A PROPOSAL FOR ASSESSING MATHEMATICS THINKING – A LESSON LEARNT FROM THE JAPANESE OPEN-ENDED APPROACH

Lee Ngan Hoe
Nanyang Technological University, Singapore

Abstract: The role of assessment in informing the public and parents of the outcome of instruction has, at times, become an obstacle to assessment reforms. In Singapore, for example, the public has come to accept the norm-referenced way of reporting scores. On the other hand, assessments tools offered by researchers for the teaching of thinking are often criterion-referenced. There is, therefore, a need to provide a bridge to the two different kinds of assessment tools in order for those that promote thinking to be better received by parents and the public. This is crucial as they are also partners in the education process. The Japanese Open-Ended Approach to teaching mathematics has been developed and researched since 1971. The approach promotes critical and creative thinking within the context of mathematical problem solving. The well-developed assessment tool that has evolved from the approach reflects this emphasis on critical and creative thinking. However, as in many other cases, it is more inclined towards criterion-referencing. Based on this assessment tool, proposed by the Japanese Open-Ended Approach, this paper offers a reporting format that is aimed at helping to acquaint parents and the public with the assessment method, and, hopefully, to gain wider acceptance among them of new assessment methods to come.

The Japanese Open-Ended Approach

The Japanese Open-Ended Approach to teaching mathematics is a product of the Japanese developmental research concerned with methods of evaluating higher-order thinking in mathematics education. The researchers believed that the “open-ended” teaching approach “had the potential to improve mathematics teaching and learning in Japanese schools” (Becker and Shigeru, 1997).

The Problems

The problems that our students commonly meet in traditional mathematical courses are those with unique answers that have to be determined in only one way. Often, students are also not given opportunities to pose their problems but are expected to solve ‘ready-made’ problems. This, as Steen (1989) noted, has often led students to perceive mathematics as a “dull” enterprise. On the contrary, in the Open-Ended Approach, mathematical problem solving are characterised by the following:

- The process is open, i.e. a diversity of approaches to solving a problem is encouraged.
- The end products are open, i.e. there can be multiple correct answers.
- The ways of formulating problems is open, i.e. students are encouraged to formulate other problems in the process.

The Lessons

Each lesson consists essentially of the following five stages:

- Introducing the problem or topic – Based on a pre-selected objective, the teacher presents or poses a problem on an appropriately chosen media.
- Understanding the problem – Teacher initiates discourse, through explanation and appropriate questioning, to ensure that students are clear about what is expected of them.
- Problem solving by students – Equipped with worksheet(s), students are encouraged to use their natural ways of thinking to work individually and/or in small groups. At the same time, the teacher moves round the class purposefully to identify appropriate thinking processes and approaches to be highlighted during class discussion.
- Comparing and discussing – By having students to present their work on the board, the teacher guides the class through comparing and discussing the responses according to their mathematical features and quality.
- Summary of the lesson – The teacher's final and crucial role is to fit the pieces, arising from the discussion, to form the big picture that the students could relate to the lesson's objective.

Creating Problems and Classroom Activities

One of the difficulties in implementing the Open-Ended Approach is to make or prepare meaningful mathematical problem situations. The Japanese researchers and classroom teachers have overcome the problem by working collaboratively to develop some guidelines for creating problems (Becker & Shimada, 1997). The open-ended problem that is developed is then gauged for its appropriateness based on the following considerations:

- Is the problem rich in mathematical content and is it valuable mathematically?
- Is the mathematical level of the problem appropriate to the students?
- Does the problem include some mathematical features that lead to further mathematical development?

Assessment Approach

The teachers' sources of information for assessing students' learning are:

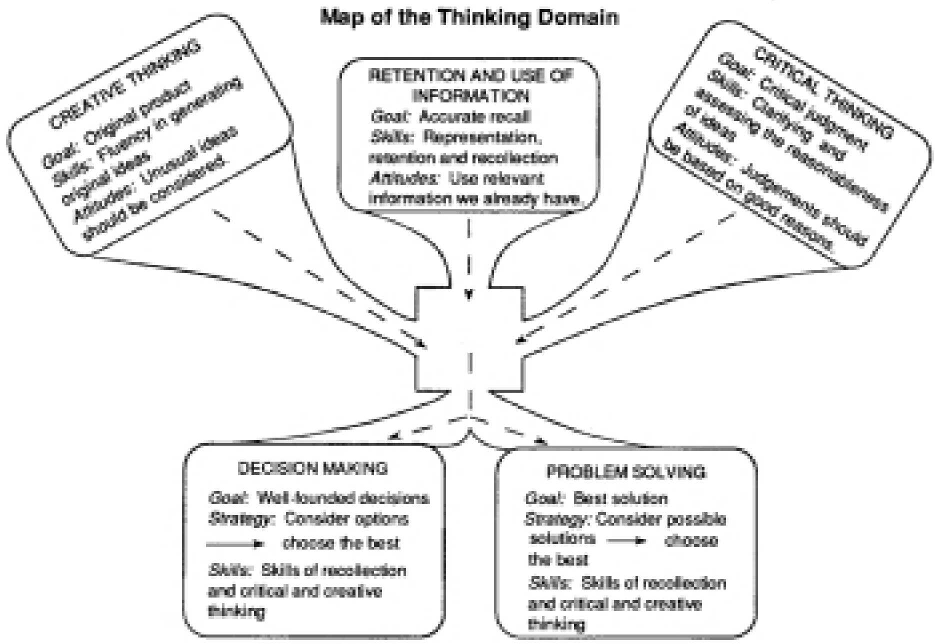
- Students' worksheets;
- Observation of students during individual and/or group work; and
- Observation of students during presentations and discussion.

