Title	Mathematical modelling in the primary school: Exemplifying the core components of the Singapore mathematics curriculum framework
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Mathematical Modelling in the Primary School: Exemplifying the Core Components of the Singapore Mathematics Curriculum Framework

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Outline of Presentation

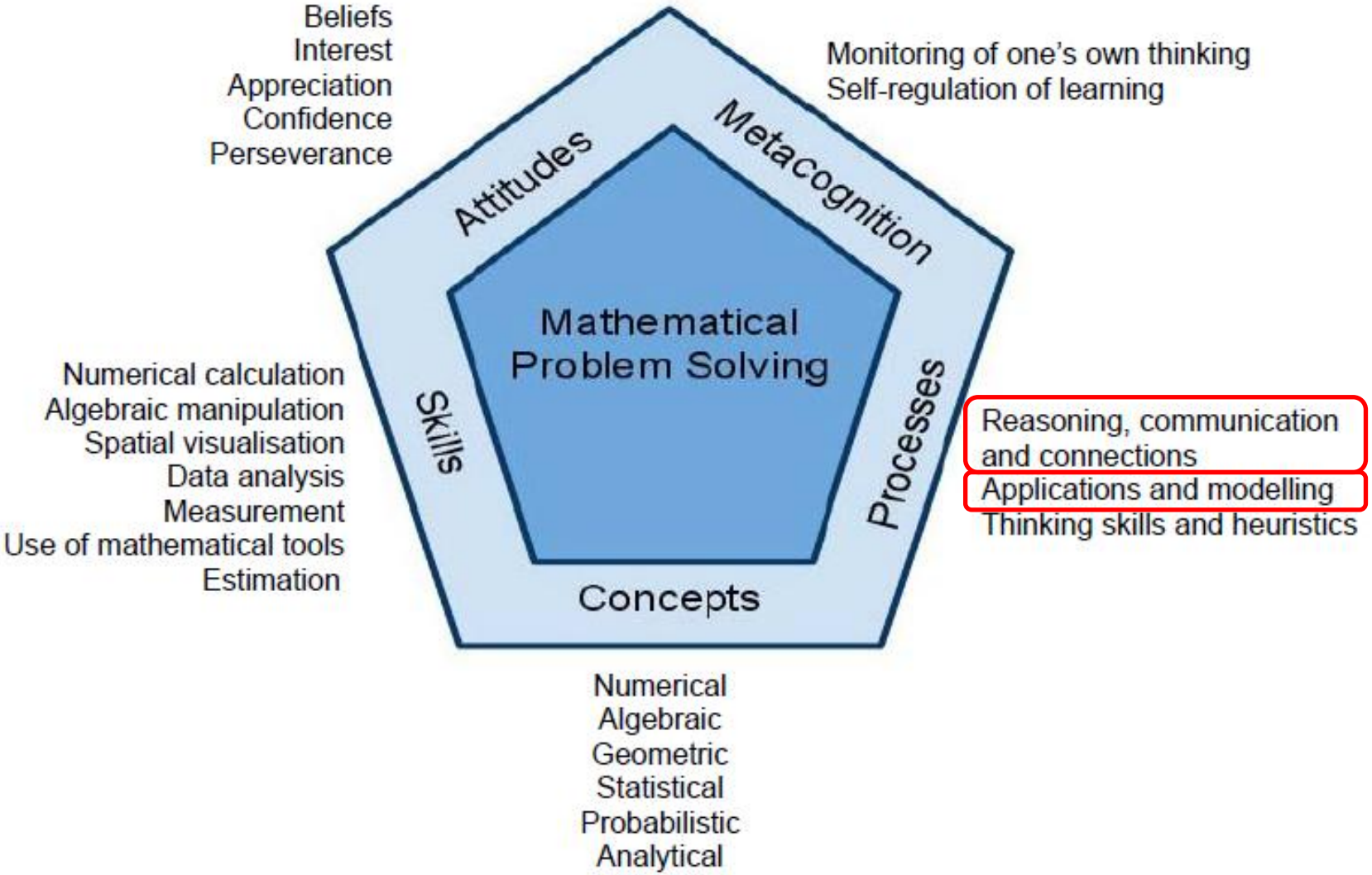
1. An introduction to the Singapore Mathematics Curriculum Framework and the Mathematical Modelling Framework

2. The Models-and-Modelling Perspective as the theoretical background

3. The Methodology

4. Exemplifying the core components of the Singapore Mathematics Curriculum Framework through pupils' samples

Background



Singapore Mathematics Curriculum Framework (2012)

