Title: Activity theory for mathematics learning in and out of the classroom?
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Source: ERAS Conference, Singapore, 19-21 November 2003
Organised by: Educational Research Association of Singapore (ERAS)

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ACTIVITY THEORY FOR MATHEMATICS LEARNING IN AND OUT OF THE CLASSROOM?

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Abstract

This paper reports on an attempt to describe mathematics teaching and learning from the perspective of cultural-historical activity theory. It offers, with the intention of stimulating discussion and critique, a reading of the theory and its potential in guiding classroom practice. The paper presents briefly the development of the activity theory through its major phases, describes its theoretical position and discusses a mediational take on cognition, language, and appropriation of meaning in the mathematics classroom. Examples considered illustrative of an activity-theoretic approach in mathematics teaching and learning are presented. In doing so, the paper hopes to clarify the central concepts and principles of the theory and to take first steps to examining its possible use in theorizing about mathematics education in and out of the classroom.

The purpose of this paper is to stimulate discussion of activity theory, or, as it has been recently called, cultural-historical theory (abbreviated CHAT). It is motivated by the relative lack of local interest in the theory. Hung (1999) is perhaps the only one to have written about the theory. The paper describes briefly the development of activity theory through its major phases. To this end, it draws extensively from Yjro Engeström’s work (Engeström, online). Classroom examples considered illustrative of an activity-theoretic approach in mathematics teaching and learning are presented alongside the description of the theory.

Historical roots of activity theory

Activity theory has its origins in Soviet psychology (Engeström, online). It grew from Lev Vygotsky’s tool-mediated conception of the development of human higher mental functions. Vygotsky’s student and later colleague, Alexei N. Leont’ev developed the notion of activity further by distinguishing between the concepts of activity, action and operation; these concepts are elaborated below. Alexander R. Luria refined the theory by emphasising human thinking is culturally mediated. Evald Il’enkov was one of a number of Soviet psychologists were involved in extending the theory. Yjro Engeström extended the tool-mediated subject-object relationship to include other mediational means – these means are detailed later. Michael Cole added to the conceptualisation of activity theory by adding the notions of simultaneous coexistence and interaction of various different cultures to it.
**Hierarchical structure of activity**

Loent’ev conceptualises *activity* as a collective system driven or defined by its object. The object of the activity is realised through individualised *actions* driven by goals. Actions are in turn realised by means of routinised or automatic *operations*. Figure 1 shows the transitions between the different levels of an activity are common. For example, where an automatic operation is hindered, as when a student fails to solve a “routine” (say, quadratic) equation, the otherwise automatic operation breaks into consciousness to the level of goal-directed actions focused on solving the equation. On the other hand, a goal-directed action may become routinised as automatic operation through familiarity or competence in the action. Additionally, the same goal-directed action can be used in different activities while different actions may be undertaken to meet the same goal. Activity theory takes the subject’s (a student or a group of students) perspective as central.

<table>
<thead>
<tr>
<th>Level</th>
<th>Oriented</th>
<th>Carried out by</th>
</tr>
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<tbody>
<tr>
<td>Activity --- Object/Motive --- Community</td>
<td></td>
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<tr>
<td>Action --- Goal --- Individual/Group</td>
<td></td>
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<tr>
<td>Operation --- Condition --- Routinised</td>
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</tbody>
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Figure 1  Structure of an activity  (Adapted from Engeström, online)

A point to note is that actions, by themselves, may appear “meaningless” or “senseless”. Leont’ev (1981) illustrated it with the example of a group of people out hunting for food. The action of scaring the prey away by one member of the hunting group would appear strange to an “observer’ if analysed on it own. That action is meaningful only when viewed as part of the collective activity (of hunting) - it might be considered strategic as the intention is to chase the prey into the hands of the waiting members. Thus an individualistic analysis the subject’s action by itself would be inadequate; the (integral) unit of analysis for human activities is the activity in its entirety.

**Activity systems**

Engeström (online) expanded on Vygotsky’s idea of a subject (person) not dealing directly with an object but via mediating tools or artefacts (Figure 2). Engeström introduced the mediating elements of *rules* for mediating between subject and community and of *division of labour* for mediating between object and community (Figure 3).
In Engeström’s model, the community consists of multiple individuals or subgroups who hold the same object of the activity and who are distinct from other communities. In his formulation, “division of labour” refers to “both the horizontal division of tasks between the members of the community and to the vertical division of power and status”. The “rules” refer to “the explicit and implicit regulations, norms and conventions that constrain actions and interactions within the activity system”. The subject, via the mediational means such as the mediating artefacts of physical and cultural tools (e.g., language), transforms the object of the activity into an outcome.