However, as students' responses and reactions to an open-ended problem are rather varied, teachers draw up a list of expected responses as part of their lesson planning. These are classified according to their mathematical features and qualities. The following criteria are then used to evaluate students' achievement (Becker & Shimada, 1997):

- Fluency – How many solutions can each student produce?
- Flexibility – How many different mathematical ideas are discovered/used correctly by the student?
- Originality – To what degree are the student's ideas original?
- Elegance – To what degree is the student's expression of ideas elegant?

Thinking Processes

Robert Swartz's (1990) map of the thinking domain offers a schematic representation of the thinking process:



Swartz has classified the thinking skills into three broad categories, namely, creative thinking, retention and use of information, and critical thinking. Creative thinking skills refer to skills at generating ideas, retention and use of information skills refer to skills that foster learning for understanding and the active use of knowledge, while critical thinking skills refer to skills at assessing the reasonableness of ideas. Swartz makes a distinction between these thinking skills and the goal oriented thinking processes, which are decision making and problem solving.

However, Swartz (1994, p. 8) feels that "[T]eaching the thinking skills … without helping students learn how to use them in decision making and problem solving accomplishes only part of the task of teaching thinking. Teaching strategies for problem solving and decision making, without teaching students the skills needed to use these strategies effectively, is similarly limited. If we teach lessons on individual thinking skills and lessons on decision making and problem solving, we can show how these thinking skills are connected with good decision making and problem solving. Students will then have the thinking tools they need to face their most challenging tasks in using information and ideas."

Further examination of Swartz's map of the thinking domain reveals yet another component of thinking – the part that links the thinking skills to the thinking processes. Swartz (1990) calls this part of the thinking metacognition – "a crosscutting superordinate kind of thinking relevant to all the others", which refers to "one's knowledge about, awareness of, and control over one's own mind and thinking".

Open-Ended Approach And Assessing Students' Mathematical Thinking

In evaluating students' fluency in their responses, the number of different correct answers, different solution approaches, or problems that are formulated is considered. Fluency is assessed quantitatively by simply counting the number. With reference to Swartz's map of the thinking domain, this is a measure of the level of creativity displayed by the student.

Flexibility of student's responses deals with the mathematical quality of the responses, indicating the number of different mathematical ideas discovered or correctly applied by the student. This is a reflection of the student's ability to retain and use information, as well as his/her ability to critically assess the reasonableness of ideas.

In the Open-Ended Approach, originality is accorded a very high value. Originality recognises unique ideas generally not found by other students. It is an acknowledgement of high mathematical quality. The measure of originality is a display of student's ability to integrate both creative and critical thinking skills to discover new or unique ways to tackle a given task reasonably.

Elegance deals with the degree of elegance in student's expression of thinking in mathematical notation. It is a communication of the orchestration of all the thinking skills in the process of mathematical problem solving. Elegance reflects the power of mathematical modeling and the language of mathematics.

Though some of the four criteria involve a certain amount of subjectivity, such as the case of flexibility, originality and elegance, they provide a good and practical set of guidelines to assess students' higher order thinking in mathematical thinking. In fact, Becker and Shimada (1997) believe that the Open-Ended Approach "has implications for improving assessment in (mathematical) problem solving".

The Singapore Culture On Assessment

The competitive nature of the society in Singapore resulted in parental and societal obsession over scores for any assessment task, be it formative or summative. The need for students to excel over their peers also lead to the emphasis for norm-referenced assessment over criterion-referenced assessment. Consequently, for any new assessment practice to gain acceptance and be widely promoted, it needs to make provisions for such concerns, while, at the same time, not ignore the pedagogical needs of assessments.