Mathematical Modelling

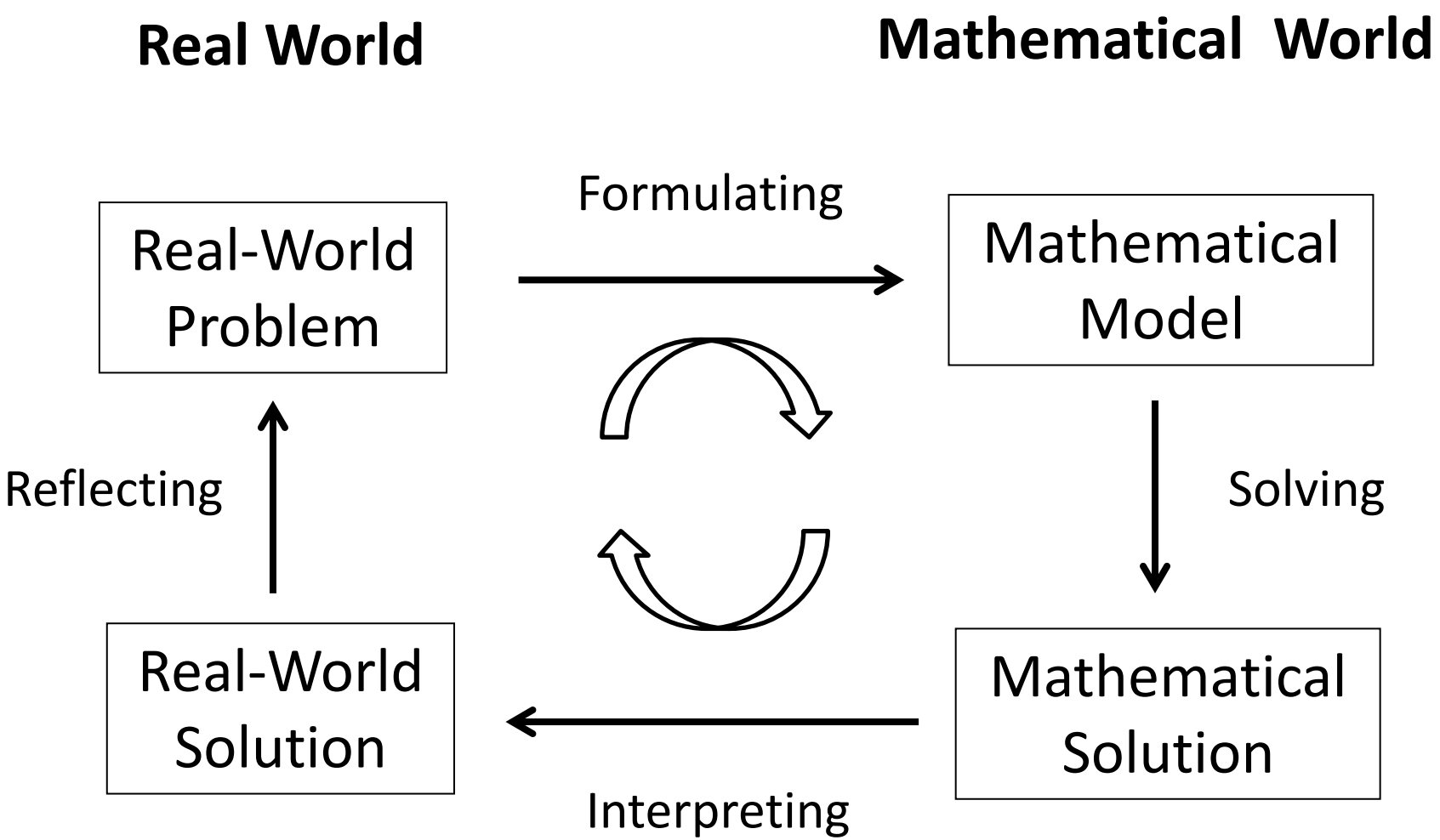
The process of formulating and improving a mathematical model to represent and solve real world problems.

Applications and modelling allow students to connect mathematics that they have learnt to the real world, enhance understanding of key mathematical concepts and methods, as well as develop mathematical competencies. Students should have opportunities to apply mathematical problem-solving and reasoning skills to tackle a variety of problems, including open-ended and real-world problems. (MOE, 2012, p.17)

Mathematical Modelling

…students should learn to deal with ambiguity, make connections, select and apply appropriate mathematical concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data.

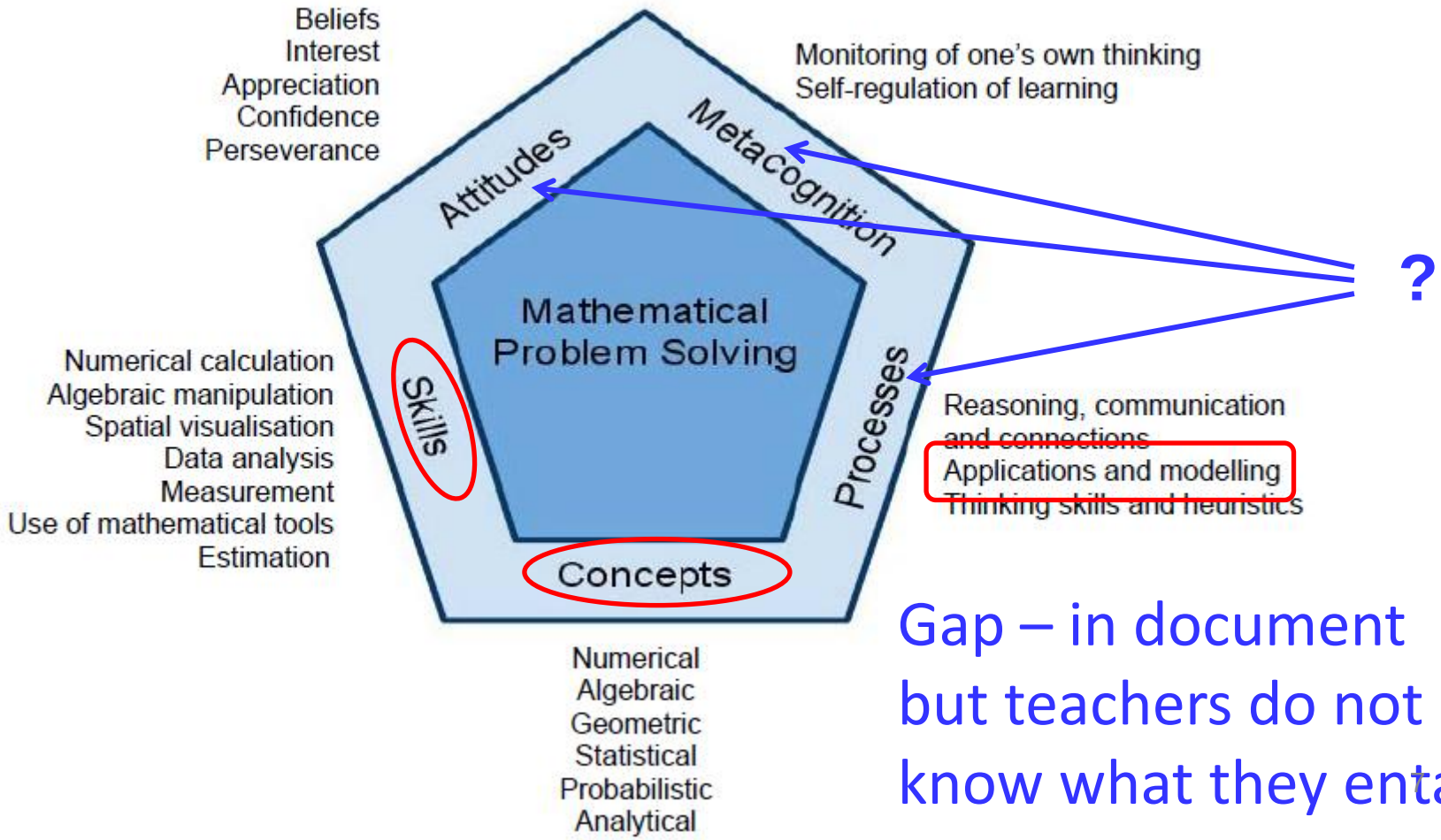
(MOE, 2012, p.18)



