Bures fidelity of displaced squeezed thermal states

Xiang-Bin Wang,* C. H. Oh,† and L. C. Kwek‡
Department of Physics, Faculty of Science, National University of Singapore, Lower Kent Ridge, Singapore 119260, Republic of Singapore

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Fidelity has always been an important concept in quantum optics. Recently, it was found that fidelity can also play a key role in quantum information and communication theory. Fidelity can be interpreted as the probability that a decoded message possesses the same information content as the message prior to coding and transmission. In this paper, we give a formula of Bures fidelity for displaced squeezed thermal states directly by the displacement and squeezing parameters and briefly discuss how the results can apply to quantum information theory. [S1050-2947(98)05711-4]

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Bures fidelity has been an important concept in the field of quantum optics (see, for example, Ref. [1]). Recently, its importance has also been demonstrated in quantum information and communication theory. An important tenet in classical information theory is the rigorous establishment of the Shannon noiseless coding theorem, in which one shows that the Shannon entropy can be interpreted as the average number of bits needed to code the output of a message source under ideal conditions. The analogous quantum version of the Shannon coding theorem is the Schumacher quantum coding theorem [2]. In the quantum version, one introduces the idea of fidelity, which can be interpreted as the probability that a decoded message carries the same information as the message prior to coding. More specifically, one can prove the Schumacher noiseless coding theorem, which states that if M is a quantum signal source with signal ensemble described by the density operator ρ then $\forall \quad \delta,\epsilon > 0$:

(i) If $S(\rho)+\delta$ qubits are available per M signal, then, for sufficiently large N, groups of N signals from the signal source M can be transposed through the available qubits with fidelity $F>(1-\epsilon)$.

(ii) If $S(\rho)-\delta$ qubits are available per M signal, then, for sufficiently large N, groups of N signals from the signal source M can be transposed through the available qubits with fidelity $F<\epsilon$. $S(\rho)$ denotes the von Neumann entropy for the signal.

Suppose a quantum signal source M generates a signal state $|i_A\rangle$ with probability $p(a)$ and the density operator ρ is described by the equation

$$\rho=\sum_a p(a)|a_M\rangle\langle a_M|, \tag{1}$$

one can define the Schumacher fidelity F as the overall probability that a signal from an ensemble M can be transmitted to M' using the relation [2,3]

$$F=\sum_a p(a)\mathrm{Tr}(\pi_a\rho_a'), \tag{2}$$

where $\pi_a\equiv|a_M\rangle\langle a_M|$ and ρ_a' denotes the density operator of the final signal in M'. This definition applies strictly to pure states and it is generally not clear how it can be applied to mixed states.

Closely related to the problem of coding is the process of entanglement purification protocol (EPP) and quantum error-correction codes (QECC) [4,5]. These protocols essentially shield quantum states from the environment. In EPP, maximally entangled states are extracted (or purified) from a mixed states while in QECC, an arbitrary quantum state is transmitted at some rate through a noisy channel with minimal degradation. Central to the idea of entanglement is the need to define a measure of entanglement. Bennett and others have proposed a measure of entanglement using the von Neumann entropy. However, it is sometimes difficult to compute and obtain a closed form using their definition. Recently, Vedral and others have studied a wide class of measures suitable for entanglement and they have proposed the Bures metric as an example of a possible means of quantifying entanglement or fidelity [6].

It is well known that experimentally a squeezed electromagnetic field [7] provides a means of overcoming the standard quantum limit for noise imposed by vacuum fluctuations. Furthermore, although the number-state channel is an optimal channel for quantum communication theory, it is often more realistic to consider the quadrature-squeezed channel [8] experimentally for several reasons. Firstly, one cannot faithfully reproduce the number eigenstates easily and secondly amplification of a quadrature-squeezed channel can be realized experimentally using a phase-sensitive amplifier. Clearly, one should therefore investigate the plausibility of applying squeezed or displaced squeezed thermal states to quantum information and communication theory.

