics, this knowledge should include one's beliefs of own mathematical ability, affective characteristics such as motivation and anxiety, and the relationship between mathematics achievement and achievements in other subject areas. Awareness of one's weaknesses or strengths in certain topics of mathematics, realizing that one is careless in computation and tends to make computational mistakes, and recognizing that one is weak in processing spatial and visual information, are some examples of this aspect of knowledge.

Knowledge about task variables implies that the individual knows the type of cognitive demands required in an instructional situation. Students' awareness of the different effects of text structures such as vocabulary, syntax, and structure, on the difficulty of word problems is an example of task knowledge. There is empirical evidence which shows that students are aware of the type of cognitive demands in solving word problems. For example, Garofalo & Lester(1985) cited findings from their research which showed young children believed that (a) the type of numerical information in a word problem is an indicator of the difficulty of the word problem, (b) word problems are harder to solve than computation problems, (c) word problems can be solved by directly applying one or more arithmetic operations and (d) the correct operations depend on identifying the right "key words".

When an individual knows what strategies to use during an instructional situation to obtain the best results, then he is said to possess knowledge about strategy variables. Slife, Weiss, and Bell (1985) found that students who were mathematically weak but with IQ similar to regular students, had less knowledge about their problem-solving skills. Peterson and associates (Peterson, 1988; Peterson & Swing, 1982; Peterson, Swing, Stark, & Waas, 1984) investigated the effects of various metacognitive processes students used during a normal classroom mathematics instruction. In their studies, they videotaped the lessons and employed a stimulated recall procedure to probe students' cognitive processes when they were engaged during teacher instruction and also during seatwork. Among the many findings, they found that students were knowledgeable about cognition as shown by the various strategies used, such as, applying information at specific level, reworking problems, rereading the text found in problems, relating new information to prior knowledge, trying to understand the lesson or trying to solve a problem by using a specific operation.

Regulation of cognition

The regulation aspect of metacognition involves "decisions one makes concerning when, why, and how one should explore a problem, plan courses of action, monitor one's actions, and evaluate one's progress, plans, action and results" (Callahan & Garofalo,1987; p. 22). It is believed that this regulation process is controlled by one's cognitive knowledge. For example, if a student believes that he/she tends to make computation mistakes, then the student has to be cautious whenever there are computation operations to perform and should attempt to monitor the operations carefully, to check and to evaluate the answer.

This self-regulation process is important for successful problem solving. Schoenfeld(1985) after analysis of college students' protocol of their problem solving processes, concluded that "the absence of monitoring and assessment at control level can guarantee failure in problem solving" (p.316). Despite its importance, this cognitive procedure is not clearly demonstrated by young children and college students. Garofalo & Lester (1985) in their research found that young children did not routinely analyze information provided in the problem and did not monitor progress or validate the results. Similarly, Stifle et al. found that young students who were not mathematically inclined did not monitor their progress during problem solving. College students too, were not very efficient in regulating their problem solving behaviours. Schoenfeld (1985) found that the overall quality of college students' monitoring, assessing and executive decision-making in problem solving were relatively poor.

Metacognitive strategies in problem solving

Using Polya's (1957) heuristic problem-solving model as a foundation, Lester and associates (Lester,1985; Garofalo & Lester, 1985) proposed a cognitive-metacognitive framework for performance in various mathematical tasks. The framework consists of four cognitive components of orientation, organization, execution and verification. The four components correspond to Polya's four phases of problem solving of understanding, planning, carrying out the plan, and looking back. However, Lester differentiates his framework from Polya's as he believes that his "model purports to describe the categories of the cognitive component in terms of points during problem solving where metacognitive actions might occur" (Lester, 1985; p. 62). Table 1 shows a summary of the structure of the framework (for full details, see Lester, 1985).

Each component is controlled by metacognitive decisions made by the individual and the type of decisions will depend on his/her own knowledge of metacognition. Thus individual's metacognition knowledge of person variables, task variables and strategy variables will influence the individual's action in the four components. For example, in the cognitive component of orientation, an individual may want to rephrase the text in order to help him/her to understand the problem situation or if the individual believes that he is better at processing visual information, he/she may reorganize and represent the text information visually. Thus, an individual with better metacognitive knowledge can use his/her "executive decisions" for better planning, execution, and monitoring of the problem solving process and, hopefully, achieve a higher success in solving problems.

