PROBING CHILDREN'S STRATEGIES

IN MATHEMATICAL PROBLEM SOLVING

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Paper presented at the 1994 AARE Conference, University of Newcastle
27 November to 1 December 1994

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Background

When investigating the role of knowledge in problem solving, Simon (1980) suggested that although a person may possess the relevant knowledge to solve a problem in a given situation, there is no guarantee that the knowledge will be accessed and applied when needed. Kroll and Miller (1993) in their study of the problem solving research literature state that to solve problems efficiently students must possess relevant knowledge and be able to coordinate their use of appropriate skills. Burkhardt (1988) claimed that there may be a five year gap between learning mathematics imitatively and using it freely, effectively and autonomously in solving a problem. Bastow, Hughes, Kissane & Mortlock (1990) suggest:

Knowledge and application of some mathematical content are required in an investigation but if they are kept to levels at which the students have competence and confidence, then the focus can be the use and development of processes. An appropriate level for content may need to be at least two years below the student's current level of formal study. (p.2)

Stacey and Bourke (1988) suggested that the required computational skills in a mathematical problem solving task should be distinctly at a level below those currently held by the problem solvers. However, Kulm (1990) stated that some researchers, in an attempt to make sure that a problem is really a problem (that is, the student has no algorithm or standard method readily available), have used mathematical content that is a year or two ahead of the students' mathematical experience.

As obvious as it may seem, it is important to emphasise that mathematics content plays an essential role in mathematical problem-solving success (Lester & Kroll, 1990). Although there have been many suggestions about the mathematics content of mathematical problems there appears to be little or no specific research to identify and quantify the apparent gap, or lag, between a student's ability to carry out the mathematical calculation and to solve a mathematical problem requiring the same mathematics content knowledge.

Prior to any overt instruction in mathematical problem solving, when students are asked to solve a wide range of mathematical problems, they often exhibit `raw' heuristics (Silver, 1985). Some individual students show differences in their tendency to use certain heuristic processes `au natural' i.e. without specific instruction (Silver, 1985) while others have no evident `raw' heuristic tendencies. This observation suggests that there may be a range of different heuristics that students of different year levels are comfortable with and hence shed some light on the instructional needs of problem solvers at different year levels of schooling. Most mathematics syllabus suggest a whole list of heuristics for teachers to teach. As an example the mathematics syllabus for primary and secondary schools in Singapore (Ministry of Education, 1990a; Ministry of Education, 1990b) state the following as some of the heuristics for problem solving:

act it out, restate the problem in another way, use a diagram/model, use tabulation, make a systematic list, look for a pattern, work backwards, use before-after concept, use open sentence, use equation, make supposition, simplify the problem, solve part of the problem and think of a related problem. (p.58)

There are, however, no guidelines provided for teachers about when to

introduce particular heuristics to students.

It is often difficult to diagnose the difficulties experienced by students in solving mathematical problems by examining their written solutions. It may be more fruitful, when diagnosing errors, to interview students, noting their verbalisations and thought patterns about the specific problems with which they were confronted. It cannot be assumed that when an incorrect answer is given to a mathematical task that the error occurred because the student lacked the necessary mathematical knowledge or skill (Newman, 1977). In written assignments a double interview technique may be used to diagnose errors which a student has made. A key assumption in this double interview technique is that the type of errors students make will be consistent from one session to another.

Clarkson (1986) conducted a small scale study to validate the above assumption and found that most students were consistent in the types of errors they made. He also noted that careless errors made during the first session were often self-corrected by the students during the interview session. Kaur (1993) documented 91% consistent errors in a study using the double interview technique to diagnose errors which students made in a paper and pencil assignment. Retrospective analysis has often been criticised for the unreliability of the accounts of behaviour, including all the cognitive processes used, which are constructed for a problem solver after an attempt to solve a particular problem (Lester, 1982). Nevertheless, it seems possible that one-to-one interviews, despite their limitations, do give greater insights into students' thinking and difficulties which would not be possible purely from an analysis of a paper and pencil answer.