Recently, Twamley [9] has calculated the Bures fidelity for squeezed thermal states. Due to some technical difficulties, the displaced squeezed states was not considered in his article. Very recently, Scutaru [10] proposed an approach to calculate the Bures fidelity for systems with a quadratic Hamiltonian. Unfortunately, a closed form for the matrix el-

*Electronic address: scip7236@leonis.nus.edu.sg
†Electronic address: phyohch@leonis.nus.edu.sg
‡Electronic address: scip6051@leonis.nus.edu.sg

ements of the density operator is not explicitly given and the final result does not relate fidelity directly with the squeezing and displacement parameters. In fact, Paraoanu and Scutaru have obtained, in a more recent paper [11], an explicit form for the Bures fidelity for two displaced thermal states. In this article, we show an alternative method in which we can actually calculate the fidelity of displaced squeezed thermal states by simply using the Baker-Campbell-Hausdorff (BCH) formula. We have also calculated a closed-form result for the Bures fidelity. This fidelity is expressed directly in terms of the parameters found in the density operator for two displaced squeezed thermal states.

Squeezed states occur in a myriad of nonlinear optical phenomena such as optical parametric oscillation and four-wave mixing [12]. The single-mode squeezed states can be generated from the vacuum by the action of the squeezed operator S,

$$S(\zeta)=\exp[\tfrac{1}{2}(\zeta^* a^2-\zeta a^{\dagger 2})], \tag{3}$$

where $\zeta=re^{i\phi}$ is a complex number with modulus r and argument ϕ, representing the squeezing parameter and ζ^* is the complex conjugate of ζ. The density operator of displaced squeezed thermal states can be defined as

$$\rho=Z(\beta)DS\Lambda S^\dagger D^\dagger, \tag{4}$$

where $D=\exp[(a^\dagger,a)\binom{k}{-k^*}]$ and $S=\exp[\frac{1}{2}r(a^2-a^{\dagger 2})]$ are unitary operators. Furthermore, in Eq. (4), the operator Λ and the normalization factor $Z(\beta)$ are given respectively by $\exp[-(\beta/2)(aa^\dagger+a^\dagger a)]$ and $(\mathrm{tr}\,\Lambda)^{-1}$ where β is the inverse temperature. [The dagger symbol in Eq. (4) denotes Hermitian conjugate.] Note that we have considered the squeezing parameter to be real since the most important parameter of a squeezed state is the squeezed factor r and not its argument ϕ [13]. The general case in which the argument ϕ is nonzero can be treated similarly. We next recall that the Bures fidelity F can be defined by the relation

$$F=(\mathrm{tr}\sqrt{\rho_1{}^{1/2}\rho_2\rho_1{}^{1/2}})^2. \tag{5}$$

For two displaced squeezed thermal states, the Bures fidelity can be expressed as

$$F=Z(\beta_1)Z(\beta_2)[\mathrm{tr}\sqrt{(D_1S_1\Lambda_1^{1/2}S_1^\dagger D_1^\dagger)(D_2S_2\Lambda_2S_2^\dagger D_2^\dagger)(D_1S_1\Lambda_1^{1/2}S_1^\dagger D_1^\dagger)}]^2 \tag{6a}$$

$$=Z(\beta_1)Z(\beta_2)(\mathrm{tr}\sqrt{\Lambda_1^{1/2}S_1^\dagger D_1^\dagger D_2S_2\Lambda_2S_2^\dagger D_2^\dagger D_1S_1\Lambda_1^{1/2}})^2. \tag{6b}$$

To simplify Eq. (6), we need to rewrite $D_1^\dagger D_2$ as

$$D_1^\dagger D_2=D_0=\exp\left[(a^\dagger,a)\begin{pmatrix} g \\ -g^* \end{pmatrix}\right], \tag{7}$$

where

$$\begin{pmatrix} g \\ -g^* \end{pmatrix}=\begin{pmatrix} k_2-k_1^* \\ -(k_2-k_1^*)^* \end{pmatrix}.$$

Thus, the formula for Bures fidelity of displaced squeezed thermal states becomes

$$F=Z(\beta_1)Z(\beta_2)(\mathrm{tr}\sqrt{\Lambda_1{}^{1/2}S_1^\dagger D_0S_2\Lambda_2S_2^\dagger D_0^\dagger S_1\Lambda_1{}^{1/2}})^2. \tag{8}$$