A Proposal For Assessing Students' Mathematical Thinking

As shared by Becker and Shimada (1997), numerical score could be awarded to each of the four criteria. Fluency could clearly be given a numerical score corresponding to the total number of responses. For consideration of flexibility, solutions or approaches are categorized such that those that have the same mathematical idea are included in the same category. The numerical score for flexibility then is the number of categories or positive responses. To look into originality, teachers will classify responses according to the level of significance. A high score will be given to an idea with a high mathematical quality. The total score for originality will then be the weighted number of positive response. Similarly, weighted scores could be designated to reflect the degree of elegance in a student's expression of thinking in mathematical notation.

At the same time, in order to address the concern of parents and some parts of the society, more information could be included in reporting the assessment of the students' achievement in the four criteria. Rather than a single score for each criterion, the range of the scores as well as the average score for each criterion obtained by the class(es) concerned could also be made available.

A Sample Task

Title Of Task – Fun With The Year 1999

Context of Task – Level: Secondary 1
Type of assessment: Summative
Topic: Real Numbers
Material: Worksheet

Pre-requisite – The task assumes the knowledge of the following:

- Order of operations
- Use of parentheses with arithmetic operations
- Concept of negative numbers
- Concept of square roots

Task – The objective of the task is to form as many of the numbers 1 to 100, in as many different ways as possible, using the digits 1, 9, 9, 9, without changing the order of these digits. Other than using the four operations and brackets, the changing of the size of the digits and the use of non-numerical symbols are also allowed. However, the use of any concepts or symbols not covered in class has to be accompanied with appropriate explanation.

Suggested Social Organisation of Task – As the task would require students to brainstorm for ideas, they could be arranged to work collaboratively either in pairs or small groups. Home involvement could also be encouraged here if provisions are made for students to justify their responses.

A Suggested Guideline for Assessment –

- *Fluency*: The score is the total number of correct expressions formed, including the different correct expressions formed for each number.
- *Flexibility*: For each of the numbers 1 to 100, a score of 2 will be awarded for 2 different expressions formed, a score of 3 will be awarded for 3 different expressions formed, and so on. The total of these scores will be the score for flexibility.
- *Originality*: For each new mathematical concept or symbol correctly justified and used, a score of 5 will be given. The total will be the score for originality.
- *Elegance*: In the justification for each new mathematical concept or symbol used, a full mathematical treatment will be awarded a score of 10 for its elegance. A score of 5 could be awarded for a case where students offer an incomplete mathematical treatment to the justification, reflecting only partial elegance in their expression of thinking mathematically.

Fun With The Year 1999 is a purely arithmetic task in a setting that requires students to generate their own expressions rather than merely compute answers to a given set of problems. In other words, students are placed in an active mode, rather than a passive mode, in demonstrating what they have learnt. It tests for facility with the number facts, including negative integers, using a form of exhaustive thinking, thus promoting an environment for creative thinking. Students also have to work backwards, asking themselves how they can combine the 4 digits to create the required numbers. This provides a certain level of authenticity to the task, in which students create within constraints and limitations. Furthermore, as alternative solutions are encouraged, students' fluency and flexibility in their mathematical thinking are promoted.

In addition, as students are free to introduce new symbols and concepts, the task also motivates students to discover and learn ideas. In fact, the task so readily lends itself to subsequent instruction on Indicial Laws, and Combinatorics and Probability. As students have only been taught squares and square-roots of integers, the use of other rational powers to obtain the required expressions provides an opportunity for a preview of and connection between the topics.

Conclusion

As we reform our curriculum to meet the challenges of the new millennium, reforms in assessment must occur in tandem. School teachers need assessment models to help them realise the new curriculum. The vision for the curriculum of the twenty-first century is one which emphasises thinking and independent learning – a vision that is shared by most over the world, as reflected by the surge in interest in the teaching of thinking. Thus, assessment models should also reflect such a shift in emphasis. However, reforms require a change of mindset, for both the public and educational professionals. Assessment models that provides a link between the new and the old help to ensure a smooth transition and greater level of acceptance. The Open-Ended Approach might provide a partial answer to such a quest.

Reference

Becker, J.P. & Shimada, S. (Eds.) (1997). *The open-ended approach: A new proposal for teaching mathematics.* Translated from the 1977 Japanese version by Shigeru Shimada and Shigeo Yoshikawa. Reston, VA: National Council of Teachers of Mathematics.

Steen, L.A. (1989) Teaching mathematics for tomorrow's world. *Educational Leadership, 47*(1), 18-22.

Swartz, R.J., & Perkins, D.N. (1990). *Teaching Thinking: Issue & Approaches.* CA: Critical Thinking Press & Software.

Swartz, R.J., Parks, S. (1994). Infusing The Teaching of Critical and Creative Thinking Into Content Instruction. CA: Critical Thinking Press & Software.