The Study

This study explores two hypotheses. They are:

1.That there is a time lag between the age at which problem solvers are able to carry out certain mathematical calculations and the age at which they are able to use the same mathematical skills to solve problems.

2.That most students are only able to select from a limited range of problem solving strategies or heuristics and that there is a developmental sequence throughout which more sophisticated strategies become available to the student.

Methodology

The test instruments

Two test instruments are used in the study:

(i)a Problems Test (Test 1) comprising 9 items, and

(ii)a Computations Test (Test 2), also comprising 9 items.

Each of the nine items in the Problems Test (Test 1) is followed by a questionnaire which the student has to complete after working the item to establish background information about the problem and the student's attitude to it. The Computations Test (Test 2) comprises 9 items, each consisting of exactly the same mathematics as that required to solve each of the corresponding problems i.e. item 1 of the Computations Test is based on the mathematics of item 1 of the Problems Test. Similarly each of the items 2 to 9 of Test 2 replicates the mathematics content of each of the Problems 2 to 9 in Test 1. The mathematics content was assessed to be that of Year 5 level mathematics in Singapore schools (Ministry of Education, 1990a).

Subjects

A total of 626 students from three government schools in Singapore took part in the study. The numbers by year levels and sex were:

Year	No. of girls	No. of boys	Total
5	65	91	156
6	67	90	157
7	89	68	157
8	78	78	156
Total	299	327	626

Procedure

Test 1 was administered to the students on day 1 and Test 2 on the second day of their participation in the study. Both tests were administered under examination conditions. The students were not told in advance about the tests. They were given sufficient time to complete the tests.
Based on the analysis of the data collected using the two instruments, a total of 139 subjects were interviewed. The numbers by year level were:

Year	No.
5	37
6	37
7	30
8	35
Total	139

The interview structure was derived from the Newman Error Analysis Guideline (Newman, 1983) and Ransley's (1979) problem solving model. The interviews were conducted one-to-one and were audio-taped.

Scoring and Data Collation

Both tests had a maximum score of 36 marks i.e. 4 marks for each item. Test 2 was easily scored (by hand) as the answers were either right or

wrong and no consideration was given to the working. Test 1 was scored using a focussed holistic scoring scheme adapted from Charles et. al., (1987, p. 35).
In order to compile a composite picture of each subject's performance on both the instruments used, a data sheet was used to record all relevant details from the test scripts. For the subjects who were also interviewed, an interview data sheet was used to record all relevant details from the audio-tapes.

Data Analysis and Results

Test Scores Data

The Complete Statistical System: Statistica (1991) computer package was used to analyse the test scores of the subjects. The means and standard deviations of the two tests for the 4 year levels are as follows:

Problems TestComputations Test
(max = 36)(max = 36)
Mean (s.d.)Mean (s.d.)

Year 519.58 (6.94)33.10 (2.31)
(n=156)

Year 622.04 (7.44)34.20 (1.95)
(n=157)

Year 723.87 (6.40)34.33 (2.02)
(n=157)

Year 826.04 (5.63)34.68 (1.77)
(n=156)

The means of the Problems Test ranged from 19.58 to 26.04 marks while that of the Computations Test ranged from 33.10 to 34.68 marks. The means of both tests increased with the corresponding year levels. The increase was more for the Problems Test than that for the corresponding Computations Test across the year levels.

The effect sizes (Cohen 1969, pp. 18-25) for the year levels on both test scores were worked. They are as follows:

Effect Sizes for Year Levels on Problems Test Score
YearEffect Size (d)5-
60.35

70.62
80.93

-
0.25-
0.540.34

Effect Sizes for Year Levels on Computations Test Score
YearEffect Size (d)5-
60.48
70.53
80.68

-
0.07-
0.240.17

Effect size, d = M6 - M5where M6 - mean of Year 6
SD5 M5 - mean of Year 5
SD5 - s.d. of Year 5
when Year 5 is taken as the baseline.