Singapore Mathematics Curriculum Framework (2012)

Local Research

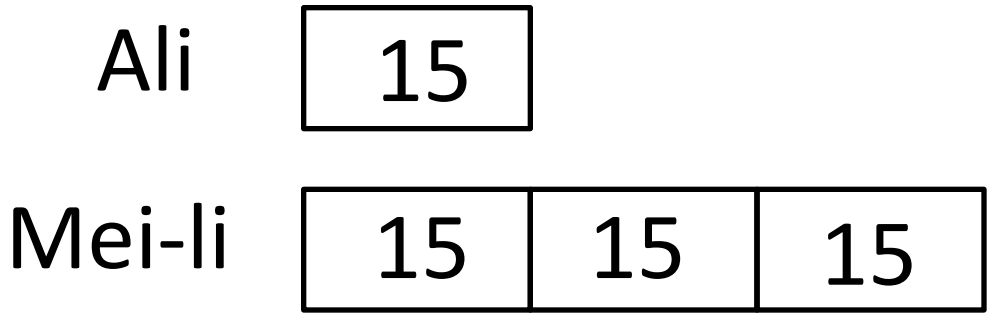
- Mathematical problem solving – mainly the IRE teaching approach (Chang, Kaur, Koay & Lee, 2001; Fan & Zhu, 2007; Foong, 2004; Ho, 2006; Kaur, 2003).



Gap – in document but teachers do not know what they entail

The Model-Drawing Method

Ali has 15 marbles.
Mei-li has 3 times the number of marbles Ali has.
How many marbles has Mei-li?



$3 \times 15 = 45$

- Heuristic
- Direct
- Single correct answer

Evidence that pupils can model situations (English, 2003, 2006, 2009; Mousoulides, Sriraman & Christou, 2006).

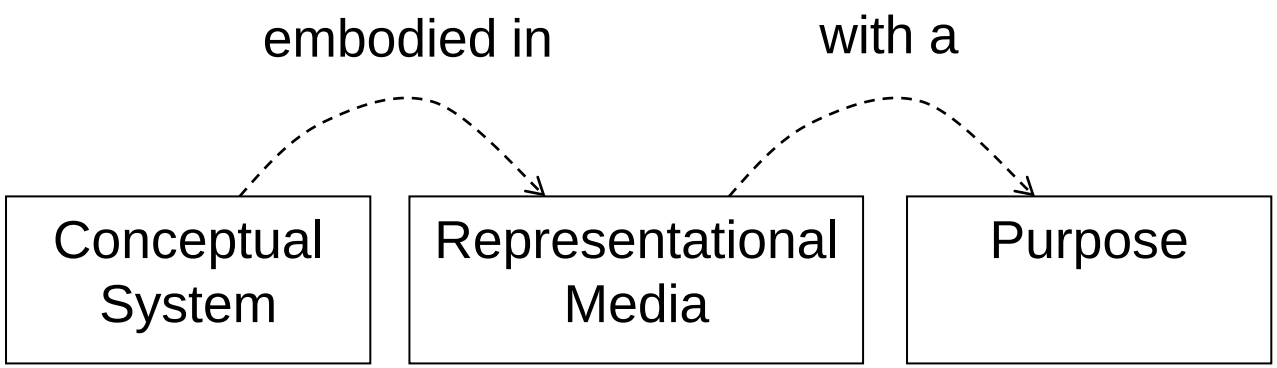
Black & Wiliam (1998)

Research results do not appeal to teachers.

Teachers need a variety of living examples of implementation…They need to see examples of what doing better means in practice.

The Models-and-Modelling Perspective (MMP)

- Children develop models (conceptual representations) expressed as spoken language, written symbols, concrete diagrams, pictures, or other representational media (Lesh & Doerr, 2003).



Express
Test
Revise

The structure of a model (Carmona, 2004)

Methodology

- Multi-tiered Teaching Experiment & Design Research Methodology (speaker: Dawn)

- Design of modelling task (The Most Efficient Bus Route) – adapted from modelling principles of Lesh et al. (2003).

- Subjects – a class of mixed-ability Primary 5 pupils in small working groups of 4 and 5.

- Selected groups were video-recorded.

- 3 x 1h sessions

- Teacher facilitated the modelling session taking them through the modelling process.

Determining the most efficient bus route

Ms Chang recently moved to Block 297C Punggol Road. She is going to start teaching at Punggol Primary school next week and needs to know how to travel to the school. However, the MRT is always too crowded for her to take and it also requires her to take a feeder bus which results in inconvenience. Ms Chang realizes that there are three bus services that ply different routes to her school. Help her to find the most efficient route to travel by bus from her home to the school. The location of her home is marked in the diagram. Currently the three bus services that are available for Ms Chang to choose are Service 124, Service 62 and Service 89. The routes for Service 124, Service 62 and Service 89 are marked as blue, yellow, and pink lines respectively on the map. The bus stops along each bus route are marked with stickers with corresponding colours.