Equation (8) needs some simplification before we can actually proceed with the detailed calculations. Before we do this, we need to invoke the BCH relation [12,14],

$$S(a^\dagger,a)S^\dagger=(a^\dagger,a)M; \quad S^\dagger(a^\dagger,a)S=(a^\dagger,a)M^{-1}, \tag{9}$$

where

$$M=\begin{pmatrix} \cosh r & -\sinh r \\ -\sinh r & \cosh r \end{pmatrix}$$

and

$$\Lambda(a^\dagger,a)\Lambda^{-1}=(a^\dagger,a)B. \tag{10}$$

Note that in Eq. (10), we have introduced the matrix

$$B\equiv\begin{pmatrix} \exp(-\beta) & 0 \\ 0 & \exp(\beta) \end{pmatrix}.$$

Let us define the matrix Ω as $\Lambda_1{}^{1/2}S_1^\dagger D_0S_2\Lambda_2S_2^\dagger D_0^\dagger S_1\Lambda_1{}^{1/2}$ in Eq. (8). It is instructive to note that, by using the BCH formula, we can readily express the matrix Ω in a more convenient form as

$$\Omega=\Lambda_1{}^{1/2}S_1^\dagger S_2\Lambda_2{}^{1/2}\exp\left[(a^\dagger,a)B_2{}^{-1/2}M_2^{-1}\begin{pmatrix} g \\ -g^* \end{pmatrix}\right]$$
$$\times\exp\left[-(a^\dagger,a)B_2{}^{1/2}M_2^{-1}\begin{pmatrix} g \\ -g^* \end{pmatrix}\right]\Lambda_2{}^{1/2}S_2^\dagger S_1\Lambda_1{}^{1/2}, \tag{11}$$

where B_i and M_i [according to the notation in Eq. (10)] are the matrices

$$\begin{pmatrix} \exp(-\beta_i) & 0 \\ 0 & \exp(\beta_i) \end{pmatrix}, \quad \begin{pmatrix} \cosh r_i & -\sinh r_i \\ -\sinh r_i & \cosh r_i \end{pmatrix}, \quad i=1,2,$$

respectively. The linear terms within the exponential factor in the above formula (11) can be collapsed into a simpler term by using the following results (see Appendix for a detailed proof):

$$\exp\left[(a^\dagger,a)N_1\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}\right]\exp\left[(a^\dagger,a)N_2\begin{pmatrix} z_3 \\ z_4 \end{pmatrix}\right]$$
$$=\exp\left[-\frac{1}{2}(z_1,z_2)\tilde{N}_1\Sigma N_2\begin{pmatrix} z_3 \\ z_4 \end{pmatrix}\right]\exp\left[(a^\dagger,a)N_1\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + (a^\dagger,a)N_2\begin{pmatrix} z_3 \\ z_4 \end{pmatrix}\right] \quad (12)$$

where N_1,N_2 are arbitrary 2×2 complex matrices, Σ is the matrix $\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ and z is an arbitrary complex number. [In Eq. (12), the tilde above the matrix N_1 denotes the transpose of the matrix.] In this manner, we see that

$$\Omega=\Gamma_1\rho_+\exp\left[(a^\dagger,a)(B_2^{-1/2}-B_2^{1/2})M_2^{-1}\begin{pmatrix} g \\ -g^* \end{pmatrix}\right]\rho_- \quad (13)$$

and

$$\Gamma_1=\exp\left[\frac{1}{2}(g,-g^*)\tilde{M}_2^{-1}B_2^{-1/2}\Sigma B_2^{1/2}M_2^{-1}\begin{pmatrix} g \\ -g^* \end{pmatrix}\right], \quad (14)$$

$$\rho_+=\Lambda_1^{1/2}S_1^\dagger S_2\Lambda_2^{1/2},\quad \rho_-=\Lambda_2^{1/2}S_2^\dagger S_1\Lambda_1^{1/2}. \quad (15)$$