Using Cohen's (1969) rule of thumb a d-value of 0.2 is referred to as a small-effect size. A medium-effect size is one for which d = 0.5, and a large-effect size is one for which d = 0.8. The effect sizes for the Problems Test ranged from 0.25 to 0.93 while that for the Computations Test ranged from 0.07 to 0.68. For every year level taken as a base year in turn the corresponding d-values were ascending but varying in step size with increasing year levels. This shows us that the older students were doing better than the younger ones at the two tests

The inter-tests correlations were low for all year levels and are as follows:

Year 5Year 6Year 7Year 8

Problems Test vs.0.32220.38760.42700.3856

Computations Test

The low correlations tell us that the scores of the two tests were not predictive of each other.

Interview Data

The audio-taped data of the interviews was analysed using a structure derived from the Newman Error Analysis Guideline (Newman, 1983) and Ransley's (1979) problem solving model. In this paper only summaries of the interview data to two of the problems (Rectangular shape, Stamps) used in the study will be presented. The flow charts show at which stage the subjects were unable to proceed. The causes established are also stated and the frequencies (%) given. The table accompanying the flow chart gives the breakdown of the occurrences at the various stages by year levels.

Figure 1Problem (Rectangular Shape)

Alice has 20 cm of wire. She makes a rectangular shape with the wire. The shape has the largest possible area. What is the length and width of the shape Alice made? Explain how you worked it out.

```
                                 /ffffffø
                                 ≥ Read ≥      A /fffffffffffffffffffffffø
                                 ¿fff¬ffŸ        ≥a) doesn't understand   ≥
                                     ≥           ≥   the phrase 'largest  ≥
                               /fffff¡ffffffø    ≥   possible area' (5.4%)≥
                               ≥ Comprehend √ffff¥b) finds the Question   ≥
                               ¿fffff¬ffffffŸ    ≥   complicated; 'area   ≥
                                     ≥           ≥   & perimeter to be    ≥
                                     ≥           ≥   worked at the same   ≥
                                     ≥           ≥   time'          (0.8%)≥
                                     ≥           ≥c) unable to distinguish≥
                                     ≥           ≥   between area and     ≥
                                     ≥           ≥   perimeter      (0.8%)≥
                                     ≥           ¿ffffffffffffffffffff¬ffŸ
                                     ≥                                 ≥
                                     ≥            B /fffffffffffffffø  ≥
                            /ffffffff¡ffffffffø     ≥Lacks strategy ≥  ≥
                            ≥ Select Strategy √fffff¥do not know how≥  ≥
                            ¿ffffffff¬ffffffffŸ     ≥to proceed     ≥  ≥
                                     ≥              ≥        (33.1%)≥  ≥
                           /fffffffff¡fffffffffø    ¿ffffffff¬ffffffŸ  ≥
                           ≥ Formulate the sum ≥             ≥         ≥
                           ¿fffffffff¬fffffffffŸ             ≥         ≥
                                     ≥                       ≥         ≥
                              /ffffff¡fffffffø               ≥         ≥
                              ≥ Do the maths ≥               ≥         ≥
                              ¿ffffff¬fffffffŸ               ≥         ≥
C                                    ≥                       ≥         ≥
/fffffffffffffffffffffffffø          ≥                       ≥         ≥
≥   Incorrect             ≥     /ffff¡fffffø                 ≥         ≥
```

```
≥a) inappropriate strategy√fffff¥ Solution ≥              ≥         ≥
≥   used.  Merely         ≥     ¿ffff¬fffffŸ              ≥         ≥
≥   manipulates numbers   ≥           ≥                   ≥         ≥
≥   e.g. 20 ˆ 6 =;        ≥           ≥                   ≥         ≥
≥   20 ˆ 4 = ; no         ≥           ≥                   ≥         ≥
≥   consideration of area ≥           ≥                   ≥         ≥
≥                 (20.0%)≥  /fffffff¡ffffffffffffø        ≥         ≥


≥b) lack of knowledge     ≥  ≥ Correct             ≥      ≥         ≥
≥   that a square is a    ≥  ≥[error inconsistent] ≥      ≥         ≥
≥   rectangle.     (36.1%)≥  ≥                (1.5%) ≥    ≥         ≥
≥c) careless: 5 x 5 = 20; ≥  ¿fffffff¬ffffffffffffŸ       ≥         ≥
≥   4 x 6 = 24            ≥           ≥                   ≥         ≥
≥   .. sol'n is 6cm by 4cm≥           ≥                   ≥         ≥
≥      rectangle    (0.8%)≥           _                   ≥         ≥
≥d) takes semi-perimeter  √fffffff_ STOP _fffffffffffffffff¡fffffffffŸ
≥   to be 20 cm     (1.5%)≥
¿fffffffffffffffffffffffffŸ
```

