Title	Mathematical modelling in the primary school: Elements in teacher education
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Mathematical Modelling in the Primary School: Elements in Teacher Education

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Note

This presentation has been prepared with reference to the following papers:

- Ng, K. E. D., Widjaja, W., Chan, C. M. E., & Seto, C. (2012). Activating teacher critical moments through reflection on mathematical modelling facilitation. Electronic pre-proceedings of the 12th International Congress on Mathematical Education (ICME-12) (pp. 3347-3356). Korea: Seoul.
- Ng, K. E. D., Chan, C. M. E., Widjaja, W., & Seto, C. (2013, 17-22 March 2013). *Fostering teacher competencies in incorporating mathematical modelling in Singapore primary mathematics classrooms.* Paper presented at the Innovations and exemplary practices in mathematics education: 6th East Asia Regional Conference on Mathematics Education, Phuket, Thailand.
- Chan, C. M. E., Widjaja, W., & Ng, K. E. D. (2011). Exemplifying a model-eliciting task for primary school pupils. *Southeast Mathematics Education Journal, 1*(1), 65-74.

Outline of Presentation

- Some modelling competencies
- A model-and-modelling perspective for mathematical modelling at primary level…
- Fostering modelling competencies…a teaching experiment…
- Proposed areas in teacher development

Why Mathematical Modelling in Singapore Schools?

Modelling Tasks:

- platforms for activating in-depth mathematical reasoning and communication
- make connections between school-based mathematics and the real world explicit
- present situations to learn about making assumptions, interpretations, working with ambiguity, reflecting on approaches, choices in application of mathematical knowledge and skills, and justifying decisions

[Balakrishnan (2011); English (2007); Singapore Ministry of Education Modelling Kit (2012)]

Why Mathematical Modelling in Singapore Schools?

Mathematical Literacy:

"an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens."

[Organisation for Economic Cooperation and Development (2012)]

Modelling Competencies

Some Modelling Competencies

Tied into the modelling process

[Lesh, Galbraith, Haines, & Hurford (2007); Maaß (2006)]

Real-World ➡ **Mathematical Model**

- Making assumptions for the real world problem
- Simplifying the situation based on interpretations
- Identify key variables
- Construct relations between key variables
- Mathematise relevant quantities and their relations
- Choose appropriate mathematical representations

Some Modelling Competencies

Tied into the modelling process

[Lesh, Galbraith, Haines, & Hurford (2007); Maaß (2006); Singapore MOE, 2012]

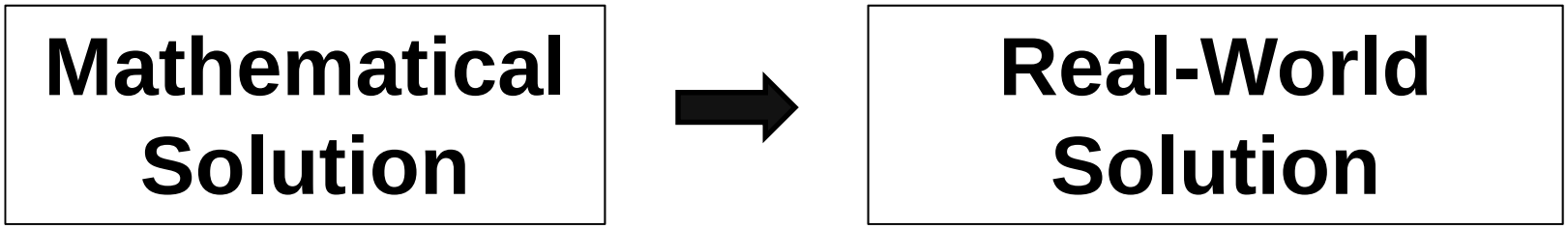
Mathematical Model → **Mathematical Solution**

- Choose appropriate mathematical knowledge and skills from existing repertoire to solve the problem as represented by the model
- Present solution clearly with sound mathematical reasoning