The depth of one's metacognitive knowledge can influence the type of metacognitive strategies one uses in problem solving. For example, in the orientation component, an individual may use different types of metacognitive strategies: "surface" strategy such as re-reading the problem, or "deep" strategy such as recalling old materials to link new materials found in the problem or "achieving" strategy such as analyzing and representing problem information in another format. Biggs (1987) defined surface strategies as superficial strategies and are reproduced through rote learning , deep strategies as meaningful ones that are "interrelated with previous relevant knowledge" while achieving strategies as those "based on organizing one's time and working space (that is) behaving as a model student" ( p.11).

**ORIENTATION**: Strategic behavior to assess and understand a problem

A. Comprehension strategies
B. Analysis of information and conditions
C. Assessment of familiarity with task
D. Initial and subsequent representation
E. Assessment of level of difficulty and chances of success

**ORGANIZATION**: Planning of behavior and choice of actions

A. Identification of goals and subgoals
B. Global planning
C. Local planning (to implement global plans)

**EXECUTION**: Regulation of behavior to conform to plans

A. Performance of local actions
B. Monitoring of progress of local and global plans
C. Trade-off decisions (e.g., speed vs accuracy, degree of elegance)

**VERIFICATION**: Evaluation of decisions made and of outcomes of executed plans

A. Evaluation of orientation and organization
   1. Adequacy of representation
   2. Adequacy of organizational decisions
   3. Consistency of local plans with global plans
   4. Consistency of global plans with goals

B. Evaluation of execution
   1. Adequacy of performance of actions
   2. Consistency of actions with plans
   3. Consistency of local results with plans and problem conditions
   4. Consistency of final results with problem conditions.

Table 1. Cognitive-metacognitive framework

(From " Metacognition, cognitive monitoring, and mathematical performance" by Joe Garofalo and Frank K. Lester, JR., 1985, Journal for Research in Mathematics Education, 16, p.171.)

Objectives of this study

This report is part of an extensive multi-phase study on investigating the learning strategies and metacognitive processes used by secondary school students and trainee teachers. Specifically, this study attempts to answer the following questions:

1. How frequently do students employ metacognitive strategies during mathematics problem solving?
2. Do students from different academic settings differ in their usage of metacognitive strategies?
3. Do students from different academic settings use different types of strategies (surface, deep or achieving strategies) ?

## Method

Subjects

Over 2500 students from nine secondary and four pre-university junior colleges participated in the Institute of Education's research on learning and teaching strategies. The students were from different streams, levels and academic tracks. Approximately one third of the students answered the Language form questionnaire on learning strategies, another third answered the Science and Mathematics form, and the rest answered the Social Studies form.

Seven hundred and seven students answered the Science and Mathematics form. Out of this, 37 sets of data were incomplete thus leaving a sample size of 670. The 670 students came from

(a) three streams, namely, Special Assistance Programme (SAP), Normal Stream (a 5-year secondary school education) and Express stream ( a 4-year secondary school education);

(b) three levels (secondary 2, Secondary 4, and Pre-University 1);

(c) three academic tracks ( Arts, Science and General).

The distribution of students for each stream, level and academic tracks is given in Appendices A, B and C.

Instrument

There were three sections in the questionnaire. The first two sections contained generic items on learning strategies, attitude towards learning and their motives for learning. The third section contained items that were specific to the content area. For example, in the Science and Mathematics form, students were asked about the frequency of usage of metacognitive strategies in solving mathematical and science problems while in the Language form students were asked about their metacognitive strategies in reading comprehension and in listening.

This study reports only on the students' returns in the Science and Mathematics form and on the section asking students about their metacognitive strategies in problem solving. The items in the problem solving section were classified into four phases, following the cognitive-metacognitive framework suggested by Garofalo and Lester (1985) with four items in each component.

(i) Orientation component. The items here concentrated on the process of reading and understanding of the problem (e.g. I analyze and try to understand the information given and draw inferences).

