CHILDREN POSING WORD PROBLEMS DURING A PAPER-AND-PENCIL TEST: RELATIONSHIP BETWEEN ACHIEVEMENT AND PROBLEM POSING ABILITY

Ho Siew Yin & Shirley Lee
Swiss Cottage Primary School
Singapore

Yeap Ban Har
National Institute of Education
Nanyang Technological University
Singapore

Abstract: Children writing mathematics word problems has been advocated by mathematicians and mathematics educators. Mathematics curricula in several countries have made the call for teachers to provide opportunities for children to solve as well as to formulate problems. Problem posing, however, is a relatively novel task among Singapore children. In this paper we describe one school's attempt in getting Primary Five children to pose problems during a traditional paper-and-pencil achievement test. The paper focuses on the relationship between the children's achievement and the problems they posed. All of the 115 Primary Five children in a school were asked to write a word problem based on a pictorial stimulus. The data was used to explore if high-achieving children pose more complex problems than low-achieving ones. The children's examination score was used to form a high-achievement group and a low-achievement group. The problems posed by children in each group were analyzed according to the number of information included in them to reveal their mathematical complexity. The results have significant implications on the use of paper-and-pencil tests to assess children's ability to think mathematically.

Mathematical Problem Posing

Mathematical problem posing is defined as the generation of new problems or the re-formulating of existing ones (Silver, 1994). It has been identified as an essential mathematical activity (NCTM, 1989) and inseparable from mathematical problem solving (Kilpatrick, 1987). Kilpatrick (1987) also suggested that problem posing is becoming increasingly important in the information age, where there is a shift from performing routine algorithms towards conceptual understanding of these algorithms and their uses. Silver (1994) describes problem posing as a feature of creativity. Thus, with the current emphasis on thinking, problem solving and information technology in the Singapore schools, mathematics educators should have an increased understanding of mathematical problem posing.

Kilpatrick (1987) lamented that mathematical problem posing has received little explicit attention in school mathematics curriculum. However, Silver (1994) observed that this is beginning to change. In the United States of America, the *Professional Standards for Teaching Mathematics* (NCTM, 1991) states that "[s]tudents should be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (p. 95). This echoed a similar call made previously in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) which identified problem posing as "an activity that is at the heart of doing mathematics" (p. 138). In Australia, the *National Statement on Mathematics for Australian Schools* (Australian Educational Council, 1991) advocates the use of problem posing in the teaching and learning of mathematics. In Singapore, the revised mathematics curriculum calls for opportunities for pupils "to extend and generate problems" (p. 17, Ministry of Education, 2000).

This paper reports on an investigation into the relationship between primary school pupils' achievement test scores and their problem posing ability.

Mathematical Problem Posing Tasks

There are many different types of problem posing tasks. Silver (1994) classified problem posing activities as occurring before, during or after problem solving. Stoyanova (1998) classified problem posing tasks as free, semi-structured or structured. A task is considered free if students are asked to generate a problem from a given, contrived or naturalistic, situation. For example, students are asked to make up problems for a mathematics competition. A task is semi-structured if students are given an open situation and are asked to explore the structure and to complete it by applying their existing knowledge, skills and concepts. For example, students are asked to make up as many problems as possible that fit a number sentence such as 3 x 5. A task is structured if it is based on a specific problem. For example, students are asked to pose as many questions as possible based on this problem: Last night there was a party and the host's doorbell rang ten times. The first time the bell rang only one guest arrived. Each time the doorbell rang after that, three more guests arrived than had in the previous ring (Stoyanova & Ellerton, 1996).

Yeap (2000) classified problem posing tasks according to the nature of numerical information given in the task. In Yeap's (2000) framework, the numerical information in a problem posing task can be in concrete, iconic or symbolic forms. In concrete tasks, numerical information is in the form of some concrete materials such as three blue pens and three red pencils. In an iconic task, numerical information is in pictorial forms as a graph chart. In symbolic tasks, numerical information is in the form of symbols, numerals or words. The numbers given in a task can be (1) not in context and isolated, (2) not in context but related, (3) in context but unrelated, (4) in context and related. Sometimes the numbers given are to be part of the posed problems. In other tasks, the numbers given are the answers to the posed problems.