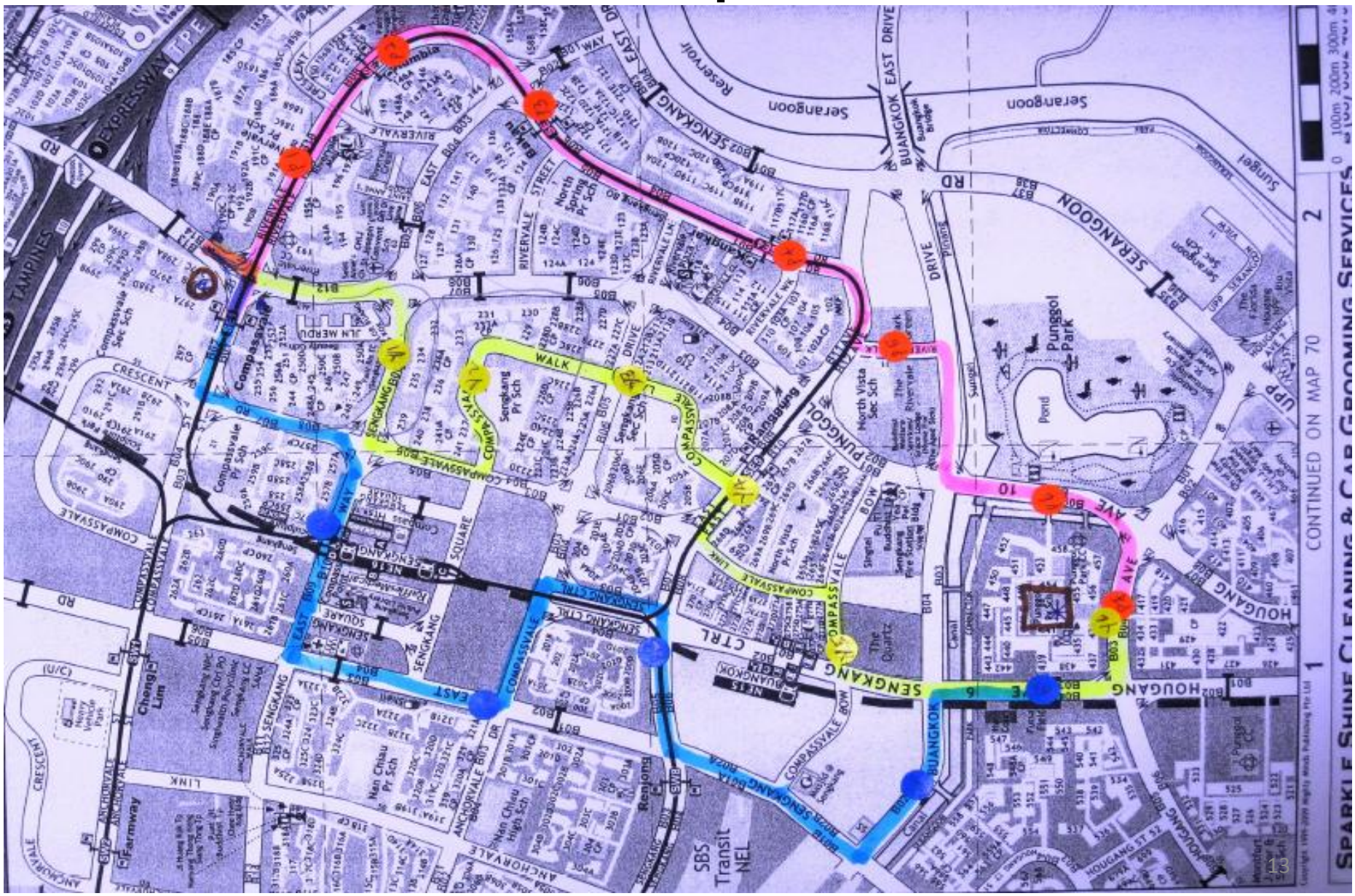
Your task is to give Ms Chang a proposal consisting of the following:

1. How your group determines what is meant by the "*most efficient*" bus route
2. *Assumptions* about the problem your group made in order to help Ms Chang
3. The *mathematics* used to decide which route is the most efficient
4. How your group *justifies* that the selected route is the most efficient
5. The final *recommended* route for Ms Chang

For us to better understand your work, you can attach the following to your proposal:

(a) A map containing the chosen bus route.
(b) The information you found useful for this task

Map


SPARKLE SHINE CLEANING & CAR GROOMING SERVICES
CONTINUED ON MAP 70
Punggol Park
Pond
Serangoon Reservoir
Serangoon
BUANGKOK EAST DRIVE
SERANGOON RD
SENGKANG EAST WAY
RIVERVALE STREET
COMPASSVALE WALK
COMPASSVALE CRESCENT
COMPASSVALE LINK
COMPASSVALE SQUARE
SENGKANG SQUARE
SENGKANG CTRL
HOUGANG AVE 10
HOUGANG AVE 6
PUNGGOL DRIVE
BUANGKOK LINK
ANCHORVALE LINK
ANCHORVALE CRESCENT
TAMPINES EXPRESSWAY
SBS Transit NEL
Compassvale Pr Sch
Compassvale Sec Sch
Sengkang Pr Sch
Sengkang Sec Sch
North Vista Pr Sch
North Vista Sec Sch
North Spring Pr Sch
Nan Chiau Pr Sch
Nan Chiau High Sch
Renjong
Cheng Lim
Farmway
The Quartz
Buangkok Bridge
Sungei
Canal
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Features

1. The benefit of using authentic problem situations to provide not only real world contexts but deriving real world solutions.

2. The opportunity for model exploration and application where pupils can build, consolidate and refine their conceptual systems.

3. The opportunity for multiple interpretations and approaches which enables others to scrutinize, to interpret, and re-interpret the problem information.

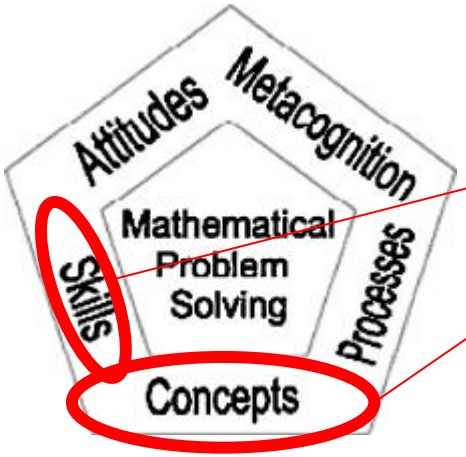
Features

4. The opportunity for social development especially when pupils have a shared responsibility to problem resolution.

5. It calls for multifaceted end products whereby pupils can adopt various modes of representations suggesting representational fluency.

