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Problem Solving in Regular Classrooms: The Use of Pattern Observation

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Introduction

The notion of mathematics as a 'science of patterns' has gained currency among mathematics educators in recent years (Schoenfeld, 1992). Key documents such as Everybody Counts (National Research Council, 1989), Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) and Reshaping School Mathematics (National Research Council, 1990) consistently reflect this view. The view that "[m]athematics is a living subject that seeks to understand patterns that permeate both the world around us and the mind within us (National Research Council, 1989, pp. 84)" is echoed in these documents. Schoenfeld (1992) characterized mathematics as "an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns - systematic attempts, based on observation, study and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically (pure mathematics) or models of systems abstracted from real world objects (applied mathematics) (pp. 335)".

The intended mathematics curriculum in Singapore primarily aims to enable students to develop their ability in problem solving (Ministry of Education, 1990). The curriculum advocates the integration of problem solving in mathematics instruction to develop their mathematical thinking. Similarly in Australia, the 1990 National Statement on Mathematics for Australian Schools, one of the goals stated is that students should develop their ability to use mathematics in solving problems individually and collaboratively (Australian Education Council, 1990).

There is sufficient anecdotal evidence that students tend to view mathematics as a collection of tools and algorithms to be used to obtain definite quantitative answers to given problems. Most would also tend to believe that mathematics is an individual endeavour.

The Study

This paper describes an attempt to teach mathematics to incorporate the views on mathematics reflected by Schoenfeld (1992). The teaching experiment was conducted on a group of Secondary 3 (Year 9) students in Additional Mathematics[1] for a period of six months. This experimental group consisted of students of low mathematical ability as reflected by achievement test score. There were two control groups. One control group comprised students with similar achievement test scores as students from the experimental group while the other group consisted of students with higher achievement test scores. These scores were obtained from the previous year's mathematics examination results.

The main part of this paper describes several activities used to teach one topic during the teaching experiment. The rest of the paper summarizes the post-test results.

1 Additional Mathematics is an optional second mathematics subject offered to Year 9 and Year 10 students in Singapore. All Singapore students offer Elementary Mathematics.

Sample Activities

The following are activities used during the instruction on the topic of Binomial Theorem.

Lesson 1 (2 periods)

Groupwork. Students worked collaboratively in groups of four to expand several algebraic expressions.

Expand each of the following:

$(1+x)^2$

$(1+x)^3$

$(1+x)^4$

$(1+x)^5$

$(1+x)^6$

$(1+x)^7$

and more, if necessary.

The main aim is to discover a pattern to help you write down an expansion such as $(1+x)^{15}$, for example, without actually doing the expansion.

Lesson 2 (1 period)

Whole Class Discussion. If there were groups that had discovered a pattern then these groups would have presented their findings. If no group had discovered any trends then the teacher would have highlighted the Pascal Triangle and its generation.

The following triangle of numbers form a pattern.

1
1 1
1 2 1
1 3 3 1

Observe and continue the pattern. This is the Pascal Triangle.

Lesson 3 (2 periods)
Groupwork. Students, in the same groups of fours, worked collaboratively on two pieces of work.

How can the Pascal Triangle be used to find the expansion of, say, (1+x)9?

Refer to your previous worksheet and look out for patterns. Explain your reasoning.

Expand
(1+2x)2
(1+2x)3
(1+2x)4
(1+2x)5
(1+2x)6
(1+2x)7

(1+2x)8
and find out how the Pascal Triangle can be used to expand (1+2x)10. Any patterns?

Similarly expand
(4+x)2
(4+x)3
(4+x)4
(4+x)5
(4+x)6
and use the Pascal Triangle to expand (4+x)10. Any patterns?

Lesson 4 (1 period)
Whole Class Discussion/Activity. The teacher conducted a whole class activity, preceding a lesson on how to use the calculator to obtain binomial coefficients. The teacher asked a series of questions.

Teacher : Now, one boy step out of this group (pointing to one particular group).

How many different ways can this be done?

Teacher : What if I require two boys to step out of the group? How many ways can it be done?

Teacher : Now, three boys step out of the group. How many different ways?

Teacher : How about four boys from the group? How many different ways to do this?

For each question, if students were able to give an answer then they would be asked to explain their method orally. Otherwise a simple role-play would make the answers evident. The teacher subsequently showed the students how to use the calculators to evaluate these coefficients.

Individual Investigation. A piece of individual assignment was given as homework.

What if
there were 5 boys in the group ?
there were 6 boys in the group ?
there were 7 boys in the group ?
there were 8 boys in the group ?
there were 9 boys in the group ?
there were 10 boys in the group ?

Investigate. How is the result of this investigation relate to the Pascal Triangle?

Lesson 5 (2 periods)
Whole Class Discussion. The teacher selected several students to present their investigation. Most students would be able to gather some data. However, not every student were able to generalize the data gathered from the investigation. The teacher concluded the presentation session by showing the expansion of several binomial expressions. The teacher wanted the students to observe the trend involved in generating binomial coefficients and to generalize the results of the previous investigation.

Let us expand the following :
$(1+3x)4$
$(2+x)5$
$(3+2x)6$
The important thing is to look out for patterns.

Individual Written Work. The teacher used another three periods to discuss two pieces of homework assignments done by the students.

Find the first three terms in the expansion, in ascending powers of x, of (1-3x)5 and (2+x)4.

Explain, without any further working, how you could determine the coefficient of x2 in the expansion
(1-3x)5(2+ x)4.

What if the coefficients of (a) x3 (b) x4 (c) x5 are required?

Given an expression (2+3x)(1-Ωx)n.

Write two questions and answer them.

Sample of Assignments
Students were required to complete a number of tasks. Grades were given for each of these tasks. Examples of individual tasks were shown above.

Students were given their group assignments at the start of each topic.

Your group must

(a) present group findings for one of the forthcoming activities

and

(b) present, in a form suitable for notice board display,

either a piece entitled 'Pascal Triangle in Real Life',

or a piece entitled 'Just Who Is Pascal?'.

Sample of Post Test Items
The post-test items were items from standard achievement tests. The two items based on Binomial Theorem in the post-test are shown.

Question 10
If the first three terms of the expansion of (1+mx)n in ascending powers of x are 1 + 6x + 16x2, find the values of m and n.

Question 15(a)
Find, in ascending powers of x, the first four terms in the expansion of

(i) (1+2x)7
(ii) (3-x)4.
Hence obtain the coefficient of x2 in the expansion of (1+2x)7(3-x)4.

Results of Post-Test Scores

This experimental group consisted of students of low pre-test scores. There were two control groups. One control group (control group 1) comprised students with similar pre-test scores as students from the experimental group while the other group (control group 2) consisted of students with higher pre-test scores. These pre-test scores were obtained from the previous year's mathematics examination results.

The experimental group ($n = 29$) performed significantly ($p = 0.05$) better than control group 1 ($n = 39$). Control group 2 ($n = 42$), however, performed significantly ($p = 0.05$) better than the experimental group.

It is thus noted that the good students, taught in the expository style, with a strong emphasis on algorithms, continued to excel in standard achievement test. The experimental group, not expected to outdo the 'good' class, showed better performance in the achievement test than another class of similar ability.

Conclusion

The results of this teaching experiment suggest some benefits of instruction that placed an emphasis on pattern observation. Even if the students did not managed to gain a 'deep' understanding of the topic taught, they would have benefited in terms of mathematical processes. It would be interesting to observe the effects of this teaching experiment on (a) the ability to solve non-routine questions, (b) perception of the nature of mathematics.

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