Some Modelling Competencies

Tied into the modelling process

[Lesh, Galbraith, Haines, & Hurford (2007); Maaß (2006); Singapore MOE, 2012]



- Interpret mathematical results within the real-world situation (considering the scope, parameters and other non-mathematical factors)
- Generalise solution to other similar situations

Some Modelling Competencies

Tied into the modelling process

[Lesh, Galbraith, Haines, & Hurford (2007); Maaß (2006); Singapore MOE, 2012]

Real-World Solution ➡ **Real-World**

- Reflect critically on model on its merits and limitations
- Validate mathematical model based on the real-world constraints
- Improve on the model

Towards Modelling Competencies:

A Models-and-Modelling Perspective

A Models-and-Modelling Perspective

[Lesh & Doerr, 2003]

For Primary Levels

- Model-Eliciting Activity…

  "a problem solving activity constructed using specific principles of instructional design in which students make sense of meaningful situations, and invent, extend, and refine their own mathematical constructs".

  [Kaiser & Sriraman, 2006, p. 306]

A Models-and-Modelling Perspective

[Lesh & Doerr, 2003]

Model-Eliciting Activities

- Mathematising [deLange, 2006] of the real-world problem is crucial.
- “naturally allow students to develop the mathematics needed to make sense of the situation” [Kaiser & Sriraman, 2006, p. 306]

A Models-and-Modelling Perspective

[Lesh & Doerr, 2003]

Model-Eliciting Activities

- problem is interpreted, represented, and solved in multiple ways
- develop appropriate internal conceptual models
- external projection of the models are expressed in spoken language, written symbols, graphs, diagrams, and other mathematical representations
- mathematical representations continually tested and revised
- modellers aim to reach feasible solutions to explain, describe, or predict the given problem situation for various purposes within a selected set of assumptions and conditions.

[Ng, Widjaja, Chan, & Seto (2012)]

Fostering Modelling Competencies

in Students:

A Teaching Experiment

Situating Our Research

- Limited implementation of modelling activities in Singapore schools - many teachers do not know what mathematical modelling entails [Chan, 2008]

- Anecdotal evidence - teachers often faced challenges in just-in-time identification of student blockages and provision of appropriate scaffolding - particularly so when faced with open-ended, non-routine tasks [Ng, 2011]

- [Doerr, 2007] - called for a mindset change in teachers when facilitating modelling tasks - from an authoritarian teacher-control mode to a more student-directed teacher-support mode

- [Stillman, 2010] - emphasised the importance of creating a conducive classroom climate for modelling tasks where students choose and justify their approaches for model development

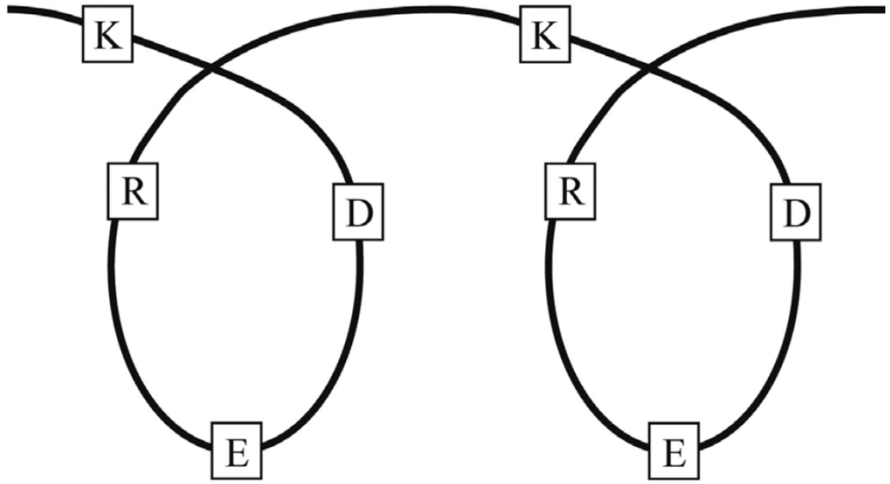
Focus of this Report as part of a Larger Study:

Building Teachers Capacity in Incorporating Mathematical Modelling in Primary Schools:

“What are the areas of focuses in teacher development towards fostering student competencies in mathematical modelling?”