A point to note in this model is that

Different subjects, due to their different histories and positions in the division of labor, construct the object and the other components of the activity in different, partially overlapping and partially conflicting ways. (Emphasis added)

Thus, by virtue of their instructional history or position, pupils hold objects for their mathematics learning that are likely to differ from the teachers’.

An activity system is not an isolated system (e.g., a student does not engaged in mathematical problem posing in isolation). It interacts with a network of activity systems (e.g., activity systems of researchers or teachers interested in mathematical problem posing) and may appropriate from them their rules (e.g., sociomathematical norms Yackel & Cobb, 1996) that determines what is acceptable as a mathematics problem) and mediating tools (e.g., recognised techniques of formulating a mathematics problems). The objects of external activity systems (e.g, the object of a student formulating an “elegant” mathematics problem rather than just any arithmetic problem) may intrude into an activity system to modify it (e.g. the student may accede to a change of object). In Engeström’s words,

“The outside influences are first appropriated by the activity system, turned and modified into internal factors … The activity system is constantly working through contradictions within and between its elements. … There is also incessant movement between nodes of the activity”
The components of the activity are seen not as static and isolated from each other but as dynamic and continuously interacting with each other components. Also, an outcome of an activity may be co-opted into a component, (say) a mediating artefact, of the activity system. Figure 4 shows a schematic example of a mathematical problem-posing activity system. The components interact in a multitude of ways to produce the outcome. For example, the rules would describe the obligations of the pupil to different members of the community (e.g., pupil to teacher, classmates, parent or researcher) and contribute to the definition of the object of the activity.

Engeström identified four levels of contradictions in a network of interacting activity systems:

- **Level 1**: Primary inner contradiction *within* each constituent component of the central activity.
- **Level 2**: Secondary contradictions between the constituents of the central activity as new elements enter the activity system from the outside.
- **Level 3**: Tertiary contradiction between the object/motive of the dominant form of the central activity and the object/motive of a culturally more advanced form of the central activity.
- **Level 4**: Quaternary contradictions that emerge between the central activity and its neighbouring activities in their interaction.

In the case of a pupil engaged in posing mathematics problems, an example of the four levels of contradictions could be as follows:
A first level contradiction arising from the pupil’s “faulty” mathematical tools (e.g., using an incorrect problem analogue) to mediate in achieving the object (here, a “good” mathematics problem) of problem posing.

A second level contradiction arising from the use of a new strategy or method of problem posing (e.g., Walter and Brown’s “What-if-not …”) that is introduced from the outside (e.g., teacher’s activity system).

A third level contradiction arising when a culturally more advanced object of pupil self-initiating the problem posing efforts is brought into the activity; this object is in conflict with the object of the activity of problem posing that is directed by the teacher.

A fourth level contradiction arising when the problem-posing activity system of professional mathematicians is to be introduced to the activity of students posing school mathematics problems.

Object and Activity
From the activity-theoretic perspective, an activity is characterised by its object. It is the object of an activity distinguishes it from another activity. There is no such thing as an “objectless” activity. However, getting at the subject’s “real” object (or motive?) may not be easy because it is tacit to the person and is not easily available for scrutiny by that person. (Wertsch, 1985) Additionally,

“Objects can be transformed in the course of an activity; they are not immutable structures…. Objects do not, however, change on a moment-by-moment basis. There is some stability over time, and changes in objects are not trivial; they can change the nature of an activity fundamentally” (Nardi, 1996, p. 74).

To illustrate, for a student sufficiently engaged in making up a challenging mathematics problem, the object of the activity could be relatively stable over a 30-minute lesson. Where the student could not sustain the interest, the object of activity might “degenerate” to one of posing just any problem that comes to mind. It is possible that the “real” object might remain tacit to the student and thereby the teacher.

Learning Mathematics In and Out of the Classroom?
An activity theory approach offers a systematic way of describing and understanding pupils learning mathematics. From an activity theory perspective, mathematics learning takes place in and out of the classroom, regardless. The enterprise of mathematics teaching and learning is seen to take place in a social setting and its object is mediated by various means.
Consider a classroom exchange between pupil (P) and teacher (T) in the activity of learning mathematics. How might an observer make sense of their exchanges to solve the mathematics problem below?
John drove from Town A to Town B at an average speed of 100 km/h. Ali drove from Town B to Town A at an average speed of 50 km/h. They started their journey at the same time. Find the distance from Town A when they meet each other.