6. The opportunity for optimal mathematical development where they engage in important mathematical processes.

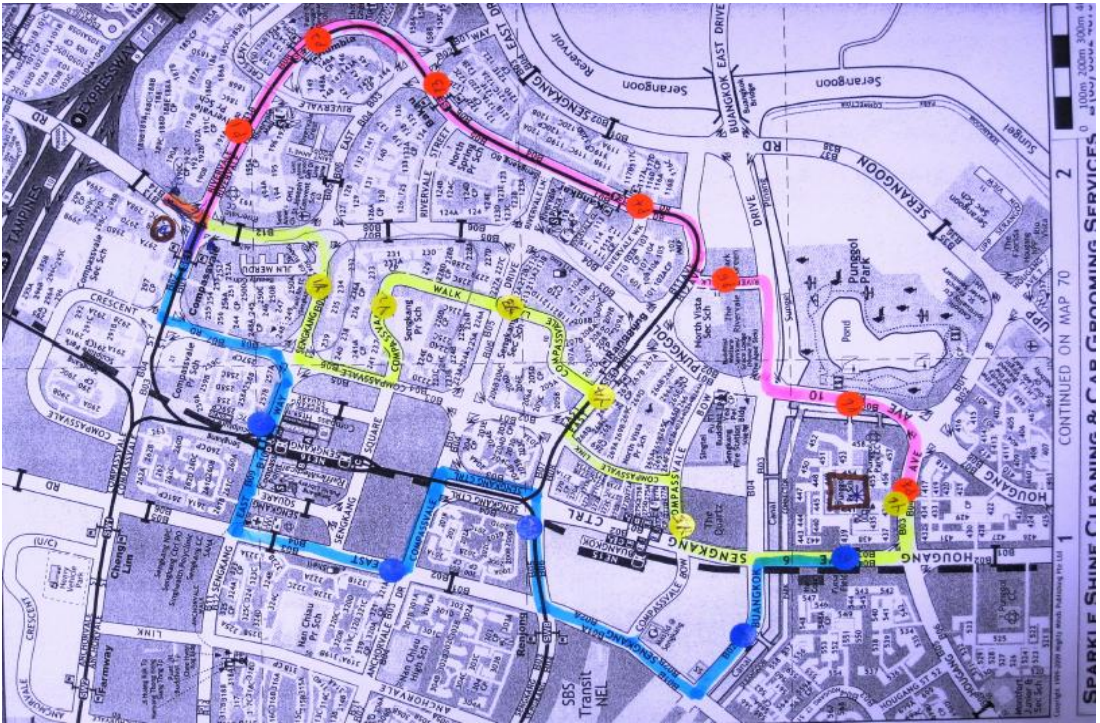
Exemplifying the Core Components



Concepts & Skills

Understanding what the measurable attribute is and determine the units and processes used in measuring the attribute

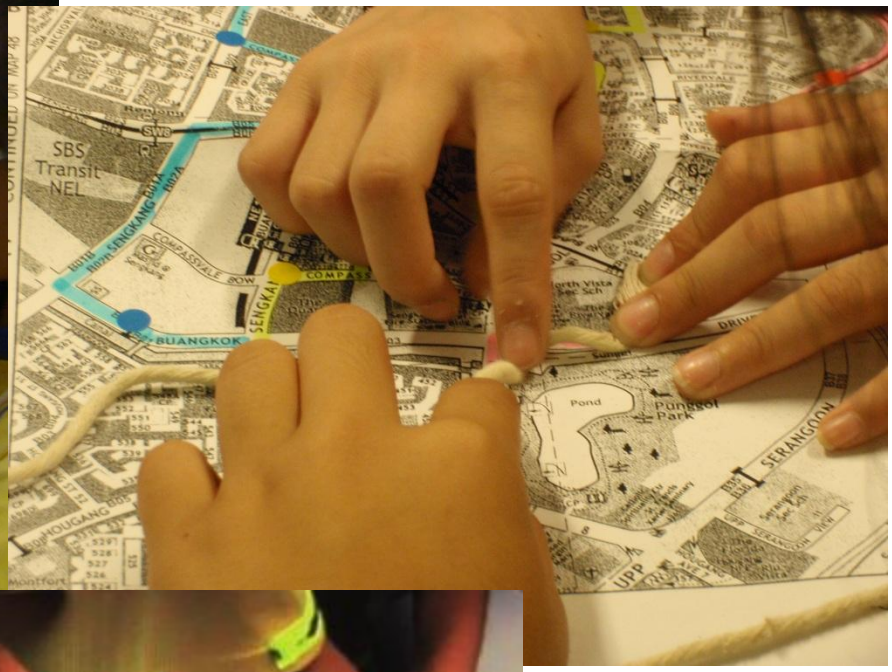
Use of measurement instrument and accuracy



S4	So we need to check the … so we need to check the length.
S2	That's why lah, we must check the string.
S3	Er the ruler, the string.
	…
S1	Where? Where is it? From here to here or here to here?
	…
S2	Then why you put the measuring in inch?
	…
S3	Yellow. 42 like that.

- Measure
- Length
- Ruler
- String
- Distance between two points
- Units of measurement

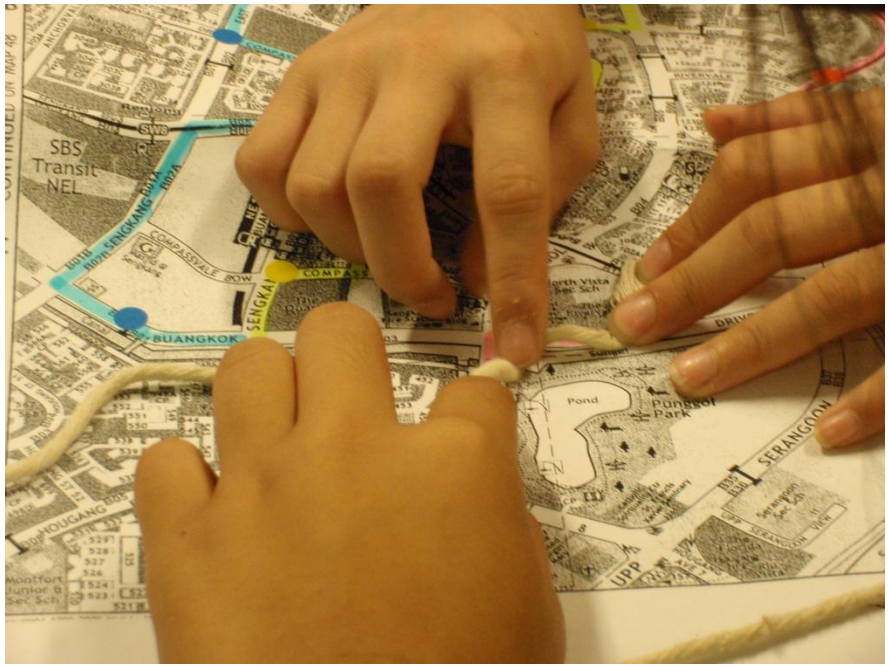
- Speed
- Cost


SBS
Transit
NEL
BUANGKOK
Pond
Punggol
Park
SERANGOON

Exemplifying the Core Components

Processes

Making assumptions
Formulating mathematical relationships
Solving / Mathematical Solution
Interpreting the real-world situation



Mathematical reasoning, communicating of ideas and making connections

Making Assumptions

S1	What is assumption?
	(pause)
S2	So we just …
S1	What is assumptions? ( ), what is assumptions?
S2	You have dictionary?