YearNo.(%)

CcCdCorrect537 (28.5)1(0.8)--17(13.1)9 (6.9)10 (7.7)

---636 (27.7)1(0.8)--13(10.0)7 (5.4)13(10.0)

-1(0.8)1(0.8)729 (22.3)2(1.5)1(0.8)-8 (6.2)6 (4.6)10 (7.7)

1(0.8)1(0.8)-828 (21.5)3(2.3)-1(0.8)5 (3.8)4 (3.1)14(10.7)

--1(0.8)Total130(100.0)7(5.4)1(0.8)1(0.8)43(33.1)26(20.0)47(36.1)

1(0.8)2(1.5)2(1.5)

Wei Min has to post a parcel costing $23. He has plenty of $5 and $2 stamps but no others. How many of each kind of stamps would he use so that not more than 8 stamps are used altogether? Show all your working.

```
                     /ffffffø
                     ≥ Read ≥    A /fffffffffffffffffffffffø
                     ¿fff¬ffŸ      ≥a) doesn't understand  ≥
                         ≥         ≥   the phrase 'not more≥
                   /fffff¡ffffffø  ≥   than 8 stamps'      ≥
                   ≥ Comprehend √ff¥                 (4.8%)≥
                   ¿fffff¬ffffffŸ  ≥b) doesn't understand  ≥
                         ≥         ≥   if he can use both  ≥
                         ≥         ≥   $5 and $2 stamps    ≥
                         ≥         ≥                 (1.6%)≥
```

```
                          ≥         ¿ffffffffffffffffffffff¬Ÿ
                          ≥          B/fffffffffffffffø    ≥
                          ≥           ≥Lacks strategy ≥    ≥
                /ffffffff¡ffffffffø  ≥do not know how≥    ≥
                ≥ Select Strategy √ff¥to proceed     √ffø ≥
                ¿ffffffff¬fffffffŸ  ≥        (16.1%)≥  ≥ ≥
                          ≥           ¿fffffffffffffffŸ  ≥ ≥
               /fffffffff¡ffffffffø C/ffffffffffffffffffø ≥ ≥
               ≥Formulate the sum√ff¥Unable to         ≥ ≥ ≥
               ¿fffffffff¬fffffffŸ  ≥translate problem≥ ≥ ≥
                          ≥          ≥into a maths sum ≥ ≥ ≥
                 /fffffff¡fffffffø   ≥            (3.2%)≥ ≥ ≥
                 ≥ Do the maths ≥   ¿fffffffff¬fffffffŸ ≥ ≥
                 ¿fffffff¬ffffffŸ            ≥         ≥ ≥
D                         ≥                  ≥         ≥ ≥
/ffffffffffffffffø    /ffff¡fffffø           ≥         ≥ ≥
≥   Incorrect    √ffff¥ Solution ≥           ≥         ≥ ≥
≥a) inappropriate≥    ¿ffff¬fffffŸ           ≥         ≥ ≥
≥   strategy,    ≥         ≥                 ≥         ≥ ≥
≥   mere mani-   ≥         ≥                 ≥         ≥ ≥
≥   pulation of  ≥         ≥                 ≥         ≥ ≥
≥   nos. (17.7%) ≥         ≥                 ≥         ≥ ≥
≥b) does not find≥ /fffffff¡ffffffffffffffø     ≥         ≥ ≥
≥   a combination≥ ≥  Correct            ≥    ≥         ≥ ≥
≥   of $5 and $2 ≥ ≥ [error inconsistent] ≥   ≥         ≥ ≥
≥   to get       ≥ ≥              (17.7%)≥    ≥         ≥ ≥
≥   exactly $23, ≥ ¿fffffff¬ffffffffffffffŸ     ≥         ≥ ≥
≥   feels that   ≥         ≥                 ≥         ≥ ≥
≥   $24 postage  ≥         ≥                 ≥         ≥ ≥
≥   will do as it≥         ≥                 ≥         ≥ ≥
≥   is more than ≥         ≥                 ≥         ≥ ≥
≥   $23          ≥         ≥                 ≥         ≥ ≥
≥[Soln given was ≥         ≥                 ≥         ≥ ≥
≥ 4 $5 and 2 $2] ≥         ≥                 ≥         ≥ ≥
≥          (38.7%)≥        ≥                 ≥         ≥ ≥
¿fffffffff¬ffffffŸ         ≥                 ≥         ≥ ≥
          ≥                                  ≥         ≥ ≥
          ¿fffffffffffff  STOP  ffffffffffffffff¡fffffffff¡fŸ
```