P: (Rather indignantly) They don’t meet.
T: Why?
P: Because they would pass each other by, that’s why!
T: Well, OK. Suppose they say “Hello” to each other as they pass each other. How far would that be from Town A?
P: Halfway.
T: Tell me how you got that.
P: The first person to get halfway waited for the other.
T: Well, OK. Suppose that person continues on when he got halfway.
...

What would be a minimal meaningful context for understanding and interpreting the pupil’s actions and outcomes (i.e., thinking made visible) as he or she undertake the activity of learning mathematics? In the above pupil-teacher exchange, the pupil’s responses could be said to be mediated by his or her real-world experiences of car trips and what it means to “meet”. These were experienced outside of school. The pupil’s lack of the established ways of working with such problems could be seen as a contradiction arising from the component of mediational tools or artefact (e.g., language as in the meaning of “meet”). The contradiction could also be seen as a “failure” of the subject’s current cognitive organisation to “adapt” to the new demands of the environment (e.g., not knowing the assumptions peculiar to solving such problems and the way the problem statement is couched). It is hope that in working his or her way through a series of contradictions, the pupil would develop the “right mathematics or cognitive tools” to solve the problem.

Alternatively, it could be said that this was bright pupil whose “real” object was “to clown around”. It could be that the object was simply “an outright refusal to join in the boring game of solving contrived problems”, thereby making the responses less than meaningful by accepted norms and ways of doing mathematics. The pupil has mapped out the meaning of the activity for herself or himself by the object he or she has defined for the activity.

To take the discussion of activity theory in mathematics learning further, we could consider the teacher as a subject who works within his or her own activity system (see Figure 5). The teacher’s object might be enculturation into a way of doing mathematics in which there are enough shared meanings and established mathematical techniques to initiate action to solve the problem. In this case, the teacher’s explicit interventions could be seen as a case of interactions with other activity systems which attempt to transform the existing pupil’s activity.
The above discussion suggests that there are many ways of interpreting meaningfully the goal-directed actions of the pupil (or teacher) because of the infinitely many ways that the context of the exchange could be dynamically and continuously constituted. From an activity theory perspective, the minimal meaningful context or the unit of analysis is the activity itself (Nardi, 1996). It is clear that attempting to characterise the pupil’s or teacher’s responses individualistic terms would be inadequate. Internal (or mental) activities cannot be analysed separately from external activities because they transform each other (Nardi, 1996). In activity theory, “context” appears to be posited as that which takes place in an activity system composed of subject, object, actions, operations and mediating artefacts, as well as the dynamics among all the components of the activity system. Thus conceptualised obviate the challenge of identifying which aspects of a situation is “relevant” as context. The common usage of context has a static sense to it in that it refers to given aspects of the pre-set environment for an event or action that is called upon to explicate the meaning of that event or action. Context in the dynamic sense refers the setting being constituted by persons (including their mental states) in interaction with the external environment.

**Conclusion**

To recall, the purpose of this paper is to initiate discussion and clarify the tenets of activity theory and its central concepts. For example, the Russian notion of “activity” (which does have an equivalent in English) and the idea of “object” or “motive” warrant further deliberation. The concept of *activity systems* can be useful for making sense about a child’s learning of mathematics inside or outside of the classroom. It is an explanatory device that allows for a micro-to-macro (or vice versa) view of the endeavour of learning mathematics. The analysis could start with the broad picture of the pupil as a member of social group and move inwards into the fine-grain features of the pupil’s individualistic psychological (e.g. cognitive and affective) functioning. The contradictions that arise from pupils’ thinking as they are made visible to an observer, e.g., teacher or researcher, could perhaps be traced to
the community of other pupils and teachers in terms of rules and expectations of what is acceptable and in forging the object of learning mathematics,

- the division of labour in terms of roles for pupils and teachers in the activity system of a pupil,

- the mediating artefacts in terms of mathematical language and tools, cognitive tools (e.g., strategies, ways of remembering), and mathematical knowledge, for constructing a mathematics problem, or

- the interactions among the components.

Other tasks ahead include mapping out the activity systems for a mathematics teacher, a mathematics education researcher, a pupil learning mathematics out of school (recall Nunes, Schliemann & Carraher’s (1993) street and school mathematics). Doing so should contribute to establishing a framework for conducting research into the interacting activity systems of pupils, teachers, parents and researchers. There could also be research on evolving activity systems, e.g., that of a pupil’s activity system as it works its way through the contradictions or the development of research methodologies that will meet the complexities of interacting activity systems of mathematics learning arising from the conceptualisation of context as dynamically constituted.

References