Let us now consider another operator

$$\Omega'=U\rho_+\rho_-U^\dagger, \quad (16)$$

where $U=\exp[(a^\dagger,a)\left(\begin{smallmatrix} l \\ -l^* \end{smallmatrix}\right)]$. If we apply the BCH formula again, we see that

$$\Omega'=\rho_+\exp\left[(a^\dagger,a)B_2^{-1/2}M_2^{-1}M_1B_1^{-1/2}\begin{pmatrix} l \\ -l^* \end{pmatrix}\right]\times\exp\left[-(a^\dagger,a)B_2^{1/2}M_2^{-1}M_1B_1^{1/2}\begin{pmatrix} l \\ -l^* \end{pmatrix}\right]\rho_- \quad (17)$$

$$\Rightarrow\Omega'=\Gamma_2\rho_+\exp\left[(a^\dagger,a)(B_2^{-1/2}M_2^{-1}M_1B_1^{-1/2}-B_2^{1/2}M_2^{-1}M_1B_1^{1/2})\begin{pmatrix} l \\ -l^* \end{pmatrix}\right]\rho_- \quad (18)$$

and

$$\Gamma_2=\exp\left[\frac{1}{2}(l,-l^*)B_1^{-1/2}\tilde{M}_1\tilde{M}_2^{-1}B_2^{-1/2}\Sigma\times B_2^{1/2}M_2^{-1}M_1B_1^{1/2}\begin{pmatrix} l \\ -l^* \end{pmatrix}\right]. \quad (19)$$

Setting

$$(B_2^{-1/2}M_2^{-1}M_1B_1^{-1/2}-B_2^{1/2}M_2^{-1}M_1B_1^{1/2})\begin{pmatrix} l \\ -l^* \end{pmatrix}=(B_2^{-1/2}-B_2^{1/2})M_2^{-1}\begin{pmatrix} g \\ -g^* \end{pmatrix} \quad (20)$$

we get

$$\Omega=\frac{\Gamma_1}{\Gamma_2}\Omega', \quad (21)$$

$$(\mathrm{tr}\sqrt{\Omega})^2=\frac{\Gamma_1}{\Gamma_2}(\mathrm{tr}\sqrt{U\rho_+\rho_-U^+})^2=\frac{\Gamma_1}{\Gamma_2}(\mathrm{tr}\sqrt{\rho_+\rho_-})^2, \quad (22a)$$

$$F=\frac{\Gamma_1}{\Gamma_2}Z(\beta_1)Z(\beta_2)(\mathrm{tr}\sqrt{\rho_+\rho_-})^2. \quad (22b)$$

Since $Z(\beta_1)Z(\beta_2)(\mathrm{tr}\sqrt{\rho_+\rho_-})^2$ has already been computed in Ref [9], we can solve the whole problem by considering the reduced calculation of Γ_1/Γ_2. Following the Twamley paper, the quantity $Z(\beta_1)Z(\beta_2)(\mathrm{tr}\sqrt{\rho_+\rho_-})^2$ in Eq. (22) can be written as

$$Z(\beta_1)Z(\beta_2)(\mathrm{tr}\sqrt{\rho_+\rho_-})^2=\frac{2\sinh(\beta_1/4)\sinh\beta_2/4}{\sqrt{Y}-1}, \quad (23)$$

where

$$Y=\cosh^2(r_1-r_2)\cosh^2(\beta_1+\beta_2)/4-\sinh^2(r_1-r_2)\cosh^2(\beta_1-\beta_2)/4.$$

From Eqs. (19) and (20), it follows that

$$\Gamma_2=\exp\left\{\frac{1}{2}(l,-l^*)B_1^{-1/2}\tilde{M}_1\tilde{M}_2^{-1}B_2^{-1/2}\cdot\Sigma\cdot\times\left[B_2^{-1/2}M_2^{-1}M_1B_1^{-1/2}\begin{pmatrix} l \\ -l^* \end{pmatrix}-(B_2^{-1/2}-B_2^{1/2})M_2^{-1}\begin{pmatrix} g \\ -g^* \end{pmatrix}\right]\right\}. \quad (24)$$