Year	No.(%)	No. (%)						
		Aa	Ab	B	C	Da	Db	Correct
5	26 (41.9)	2(3.2)	-	6 (9.7)	-	6 (9.7)	7(11.3)	5 (8.1)
6	18 (29.0)	1(1.6)	-	1 (1.6)	1(1.6)	2 (3.2)	10(16.1)	3 (4.8)

≥ 7   ≥10 (16.1)∫   - ≥1(1.6)≥ 2 (3.2)≥1(1.6)≥ 2 (3.2)≥ 3 (4.8)≥ 1 (1.6)≥
√ƒƒƒƒƒƒ≈ƒƒƒƒƒƒƒƒƒ◊ƒƒƒƒƒƒ≈ƒƒƒƒƒƒ≈ƒƒƒƒƒƒƒƒ≈ƒƒƒƒƒƒ≈ƒƒƒƒƒƒƒƒ≈ƒƒƒƒƒƒƒƒ≈ƒƒƒƒƒƒƒƒ¥
≥ 8   ≥ 8 (12.9)∫   - ≥   - ≥ 1 (1.6)≥   - ≥ 1 (1.6)≥ 4 (6.5)≥ 2 (3.2)≥
∆ÕÕÕÕÕÕÿÕÕÕÕÕÕÕÕÕŒÕÕÕÕÕÕÿÕÕÕÕÕÕÿÕÕÕÕÕÕÕÕÿÕÕÕÕÕÕÿÕÕÕÕÕÕÕÕÿÕÕÕÕÕÕÕÕÿÕÕÕÕÕÕÕÕµ
≥Total ≥62(100.0)∫3(4.8)≥1(1.6)≥10(16.1)≥2(3.2)≥11(17.7)≥24(38.7)≥11(17.7)≥
¿ƒƒƒƒƒƒ¡ƒƒƒƒƒƒƒƒƒ–ƒƒƒƒƒƒ¡ƒƒƒƒƒƒ¡ƒƒƒƒƒƒƒƒ¡ƒƒƒƒƒƒ¡ƒƒƒƒƒƒƒƒ¡ƒƒƒƒƒƒƒƒ¡ƒƒƒƒƒƒƒƒŸ

A detailed study of the subjects written responses to the problems was carried out and the following were the strategies used by the subjects from Years 5 to 8 to solve the 9 problems :

ProblemAFGCLRMPSIX

Medicine**

Plastic tile**

Rectangular shape****

Stamps*****

Travelling****

Loans***

Presents****

Handshakes*****

Cows and chickens*****

Legend :A - Algebra
F - Use of formula
GC - Guess & check
LR - Logical reasoning
M - Modelling
P - Look for a pattern
SI - Systematic investigation
X - Other (number manipulation, unable to detect method used)

For all the nine problems there were subjects relying on the three strategies: logical reasoning, modelling, and number manipulation to work a solution. It appears that some strategies were specific to the problems e.g. guess & check to the Stamps and Cows and chickens problems; and systematic investigation to the Rectangular shape Problem.