(ii) Organization component. The items here concentrated on the approach and the planning for execution of procedures (e.g. I turn an argument over in my mind a number of times before accepting it).

(iii) Execution component. The items here were directed at finding out how often the students would carry out the plan (e.g. I find that drawing diagrams helps me to solve problems).

(iv) Verification component. Here the items were directed at finding out the frequency various strategies were used to check answers and procedures (e.g. I check over my test to avoid making mistakes).

The questionnaire had been pilot tested, validated and used in a number of research studies (Chang, 1988; 1989).

Procedure

The questionnaire required students to rate each item on a 5-point Likert scale, with a score of 5 indicating a frequently-used metacognitive strategy while a score of 1 indicating a rarely-used or never-used strategy. The questionnaire was administered to the whole class by the class teacher. Most students were able to finish answering the questionnaire within a one-period lesson. The class teacher explained some phrasing of items to students who could not understand the item.

Results

There are five sets of subscores with a maximum of 20 points per set. There is a score for each of the four problem solving components (orientation, organization, execution and verification) and one score for students' problem-solving beliefs. Analysis of variance (ANOVA) with significant level of 0.05 was carried out using the mean scores as the dependent variable. Three separate analyses were conducted with different independent variables, namely, stream; level; and academic track. The results of each analysis are described below.

**Stream.** The three streams of Express, Normal and Special are applicable to secondary schools only. Data from Pre-University students were not included in the analyses.

The means of all the problem phases were found to be statistically different. In all the four phases, Normal stream students scored lower than students from the Express and the SAP stream indicating that Normal stream students had reported less frequent use of metacognitive strategies than students from SAP and Express Stream students (Figure 1). A follow-up test using Duncan's test showed that the means of SAP students and Express students were not significantly different.

The score in the verification component was higher compared to the three other phases. The means for the three phases of orientation, organization and execution were around 12.5 while the means for the verification component were around 15.5.

Based on Biggs' (1987), each item in the questionnaire was classified as either surface, deep or achieving strategy. Appendix A shows the classification of individual items together with the means of each item for the three streams of students. On the analysis of individual item, it was found that Normal stream students used surface strategies more often than deep or achieving strategies. For example, they reported that they used surface strategies like " I need to attend to the instructions carefully in order to get the required results" (mean = 3.57) more frequently than to deep strategies like "I analyse and try to understand the information and draw inferences" (mean = 3.03).