It is instructive to note that the matrices B and M are all symplectic matrices, so that we have

$$(l,-l^*)B_1^{-1/2}\tilde{M}_1\tilde{M}_2^{-1}B_2^{-1/2}\Sigma B_2^{-1/2}M_2^{-1}M_1B_1^{-1/2}\begin{pmatrix} l \\ -l^* \end{pmatrix}=(l,-l^*)\Sigma\begin{pmatrix} l \\ -l^* \end{pmatrix}=0. \quad (25)$$

With this observation, it is straightforward to see that Eq. (19) can be simplified as

$$\Gamma_2=\exp\left[-\frac{1}{2}(l,-l^*)B_1{}^{-1/2}\tilde{M}_1\tilde{M}_2^{-1}B_2{}^{-1/2}\times\Sigma(B_2{}^{-1/2}-B_2{}^{1/2})M_2^{-1}\begin{pmatrix}g\\-g^*\end{pmatrix}\right]. \quad (26)$$

To obtain the final explicit form of Γ_2, we have from Eq. (20)

$$(l,-l^*)=(g,-g^*)\tilde{M}_2^{-1}(B_2{}^{-1/2}-B_2{}^{1/2})\tilde{P}^{-1}, \quad (27)$$

where the matrix $P\equiv(B_2{}^{-1/2}M_2^{-1}M_1B_1-B_2{}^{1/2}M_2^{-1}M_1B_1{}^{1/2})$. If we plug Eq. (27) into Eq. (26), we arrive at the following formula for calculation of Γ_2:

$$\Gamma_2=\exp\left[-\frac{1}{2}(g,-g^*)\tilde{M}_2^{-1}(B_2{}^{-1/2}-B_2{}^{1/2})\tilde{P}^{-1}B_1{}^{-1/2}\times\tilde{M}_1\tilde{M}_2^{-1}B_2{}^{-1/2}\Sigma(B_2{}^{-1/2}-B_2{}^{1/2})M_2^{-1}\begin{pmatrix}g\\-g^*\end{pmatrix}\right]. \quad (28)$$

In our case, it is not difficult to evaluate the expression for Γ_1 and Γ_2 explicitly. To do this, we note that if we denote

$$\Gamma_1=\exp\left[\frac{1}{2}(g,-g^*)Q_1\begin{pmatrix}g\\-g^*\end{pmatrix}\right], \quad (29)$$

then the matrix Q_1 is simply

$$Q_1=\begin{pmatrix}\sinh\beta_2\sinh(2r_2) & \cosh\beta_2+\sinh\beta_2\cosh(2r_2)\\ -\cosh\beta_2+\sinh\beta_2\cosh(2r_2) & \sinh\beta_2\sinh(2r_2)\end{pmatrix}. \quad (30)$$

For Γ_2, a straightforward computation for the matrix P yields

$$P=\frac{1}{\Delta}\begin{pmatrix}\sinh\frac{\beta_2+\beta_1}{2}\cosh(r_1-r_2) & \sinh\frac{\beta_2-\beta_1}{2}\sinh(r_1-r_2)\\ -\sinh\frac{\beta_2-\beta_1}{2}\sinh(r_1-r_2) & -\sinh\frac{\beta_2+\beta_1}{2}\cosh(r_1-r_2)\end{pmatrix} \quad (31)$$

with $\Delta=\cosh\beta_1\cosh\beta_2+\sinh\beta_1\sinh\beta_2\cosh 2(r_1-r_2)-1$, so that if we denote

$$\frac{\Gamma_1}{\Gamma_2}=\exp\left\{\frac{1}{2}(g,-g^*)R\begin{pmatrix}g\\-g^*\end{pmatrix}\right\}, \quad (32)$$