The frequency (%) of strategies used by the subjects to solve the two problems (Stamps, Cows and chickens) are presented. They are as follows:

Table 1

Frequency (%) of strategies used by students to solve the Stamps problem

How studentsYear 5 (%)Year 6 (%)Year 7 (%)Year 8 (%)
solved the(n=156)(n=157)(n=157)(n=156)
problem

SolnSolnSolnSolnSolnSolnSolnSolnGuess & check
X/X/X/X/(random)

0.0037.826.3743.951.2756.052.5657.05Guess & check
(systematic)

0.647.695.1017.203.1813.380.643.85Guess & check
(mental)

0.0010.261.271.911.917.011.2811.54Modelling
(diagrams)

0.003.210.000.000.000.000.642.56Logical
reasoning

0.007.690.648.920.003.180.007.05Algebra -
linear
equations

0.000.000.000.003.180.002.560.00Algebra -
simultaneous
equations

0.000.000.000.000.000.000.001.92Other (number
manipulation,
unable to
detect method
used)

28.210.0011.460.008.280.006.410.64

No response; Yr 5 = 4.49%, Yr 6 = 3.18%, Yr 7 = 2.55%, Yr 8 = 1.28%

Legend:Soln X -incorrect solution
Soln / -correct solution

Table 2

Frequency (%) of strategies used by students to solve the Cows and chickens problem

How studentsYear 5 (%)Year 6 (%)Year 7 (%)Year 8 (%)
solved the(n=156)(n=157)(n=157)(n=156)
problem

SolnSolnSolnSolnSolnSolnSolnSolnGuess & check
X/X/X/X/(random)

1.287.692.5512.101.9114.010.0025.00Guess & check
(systematic)

1.285.771.2719.111.9124.840.008.33Modelling
(diagrams)

0.640.641.910.000.640.000.000.00Logical
reasoning

0.001.920.641.910.001.270.000.64Algebra - linear
equations

0.000.000.000.003.823.181.925.77Algebra -
simultaneous
equations

0.000.000.000.000.000.641.9216.03Other (number
manipulation,
unable to detect
method used)

62.820.0045.220.0038.850.0026.280.00

No response; Yr 5 = 17.95%, Yr 6 = 15.29%, Yr 7 = 8.92%, Yr 8 = 14.10%

Legend:Soln X -incorrect solution
Soln / -correct solution

Findings and Conclusions

The results of this study suggests that there is a gap or lag between the mathematical knowledge and the ability to apply this knowledge to solving problems for most school students. Certainly it appears that the required computational skills needed to solve a particular problem should be at a level below those currently held by the individual problem solver. The gap is not a fixed measurable time, however, as suggested in some of the literature, but appears to vary, both for individual students and for different types of problems and the varying strategies needed for their solution.

The interview data revealed that the general assumption that unsuccessful problem solvers lack relevant mathematical skills is not necessarily true. Students were not successful at arriving at the solution for the following reasons:

* lack of comprehension of the question posed
* lack of schema and strategic knowledge
* inability to translate the problem into a mathematical form

When a solution was obtained, many a time it was incorrect owing to:

* inappropriate strategy used
* computational errors
* lack of mathematical knowledge such as 'a square is a rectangle'
* carelessness
* conditions of the problem being interpreted incorrectly

An analysis of the problem solving strategies used by the students to solve the nine problems show that there are a few basic strategies which most students tend to use across all year levels. For both the problems (stamps, cows and chickens) a descending percentage of the students corresponding to an ascending year level resorted to number manipulation which are coping strategies (Sowder, 1988; cited in Lambdin,D.V., Kloosterman,P., & Johnson,M., 1994) such as

a) find the numbers and add (or subtract, multiply, or divide, depending on recent classroom computational work or the operation the student is most comfortable with),
b) guess at the operation to be used,

to solve the problems. These students either did not appear to comprehend the problems and recognise the givens and the goals or lacked schema and strategic knowledge (Kroll & Miller, 1993). For both the problems (Stamps, Cows and chickens) the guess and check strategy was used by a significant number of students across all year levels. However the individual developmental levels of the students guided them to guess and check either mentally, randomly or systematically.