T	Yes. there’s some information that you don’t have. You feel you don’t have enough info. You can actually make your own assumptions. You can look at decision on how you want to work through that part a bit. Okay, you may not know everything about it but you can make the assumptions.

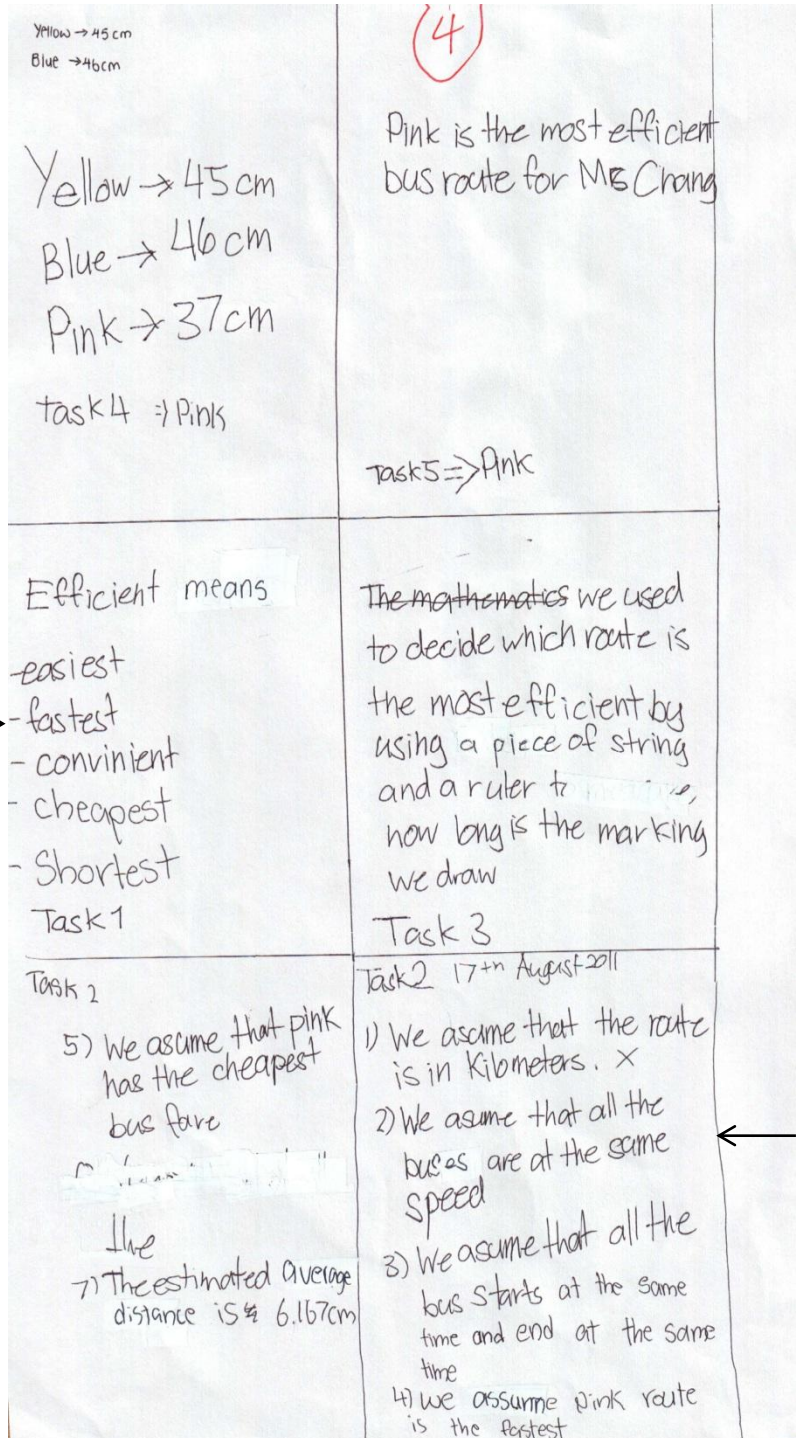
Making Assumptions

1) We assume that all the buses are at the same speed.
2) We assume that all the buses starts and end at the same time.
3) We assume that the pink route is the fastest.
4) We assume that the pink route is the cheapest.
5) The estimated average distance is 6.167cm.

Assumption

- We assume that all bus are in good condition
- We assume that all bus start at the same time
- We assume that there are no funtions.

Unpacking the meaning of “Most Efficient”



Yellow → 45 cm
Blue → 46cm

Yellow → 45 cm
Blue → 46 cm
Pink → 37cm

task 4 => Pink

(4)

Pink is the most efficient bus route for Mrs Chang

Task 5 => Pink

Efficient means

- easiest
- fastest
- convinient
- cheapest
- Shortest

Task 1

~~The mathematics~~ we used to decide which route is the most efficient by using a piece of string and a ruler to [illegible], how long is the marking we draw

Task 3

Task 2

5) We asume that pink has the cheapest bus fare

[illegible]