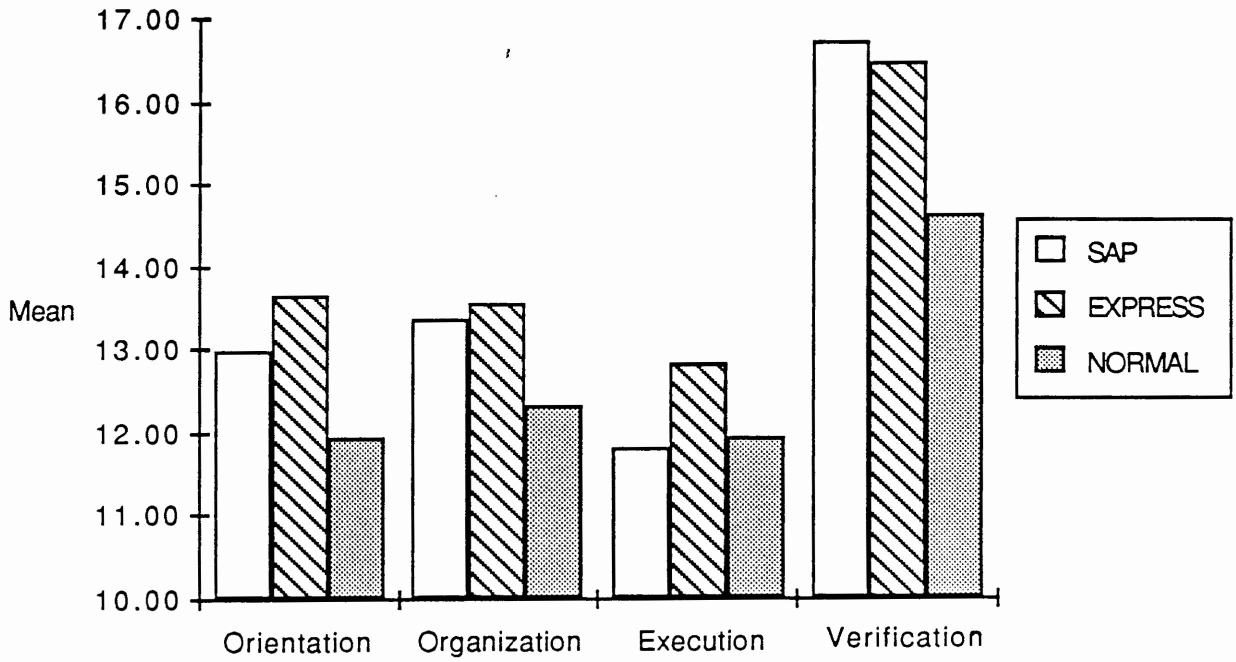



Figure 1: Means of each component by Stream

<u>Level.</u> The means of each component are shown in Figure 2. Statistically, there was no difference in the frequency of usage of metacognitive strategies between students from different levels, viz. Secondary 2, Secondary 4 and Pre-University. Again, the means for the verification component (averaging 16.0) were higher than the means of the other phases (averaging 12.5).