Research Methodology

- Design Research Methodology



[Dolk et al., 2010, p.175]

K – Knowledge
E – (Teaching) Experiment
D – Design
R – Retrospective Analysis

Research Methodology

- Iterative cycles comprising the phases of Knowledge (K), Design (D), Teaching-Experiment (E) and Retrospective analysis (R)

- Two cycles used for this study:
  - Cycle 1 → familiarizing the teacher with the features of modelling tasks, scaffolding strategies for mathematisation, and predicting possible student outcomes exemplified through the use of a researcher-designed model-eliciting task.
  - Cycle 2 → scaffold the teacher in designing a second modelling task for the same class, working out pathways for implementation, and predicting student outcomes.

- Each cycle comprised of 5-6 sessions (handholding in K & D phases, implementation of task in E phase, reflection in R phase)

The Task

Determining the Most Efficient Bus Route

Ms Chang recently moved to Block 297C Punggol Road. She is going to start teaching at Punggol Primary school next week and needs to know how to travel to the school. However, the MRT is always too crowded for her to take and it also requires her to take a feeder bus which results in inconvenience. Ms Chang realizes that there are three bus services that ply different routes to her school. Help her to find the most efficient route to travel by bus from her home to the school. The location of her home is marked in the map. Currently the three bus services that are available for Ms Chang to choose are Service 124, Service 62 and Service 89. The routes for Service 124, Service 62 and Service 89 are marked as blue, yellow, and pink lines respectively on the map. The bus stops along each bus route are marked with stickers with corresponding colours.

[Chan, Widjaja, & Ng, K. E. D. (2011)]

Focus 1 in Teacher Development:

- Striking a Balance: Questioning and Listening

Focus 2 in Teacher Development:

- Metacognitive Strategies during Facilitation

  - deliberate choice to move away from her routine prescriptive approach so as to encourage more student-directed inquiry

Focus 2 in Teacher Development:

- Metacognitive Strategies during Facilitation

  - deliberate choice to pose certain questions to help students along without telling them too much

Focus 2 in Teacher Development:

- Metacognitive Strategies during Facilitation
  - The teacher might not have realised she has to allow student metacognition to occur.
  - This requires an innate sense of flexibility in the teacher's current use of metacognitive strategies.
  - She now has to consciously level herself towards a more macroscopic plane →fostering the metacognitive behaviours of others within the fluidity of discourse dynamics; not just microscopically doing so during her own pedagogical decision making.

Focus 3 in Teacher Development:

- Fostering the Setting of Assumptions

Other Concerns in Teacher Facilitation:

- Help students to interpret their model based on the real-world situation
- Lead students to critically evaluate the validity of their models for limitations and generalizability
- Allow time for students to propose improvements of their models

Discussion

- Three areas of focuses in teacher development towards promoting positive modelling experiences among students:

  (a) striking an appropriate balance between questioning and listening during facilitation of student discussions,

  (b) use of metacognitive strategies, and

  (c) fostering the setting of assumptions in the modelling process.

Discussion

- From Doerr and English (2006)
  - teachers should adopt a pattern of listening-observing-questioning right at the beginning of the facilitation process
  - glean an understanding of students' thinking and note the different ways in which the real-world problem was interpreted and represented.

Discussion

- Stillman, Brown, and Galbraith (2010) identified the importance of use of metacognitive strategies in overcoming blockages of lower intensity during the modelling cycle and emphasised the need for teachers to be sensitive to the blockages faced by students.

Implications

- addressing teacher's use of metacognitive strategies during facilitation and enhancing student metacognition.
- equip teachers with various approaches to help students become aware of, appreciate, set, and subsequently work with assumptions (e.g., mathematical and contextual) during modelling tasks.
- The validation of the model and its subsequent revision is important as a part of the modelling cycle – promote metacognitive behaviours and examination of assumptions and parameters.

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