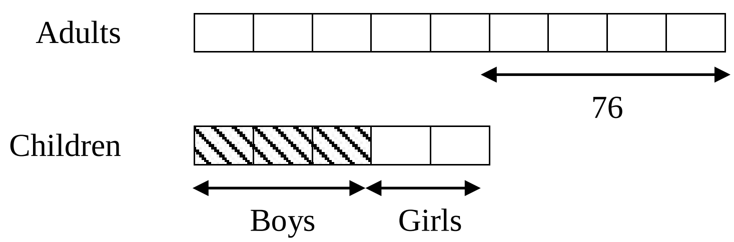
The Study

Sample

The sample comprises the entire cohort of primary five pupils in a school. One pupil was absent on the day of data collection. A total of 114 pupils participated in the study. The students were from the EM1 and EM2 streams.

Data Collection

The pupils sat for the first semestral examination, which was conducted in the middle of the year. This semestral examination was in the form of a typical paper-and-pencil achievement test. Pupils responded to 15 multiple-choice questions, 20 short-answer items and 14 word problems and problems with diagrams. In solving the problems, pupils are required to show their methods. In the short-answer item, only an answer was required. Approximately 50% of the marks for the achievement test were allocated to the word problems. The examination items include a task that requires pupils to pose a problem based on a pictorial stimulus. This item was the third last item on the achievement test. Figure 1 shows the item.



Write a ratio problem sum for this model.

Figure 1: Problem posing item

Data Analysis

The score for achievement test was taken from pupils' performance in the multiple-choice items, short-answer items and word problems. The achievement scores of the 114 pupils range from 11 to 96 out of a possible 96. Pupils with scores in the lowest third were labelled Low Achieving (n=38, average achievement score =47), those with scores in the middle third were labelled Mid Achieving (n=37, average achievement score =74), and those with scores in the highest third were labelled High Achieving (n=39, average achievement score =87).

The responses to the problem posing task were coded in two stages. In the first stage, a response was coded as successful or unsuccessful. A successful response is a problem that is plausible. An unsuccessful response includes problems that contain insufficient information or contradictory information such that these posed problems cannot be solved. Pupils who did not respond or did not write a word problem were also included in this category. Table 1 shows some examples of unsuccessful responses.

Table 1
Examples of Unsuccessful Responses

Category	Example
Insufficient Information	*The number of women who attended a concert is 76. If the number of men is the same as the number of children altogether, what is the ratio of the adult men to the women to the children?* [5112]
Inconsistent Information	*There are 15 boys and 20 girls. Their ratio is 3:2. 76 adults are men. Its ratio to women is 4:4. How many people are there watching the show altogether?* [5217]
Non-word Problem	*There are 14 units of adults and children. If there are 76 more adults than children how many children are there?* [5316]

In the second stage, a successful response was further analyzed for the number of essential information included in the posed problem. This is the minimum number of information required to solve the problem.

Table 2
Examples of Successful Responses

Category	Example
Number of Essential Information =1	*266 people went to a 'West Life' concert. The ratio of adult to children is 9:5. How many people went to the concert?* [5314]
Number of Essential Information = 2	*In a ship, there are adults, boys and girls. The ratio of adults to children is 9:5. There are 76 more adults than children. If the ratio of boys to girls is 3:2, find how many adults are there?* [5404]
Number of Essential Information = 3	*The ratio of adults to boys to girls who visited a carnival is 9:3:2. There are 76 more adults if you add the boys and girls together. How many boys went to the carnival?* [5422]

In the response "*266 people went to a 'West Life' concert. The ratio of adult to children is 9:5. How many people went to the concert?*", only one piece of information (*266 people went to a 'West Life' concert*) is essential to solve the posed problem. In the response, "*In a ship, there are adults, boys and girls. The ratio of adults to children is 9:5. There are 76 more adults than children. If the ratio of boys to girls is 3:2, find how many adults are there?*", only two (*The ratio of adults to children is 9:5. There are 76 more adults than children.*) of the three pieces of information included are essential to solve the posed problem. In the response "*The ratio of adults to boys to girls who visited a carnival is 9:3:2. There are 76 more adults if you add the boys and girls together. How many boys went to the carnival?*", three pieces of information (*The ratio of adults to boys* and *girls who visited a carnival is 9:*5. *There are 76 more adults if you add the boys and girls together*. *The ratio of …boys to girls who visited a carnival is ..3:2.*) are essential to solve the problem.