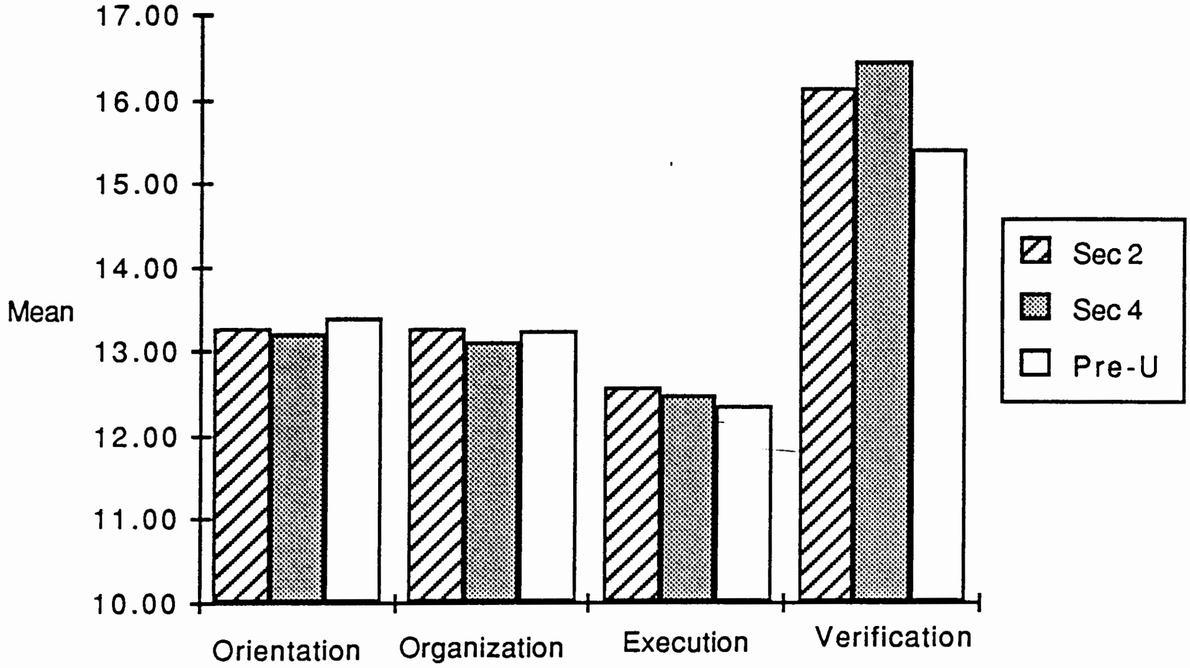



Figure 2: Means of each component by level

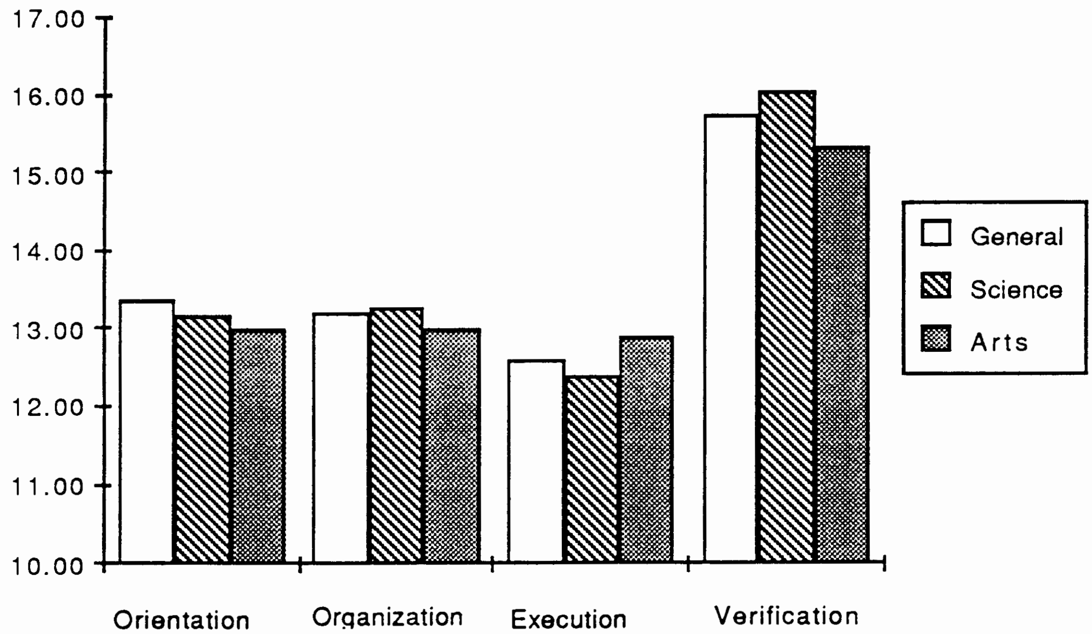



Figure 3: Means of each component by Academic Track

Academic Track. The means of the four phases are shown in Figure 3 and the means were found to be statistically not different for all the three tracks. The means for the verification component were higher than the means for the other three components (averaging 13.0).

Beliefs. Students' problem solving beliefs were investigated through four items (17, 18, 19, and 20). Figures 5, 6, and 7 show the means of the four items for stream, level and academic track. Students from the Normal stream, from Secondary Two and from General and Arts academic tracks believed that certain surface strategies were appropriate for developing problem solving skills. For example, in Item 19 they indicated that they memorized model answers more often than the other students. Similarly, in Item 20, more students from Secondary 2 , Normal stream, academic track of Arts and General, believed that there is only one best way to solve a problem. On the other hand, the Express and SAP students, students from the General and the Arts stream, and Secondary 4 and Pre-University students believed that certain deep strategies (e.g., they needed a lot of drill and practice, that it is important to be able to solve problems set in past-year examination) are important to their problem solving abilities. This is indicated by the higher ratings in Items 17 and 18.

## Discussion

From the results, the mean scores for the four components are above the half-way mark of 10 indicating that students are conscious of metacognitive strategies that are required for problem solving. Most students indicated that they practiced some of these metacognitive strategies at least half the time when they are solving problems. Although the four components are equally important for problem solving, the students practice the verification process more frequently than the other components. The low scores of the three components of orientation, organization, and execution could be due to the lack of emphasis on these three processes during the teaching of problem solving in the classroom. In the verification component, students from all streams, levels and academic tracks scored very high. However, the four items in the verification component of the questionnaire asked students on only one aspect of the verification process. The students were asked about the accuracy of their workings and the accuracy of their answers especially in an examination setting. Since the education system in Singapore is examination-orientated, it is natural for students and teachers to emphasize this aspect of verification.