the

7) The estimated average distance is 6.167cm

Task 2 17th August 2011

1) We asume that the route is in Kilometers. ×

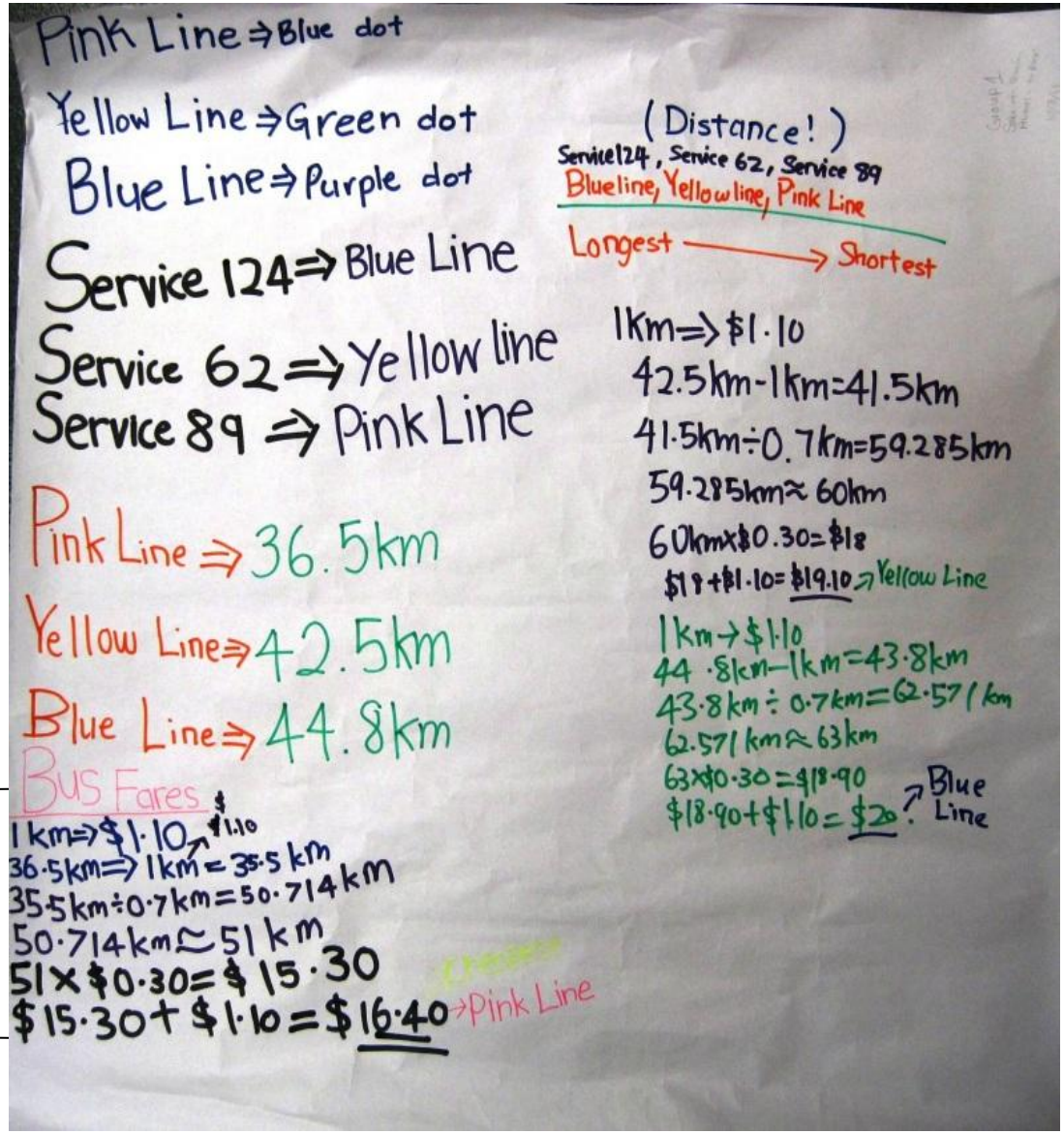
2) We asume that all the buses are at the same speed

3) We asume that all the bus starts at the same time and end at the same time

4) we assume pink route is the fastest

Making assumptions

Formulating Mathematical Relationships & Working Towards a Mathematical Solution

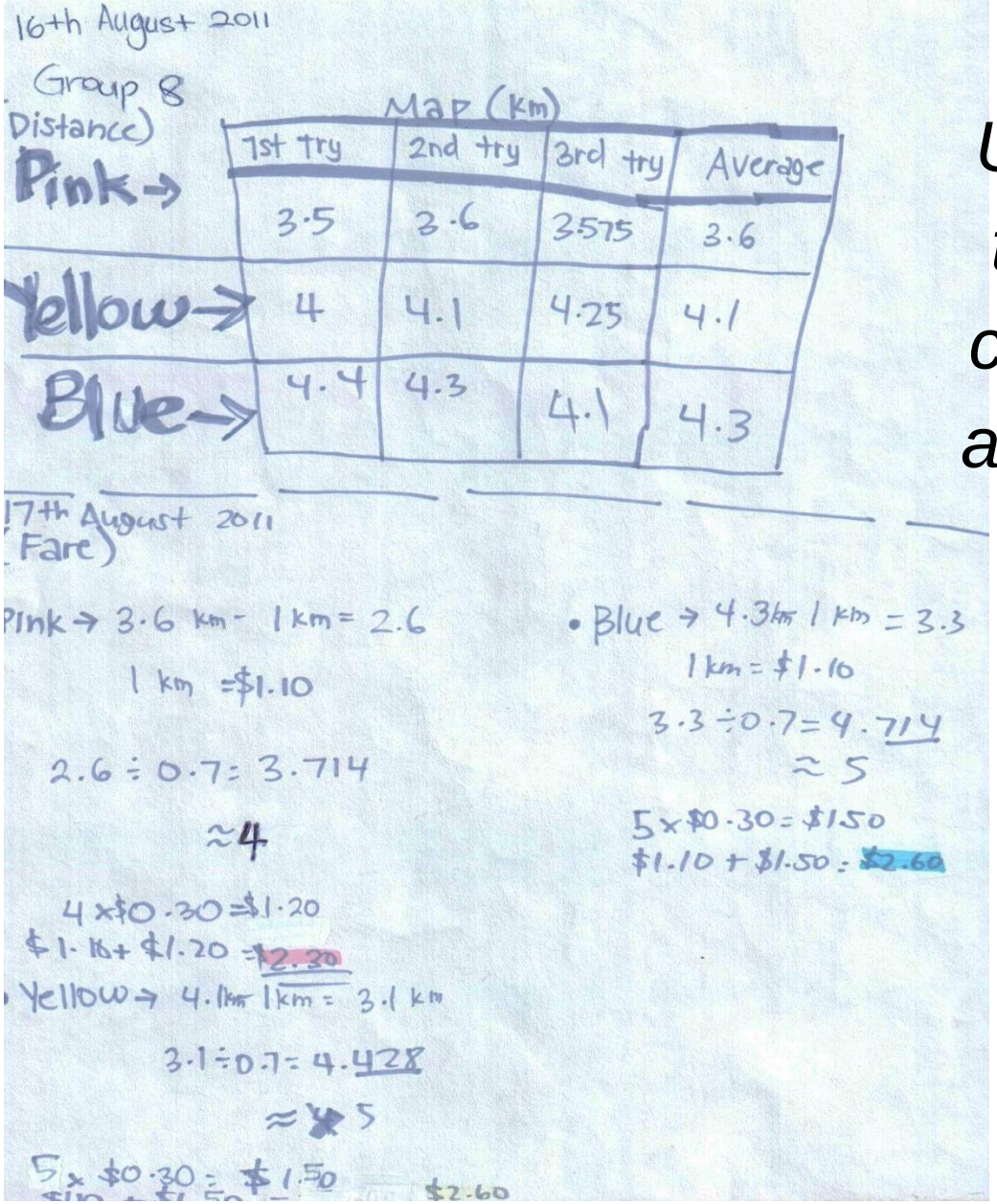


Distance vs Cost

Yellow Route

Blue Route

Pink Route



16th August 2011

Group 8

Distance)