Results

The number of pupils in each achievement level that can or cannot pose a solvable problem is shown in the contingency table (Table 3). The chi-square test revealed that there is most likely a relationship between achievement test score and ability to pose problem. The chi-square value was 11.4 (larger than 9.2), with 2 degrees of freedom, is significant at 0.01 level. The contingency coefficient, which indicates the strength of the relationship, was 0.22.

Table 3
Contingency Table to Show the Ability of Pupils with Different Achievement Profile to Pose Problems

	Low Achiever	Mid Achiever	High Achiever
Unable to pose problem	25	20	11
Able to pose problem	13	17	28

The complexity of problems posed by the pupils in the three levels of achievement was also analyzed. The contingency table (Table 4) shows the number of pupils who posed problems with one, two or three pieces of essential information. The chi-square test also revealed that there is most likely a relationship between achievement test score and ability to pose more complex problems. The chi-square value was 9.54 (larger than 9.5), with 4 degrees of freedom, is significant at 0.05 level. The contingency coefficient was 0.25.

Table 4
Contingency Table to Show the Complexity of Problem Posed by Pupils with Different Achievement Profile

Number of essential information required to solve the problem	Low Achiever	Mid Achiever	High Achiever
1	2	0	1
2	8	8	8
3	3	9	19

Discussion & Conclusion

The results show that there is significant relationship between achievement and ability to pose problems as well as ability to pose problems that contain more essential information. However, the same relationship did not exist in one of the four classes involved in the study. This class was exposed to problem posing activities. The pupils have been asked to write problems for each other to solve once a fortnight. Apart from writing word problems, no other intervention was introduced.

The number of pupils in this problem posing class that can or cannot pose a solvable problem is shown in the contingency table (Table 5). The chi-square test revealed that there is no significant relationship between achievement test score and ability to pose problem among pupils in this class. The chi-square value was 1.18 (less than 6.0), with 2 degrees of freedom, is not significant at 0.05 level.

Similarly, for the problem posing class, there is no significant relationship between achievement scores and complexity of problems posed. The chi-square value was 4.29 (less than 9.5), with 4 degrees of freedom, is not significant at 0.05 level.

Table 5
Contingency Table to Show the Ability of Pupils with Different Achievement Profile to Pose Problems in the Problem Posing Class

	Low Achiever	Mid Achiever	High Achiever
Unable to pose problem	6	5	2
Able to pose problem	7	4	5

Table 6
Contingency Table to Show the Complexity of Problem Posed by Pupils with Different Achievement Profile in the Problem Posing Class

Number of essential information required to solve the problem	Low Achiever	Mid Achiever	High Achiever
1	1	0	0
2	4	1	1
3	2	3	4

Low achieving pupils, when given the experience to write problems, were just as able to pose solvable problems as high achieving pupils. One classroom implication is whether teachers can use problem posing activities to increase the level of achievement of these pupils.

Thus, there is a significant relationship between achievement score and problem posing ability. However, this relationship is not significant when pupils have been given opportunities to pose problems on a regular basis.

Further Research

51% of the sample was unable to pose a solvable problem. Further analysis to emerge categories of difficulties will be conducted to suggest reasons for inability to pose solvable problems. It is interesting to see if any of these categories of difficulties is due to the lack of conceptual understanding. Further analysis will also be conducted to investigate the relationship between ability to solve word problems and ability to pose them. It will also be interesting to investigate how pupils respond to different types of problem posing tasks as well as how pupils in different age group respond to the same task.

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