Fig. 4: Means of Metacognitive Beliefs by Stream

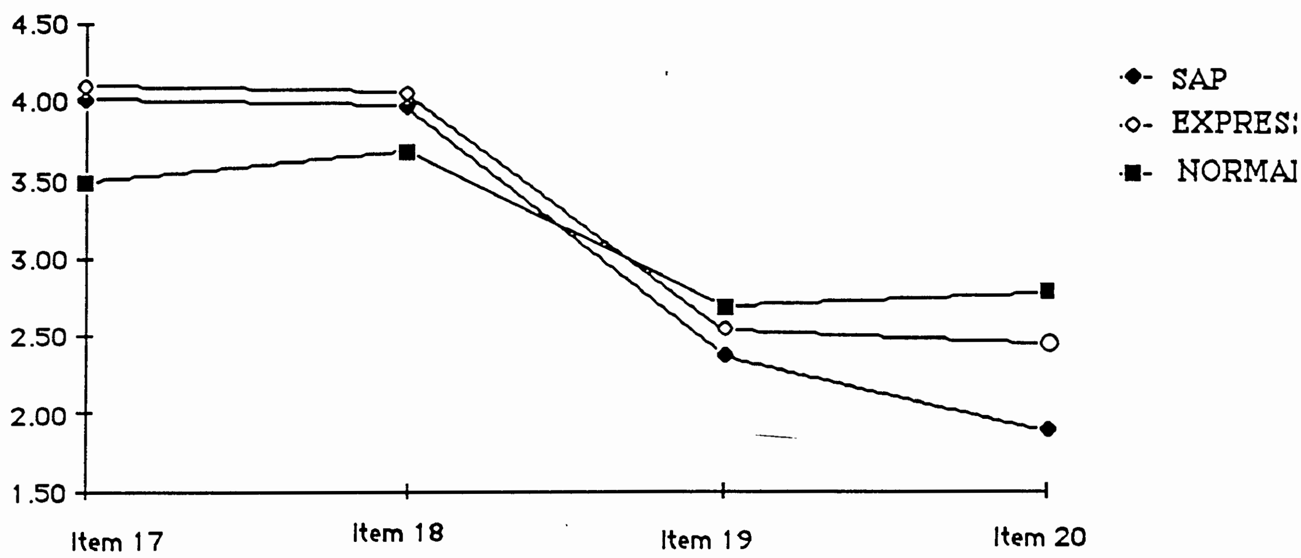
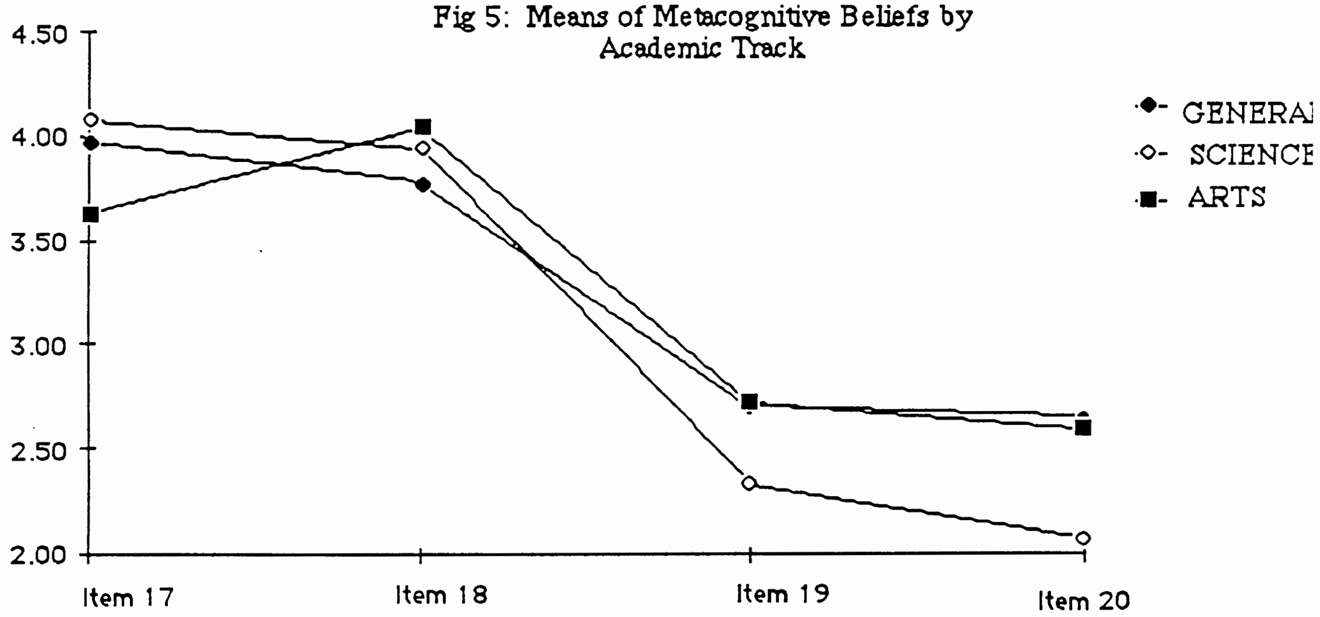