Map (km)

	1st try	2nd try	3rd try	Average
Pink →	3.5	3.6	3.575	3.6
Yellow →	4	4.1	4.25	4.1
Blue →	4.4	4.3	4.1	4.3

17th August 2011

Fare)

Pink → 3.6 km - 1 km = 2.6

1 km = $1.10

2.6 ÷ 0.7 = 3.714

≈ 4

4 × $0.30 = $1.20

$1.10 + $1.20 = $2.30

Yellow → 4.1 km 1 km = 3.1 km

3.1 ÷ 0.7 = 4.428

≈ 5

5 × $0.30 = $1.50

$2.60

• Blue → 4.3 km 1 km = 3.3

1 km = $1.10

3.3 ÷ 0.7 = 4.714

≈ 5

5 × $0.30 = $1.50

$1.10 + $1.50 = $2.60

Use of a table to compare averages

Distance vs Cost

Relating Solution to Problem Situation

S3	Our group determine the most efficient bus route by trying to find the shortest route and cheapest bus fare. We came out with the conclusion that pink route which is service 89 route is the most efficient route.
S1	All the buses would be travelling at the same speed. There would be no traffic (jams).
T	Are these your assumptions?
S1	There would be no traffic lights.
S1	The mathematics used to decide which route - we use average, money, assumptions, decimal, estimations and measurement to find out the most efficient route.
S2	We calculated the bus fares through the distance. Our final chosen route is the route of bus service 89.

Exemplifying the Core Components

Metacognition

Self-Checking

Monitoring the process

S2	Red is the pink one.
S1	Yah. Then it's 30. Eh 42.
S4	Eh er write down, write down, write down.
S3	Okay. This red colour. Eh, you need to measure blue also.
S1	Huh?
S3	Measure blue. Just in case lah.
S1	Write first, write first. Later you forgot.

Exemplifying the Core Components

Metacognition

Self-Checking

Monitoring the process

S4	Is it the – no, it's all in cm.
	…
S4	Is it, is it like 1km, 1km?
S1	Yah, could be leh.
S2	Yah. Actually should be 1km. Because a road cannot be 1cm.

Exemplifying the Core Components

Attitudes

Beliefs – Math is meaningful and related to the real world

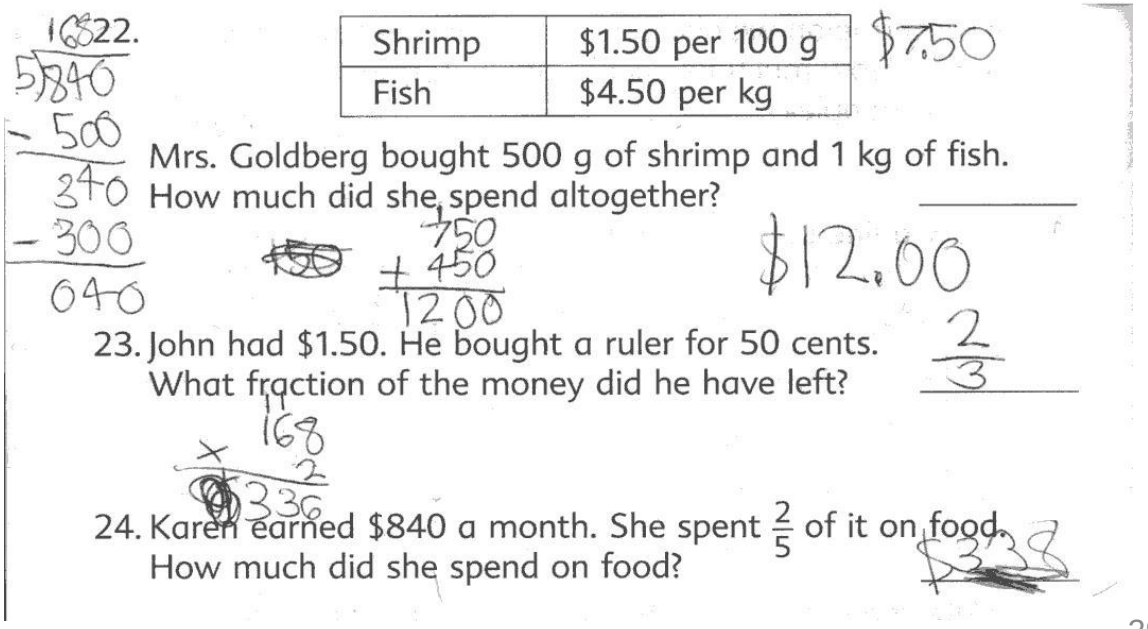
S1	What if there's….one of the, one of the shortest route has a traffic jam?
	…
S2	(Maybe we can assume that) there are no traffic lights...
S3	You know sometimes you go stop at bus stop, there are a lot of people queuing up. You know like…just like morning one we go to school you know.
S2	Maybe and probably that the bus stop got a lot of people queuing up to go up the bus then will delay.

Exemplifying the Core Components

Attitudes

Perseverance – the activity lasted 3h in all.

Contrast



Shrimp	$1.50 per 100 g
Fish	$4.50 per kg

Mrs. Goldberg bought 500 g of shrimp and 1 kg of fish. How much did she spend altogether? _____

23. John had $1.50. He bought a ruler for 50 cents. What fraction of the money did he have left? _____

24. Karen earned $840 a month. She spent $\frac{2}{5}$ of it on food. How much did she spend on food? _____

Implications

- Novice modellers are capable of completing a modelling task.

- Modelling tasks rooted in real-world settings have great potential to actualize the curriculum → elicits as well as develops pupils' thinking w.r.t. to mathematical concepts, skills, processes, metacognition and attitudes.

Thank You

Journal papers on the Most Efficient Bus Route Task involving pupils.

Chan, C. M. E., Ng, K. E. D., Widjaja, W., & Seto, C. (2012). Assessment of primary 5 students' mathematical modelling competencies. *Journal of Science and Mathematics Education in Southeast Asia, 35*(2), 146-178.

Chan, C. M. E., Widjaja, W., & Ng, K. E. D. (2011). Exemplifying a model-eliciting task for primary school pupils. *Southeast Asian Mathematics Education Journal, 1*(1), 65-74.