Some year 7 and 8 students were able to use algebra to solve the problems. Although all the year 8 students had been taught simultaneous equations at school six months prior to their participation in this study, it is interesting to note that only very few of them did try to solve the cows and chickens problem using simultaneous equations.

References:

Bastow, B., Hughes, J., Kissane, B. & Mortlock, R. (1990). 40 Mathematical Investigations. The Mathematical Association of Western Australia.

Burkhardt, H. (1988). Teaching problem solving. In H. Burkhardt, S. Groves, A. Schoenfeld and K. Stacey (Eds.) Problem solving - A world view (pp 17-42). Nottingham, England: The Shell Centre for Mathematical Education, University of Nottingham.

Charles, R., Lester, F., & O'Daffer, P. (1987). How to evaluate progress in problem solving. National Council of Teachers of Mathematics.

Clarkson, P. (1986, Dec). On consistency of classifying errors. Research in Mathematics Education in Australasia (pp 22-24). Publication of MERGA (Mathematics Education Research Group of Australasia).

Cohen, J. (1969). Statistical power analysis for the behavioral sciences. NY: Academic Press.

Complete Statistical System. (1991). CSS (Complete Statistical System): Statistica. Statsoft Inc.

Kaur, B. (1993, Sept). The relationship between mathematics knowledge and mathematical problem solving. Paper presented at the seventh annual conference of the Educational Research Association (Singapore), Singapore.

Kroll, D.L. & Miller, T. (1993). Insights from research on mathematical problem solving in the middle grades. In D.T.Owens (ed.), Research ideas for the classroom: Middle grades mathematics, 58-77. NY: Macmillan.

Kulm, G. (1990). Assessing higher order mathematical thinking: What we need to know and be able to do. In G. Kulm (Ed.) Assessing higher order thinking in mathematics (pp 1-4). American Association for the Advancement of Science.

Lambdin, D.V., Kloosterman, P., & Johnson, M. (1994). Connecting research to teaching. Mathematics in the Middle School, 1(1), 38-43.

Lester, F.K. (1982). Issues in mathematical problem-solving research. In S.

Rachlin & J.McDonald (Eds.) Problem solving in the mathematics classroom (Math monograph number 7) (pp 141-145). MCATA - Publication of the Mathematics Council of the Alberta Teachers' Association.

Lester, F.K. & Kroll, D.L. (1990). Assessing student growth in mathematical problem solving. In G. Kulm (Ed.) Assessing higher order thinking in mathematics (pp 53- 70). American Association for the Advancement of Science.

Ministry of Education (1990a). Mathematics syllabus (Primary 1-3, Primary 4-6 [Normal course], Primary 4-8 [Extended course]). Curriculum planning division, Ministry of Education, Singapore.

Ministry of Education (1990b). Mathematics syllabus (Secondary 1-2 [Special/Express course], Secondary 1-2 [Normal course]). Curriculum planning division, Ministry of Education, Singapore.

Newman, A. (1983). The Newman Language of Mathematics Kit - Strategies for Diagnosis and Remediation. Harcourt Brace Jovanovich Group (Australia) Pty Ltd.

Newman, M.A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. Research in Mathematics Education in Australasia (pp 239-258). Publication of MERGA (Mathematics Education Research Group of Australasia).

Ransley, W. (1979). Problem solving and a mathematical diagnostic interview technique. Canberra College of Advanced Education.

Silver, E.A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E.A. Silver (Ed.) Teaching and learning mathematical problem solving: Multiple research perspectives (pp 247-266). Hillsdale, NJ: Lawrence Erlbaum Associates.

Simon, H.A. (1980). Problem solving and education. In D.T. Tuma & F. Relif (Eds.) Problem solving and education: Issues in teaching and research (pp 81-96). Hillsdale, NJ: Lawrence Erlbaum Associates.

Stacey, K. & Bourke, S. (1988, Dec). Testing mathematical problem solving. In Research in Mathematics Education in Australasia (pp 12-28). Publication of MERGA (Mathematics Education Research Group of Australasia).

** Paper presented at the 1994 AARE Conference, University of Newcastle 27 November to 1 December 1994.