then a straightforward, albeit tedious, calculation yields

$$R=\begin{pmatrix}0&1\\-1&0\end{pmatrix}+\frac{2}{\Delta}\sinh\beta_1\sinh^2\frac{\beta_2}{2}\begin{pmatrix}\sinh(2r_1)&\cosh(2r_1)\\ \cosh(2r_1)&\sinh(2r_1)\end{pmatrix}+\frac{2}{\Delta}\sinh^2\frac{\beta_1}{2}\sinh\beta_2\begin{pmatrix}\sinh(2r_2)&\cosh(2r_2)\\ \cosh(2r_2)&\sinh(2r_2)\end{pmatrix} \quad (33)$$

so that the factor Γ_1/Γ_2 works out explicitly into

$$\exp\left\{\frac{1}{\Delta}(\epsilon_1+\epsilon_2)\right\}, \quad (34)$$

where

$$\epsilon_1=\sinh\beta_1\sinh^2\frac{\beta_2}{2}[(g^2+g^{*2})\sinh 2r_1-2|g|^2\cosh 2r_1], \quad (35a)$$

$$\epsilon_2=\sinh^2\frac{\beta_1}{2}\sinh\beta_2[(g^2+g^{*2})\sinh 2r_2-2|g|^2\cosh 2r_2]. \quad (35b)$$

We can easily show that that $\Gamma_1/\Gamma_2<1$ as it should be and that in the limit $g=g^*=0$, the ratio reduces to unity so that we obtain the Bures fidelity for the undisplaced squeezed states as shown in Ref. [9]. Further, we should also note that in the limit when $r=0$, we get the Bures fidelity for the displaced *unsqueezed* thermal coherent states. This Bures fidelity is the same as the result previously obtained by Paraoanu and Scutaru [11].

APPENDIX

In this appendix, we shall explicitly show the proof for Eq. (12). For simplicity and convenience, we define Ω_i as the expression

$$\Omega_i=(a^\dagger,a)N_i\begin{pmatrix}z_{2i-1}\\z_{2i}\end{pmatrix} \quad \text{for } i=1,2. \quad (A1)$$

To show Eq. (12), we need to compute $e^{\Omega_1}e^{\Omega_2}$. Since N_1 and N_2 are simply two arbitrary 2×2 matrices, in all generality they can be written as

$$N_1=\begin{pmatrix}a&d\\b&c\end{pmatrix},\quad N_2=\begin{pmatrix}e&h\\f&g\end{pmatrix}. \quad (A2)$$

We next compute the commutator for Ω_1 and Ω_2.

$$[\Omega_1,\Omega_2]=-(az_1+dz_2)(fz_3+gz_4)+(bz_1+cz_2)(ez_3+hz_4). \tag{A3}$$

On the other hand, we should note that

$$\begin{aligned}(z_1,z_2)\tilde{N}_1\Sigma N_2\begin{pmatrix}z_3\\ z_4\end{pmatrix}&=(z_1,z_2)\begin{pmatrix}a & b\\ d & c\end{pmatrix}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}e & h\\ f & g\end{pmatrix}\begin{pmatrix}z_3\\ z_4\end{pmatrix}\\ &=(az_1+dz_2,bz_1+cz_2)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}ez_3+hz_4\\ fz_3+gz_4\end{pmatrix}\\ &=(-bz_1-cz_2,az_1+dz_2)\begin{pmatrix}ez_3+hz_4\\ fz_3+gz_4\end{pmatrix} \qquad \text{(A4a)}\\ &=-(bz_1+cz_2)(ez_3+hz_4)+(az_1+dz_2)(fz_3+gz_4). \qquad \text{(A4b)}\end{aligned}$$

Consequently, using Baker-Campbell-Hausdorff formula, one gets

$$e^{\Omega_1}e^{\Omega_2}=e^{1/2[\Omega_1,\Omega_2]}e^{\Omega_1+\Omega_2} \tag{A5a}$$

$$=\exp\left[-\frac{1}{2}(z_1,z_2)\tilde{N}_1\Sigma N_2\begin{pmatrix}z_3\\ z_4\end{pmatrix}\right]\exp\left[(a^\dagger,a)N_1\begin{pmatrix}z_1\\ z_2\end{pmatrix}+(a^\dagger,a)N_1\begin{pmatrix}z_3\\ z_4\end{pmatrix}\right]. \tag{A5b}$$

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