Fig 5: Means of Metacognitive Beliefs by Academic Track

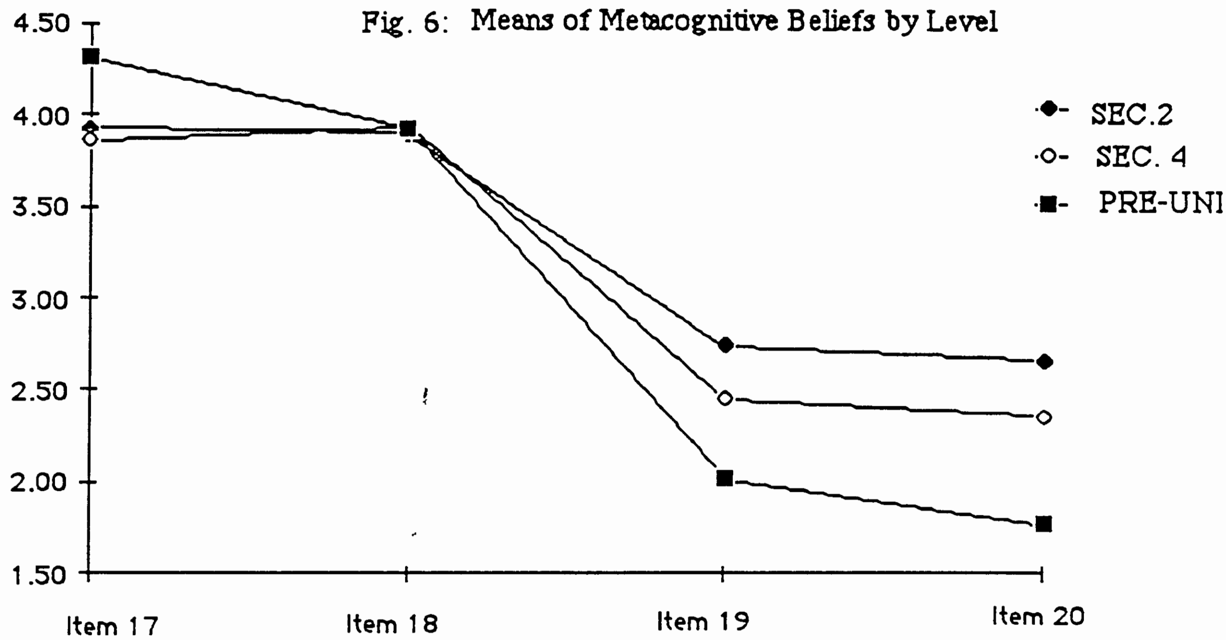



Item 17: In my revision, it is important to me to be able to solve problems set in past-year examinatior

Item 18: I need a lot of drill and practice in learning maths.

Item 19. There is only one best way in solving a problem.

Item 20: I memorize model answers.




Although the results generally showed that students do practice metacognitive strategies, certain groups of the student population were not very fluent in their usage. For example, the Normal stream students scored lower when compared to the SAP and Express stream students. This could be due to the selection process when students were streamed into SAP, Express or Normal. The Normal stream students follow a five-year secondary school education compared to the Express and the SAP stream students who follow a four-year secondary school education. The students entered the various streams based on the achievement scores in their primary school leaving examination. Thus academic ability has an effect on the frequency of usage of metacognitive strategies and other researchers have reported similar findings (Chang,1989; Peterson, 1988; Slife et al., 1985). It appears that academic ability influences metacognition in two ways: first, the lack of academic ability impedes students' knowledge of strategies and second, the lack of knowledge of metacognition leads to poor academic performance. Unfortunately, findings from this research cannot differentiate them.

Age and years of schooling are some other factors that could influence one's knowledge and application of metacognitive strategies. Awareness of metacognitive strategies starts at a very early age. Various studies on metamemory have shown that children as young as five years old are aware of strategies for recall (see Flavell & Wellman, 1977). It is also observed that older children are better at using various strategies for recall than young children (e.g., Brown, 1978). This is also true in reading comprehension. For example, Myers and Paris (1978) found out that 12 year-old students were more aware of the effects of various variables, such as their knowledge of content and their interests in the stories, on their comprehension than 8 year-old students. Biggs (1987) in his study, noticed that young students may have the awareness of the needs of monitoring and regulating their cognitive processes but may not have sufficient executive control over them. However, in this study there was no statistical difference in scores between students of different ages as students from Secondary 2, Secondary 4 and Pre-University indicated similar frequency of usage in the four components. While the level of usage remains the same across students from different levels and academic tracks, the types of strategies used differed. The younger students, the Arts and General track students and Normal stream students tended to use more surface strategies. However, the use of surface strategy should decline as the level changed to the higher level. Similar observations were also noted by Biggs (1987). This study does support the fact that students become increasingly more aware of metacognitive strategies with increasing years of schooling.

## Implications

Implications of this study can be discussed under two categories, namely, research implications and teaching implications.

(a) Research implications.

Research on metacognition should be a multifaceted task using a variety of research methods, instruments and a sample with different educational backgrounds. This study is the initial phase of the research project on effectiveness of learning strategies and is based on students' self report in a questionnaire. However, the use of questionnaire has its limitations. As such, in the second phase of the study, students will be interviewed and their protocols during a problem solving task obtained. These procedures would give us a better insight of the type of problem-solving and metacognitive strategies used. Unfortunately, this research method is time consuming and only a handful of students can be interviewed. But when the interviews are combined with the results from the questionnaire, a better picture of students' metacognition can be obtained. Also, in the second phase of the study, teachers will be asked to report their knowledge of metacognitive strategies and their frequency of teaching these strategies to students. After obtaining this baseline data, various projects on teaching metacognitive strategies can be conducted.

(b) Teaching Implications

To help students develop metacognitive strategies, the present batch of trainee teachers are exposed to a number of lectures on this aspect. They are encouraged to use various methods to achieve this. They could incorporate some of the teaching methods, activities, and approaches suggested by Callahan & Garofalo (1987), Long (1986) and Devine (1981).

Drawing from this study and other reports, lower ability students do not use metacognitive strategies as frequently as high ability students and this deficiency could lead to poor performance. They, therefore, need extra training in order to enable them to operate at the same level as the higher ability students. Various projects on teaching students thinking skills, reading skills and learning skills have been very successful. For example, Peterson and associates (cited in Peterson, 1988) conducted an extensive project which helps fourth grade students to develop thinking skills in mathematics, and noted that low ability students benefitted more from the training. She said " the thinking skills training may have provided the low ability students with the thinking skills or cognitive strategies that they did not have, but that higher ability students did have already.... Acquisition of these strategies then permitted them to learn as effectively as the higher ability students within the class" (p. 10). Attempts have been made by schools and the Ministry of Education to introduce some of these projects to students. Some examples include: the publication of a handbook, *Learning Skills in Content Area*, for secondary school teachers to be used for conducting workshops on learning skills, the introduction of DeBono's CORT programme to 25 secondary schools, and training programmes to student-teachers on effective teaching and learning skills.

## Conclusion

This study reports on the type of metacognitive strategies used by students from different academic settings in mathematical problem solving. It is generally observed that students are aware of metacognitive strategies although students from the Normal stream seem to use metacognitive strategies less frequently. There is a declining use of surface strategies and increasing use of deep and achieving strategies as the level changed to the higher level. The results of this study warrant a need to introduce the teaching of metacognitive strategies to all students especially to low ability students.

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