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Executive Functioning and Mathematics Achievement

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Abstract

The importance of executive function skills in mathematical achievement is well established, and research conducted in the last decade has witnessed a move from a focus on just measuring working memory or updating to an inclusion of other EF skills, namely inhibition and shifting. We review findings from studies that have taken different approaches to measuring EF (e.g., using single versus multiple indicators), and which apply different analytical techniques to conceptualize EF structure (e.g., exploratory versus confirmatory techniques). The consistent finding that emerges despite these differences is that updating is a significant, often unique, predictor of math achievement across a wide age range, whilst the findings relating to inhibition and switching are less conclusive. We discuss these findings in relation to age-related variance in EF structure, the nature of inhibitory and shifting task requirements, and the possibility that updating is a limiting factor or a common resource for inhibition and shifting.
The last decade has witnessed substantial growth in the number of studies examining executive functioning (EF) and its relation to functional outcomes, including academic achievement. Individual differences in EF are related to literacy, writing, and science achievement (e.g., Bull, Espy, & Wiebe, 2008; Monette, Bigras, & Guay, 2011; Neuenschwander, Rothlisberger, Cimeli, & Roebers, 2012; Roebers, Cimeli, Rothlisberger, & Neuenschwander, 2012; St. Clair-Thompson & Gathercole, 2006), although some studies report that the direct effects of EF seem especially strong for mathematics (Bull, Espy, Wiebe, Sheffield, & Nelson, 2011; Lee, Ng, & Ng, 2009; van der Ven, Kroesbergen, Boom, & Leseman, 2012). Here we consider the conceptualization and measurement of EF, key findings concerning the role of EF in math achievement emerging across studies, and the implications of these findings for EF measurement and theory, and learning in the classroom.

**What is Executive Functioning?**

EF is commonly defined as processes that control, direct, or coordinate other cognitive processes, but both its conceptualization and measurement vary across studies. Early studies focused on the central executive (CE) component of Baddeley and Hitch’s (1974) working memory (WM) model and typically assessed CE capacity using span tasks requiring simultaneous processing and storage of information. These studies showed that WM explained substantial variance in mathematics performance, more so than measures of intelligence (for a review, see Raghubar, Barnes, & Hecht, 2010).

Baddeley (1996) argued for a more expansive view of EF, and in 2000, Miyake, Friedman, Emerson, Witzki, Howerter, and Wager published a highly influential paper on the unity and diversity of the CE. They discussed EF in terms of inhibition (overriding of prepotent or dominant responses), shifting (switching flexibly between tasks or mental sets) and updating.
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(monitoring and the addition or deletion of contents from WM). We focus on this wider conceptualization of EF to determine whether, despite substantial differences in measurement and analytical perspectives, a consistent picture emerges regarding the relationship of EF to mathematical achievement (studies reviewed are listed in a supplementary document available online). Included are papers that indexed updating with either WM span or updating tasks. Although the two concepts are not identical (processing and recall versus the selective replacement of information), performance on span and updating tasks appear to be closely related (St Clair-Thompson & Gathercole, 2006; Wilhelm, Hildebrandt, & Oberauer, 2013).

How might EFs contribute to mathematics achievement?

Updating might be important for holding relevant information during the problem solving process, and in the storage and retrieval of partial results. Inhibition may be needed to suppress inappropriate strategies (e.g., addition when subtraction is required), or prepotent number representations, (e.g., with whole numbers larger numbers map to greater magnitude; when the same numbers are combined in a fraction, larger denominators represent smaller magnitude. Understanding of fractions may require inhibition of number-magnitude mappings that are applicable to whole numbers). Inhibition may also be required to suppress retrieval of number bonds (e.g., retrieving ‘12’ for 3+4=), or utilization of information from a word problem that is irrelevant to the solution. Shifting skills may help switching between operations, solution strategies, quantity ranges and notations (e.g., between verbal digits, written Arabic symbols, and non-symbolic quantity representations), and between the steps of a complex multi-step problem.

Studying the relationship between EF and mathematics is not simple as neither set of skills is developmentally static. Prior to formal schooling children acquire precursor skills to mathematical functioning, e.g., understanding numerical magnitude, counting, number
recognition, and understanding of ordinal relations. With increasing age there is greater emphasis on the development of arithmetical skills, which form the basis for being able to complete multi-step word problems, algebraic word problems, and other curriculum based tests. As well as changes in the complexity of mathematics skills, investigations of EF are complicated by evidence suggesting that the relations between updating, inhibition, and shifting are not age invariant. Studies of preschool children find that the three aspects of EF cannot be distinguished (e.g., Wiebe, Espy, & Charak, 2008; Willoughby, Wirth, & Blair, 2012). Lee, Bull, and Ho (2013) found that EF is differentiated into two domains consisting of updating and an amalgamated inhibition/shifting factor in children age 5 to 13, with a clear separation of the three EF domains not found until 15 years of age. Nonetheless, many studies are designed with an assumption of diversity, which is rarely tested.

**Exploratory and confirmatory studies of the role of EF in math achievement.**

To examine whether scores from different EF tasks are influenced by one or several underlying executive processes, researchers have utilized a variety of factor analytic techniques. Confirmatory factor analyses (CFA), unlike exploratory (EFA) techniques, provide explicit tests of whether data are best modeled using single or multiple underlying or latent factors. Studies using CFA to examine the structure of EF in preschool children find that models specifying a single factor provide the best fit to the data (Bull et al., 2011; Clark, Sheffield, Wiebe, & Espy, 2013). These studies found that the latent EF factor predicted concurrent and later math skills, that this relationship was independent of IQ, and that EF mediates the effect of language proficiency and processing speed on math achievement.

Studies of preschool children that have used EFA, using either single or multiple tasks for each construct, have identified either or both updating and inhibition as predictors of math
achievement. Updating has been found to be the only unique predictor of standardized math achievement (Monette et al, 2011) and the most important predictor of early numerical magnitude skills (Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013). In contrast, Espy, McDiarmid, Cwik, Stalets, Hamby, and Senn (2004) found that both updating and inhibition predicted performance on emerging math skills when age and other covariates were included, but that only inhibition was a significant predictor when other EF’s were included as additional covariates. Lan, Legare, Ponitz, Li, and Morrison (2011) found that the contribution of EF depended upon the specific skill being measured; updating was the only unique predictor of calculation accuracy, whilst inhibition and updating both predicted counting skills. None of these studies reported a significant role of shifting in math achievement.

Monette et al (2011) argued that the link between math ability and inhibition found in previous studies almost invariably occurs because those studies did not assess updating, making it impossible to compare the effects of updating and inhibition. Indeed, two studies that have not included updating measures found that only inhibition uniquely predicted math ability (Blair & Razza, 2007) or only inhibition remained significant with other covariates in the regression model (Clark, Pritchard, & Woodward, 2010). Bull, Espy, and Wiebe (2008) measured all three aspects of EF, and found inhibition was a significant predictor of rate of growth in math achievement, but because of missing data, excluded the updating measure. Correlational analysis at each time point indicated that, after controlling for reading ability, tasks measuring all aspects of EF correlated significantly with math ability, although this analysis did not consider the independence of these EF contributions from each other.

Recent CFA studies of children in early primary school (Lee, Ng, Pe, Ang, Hassim, & Bull, 2012; Van der Ven et al., 2012) identified two latent EF factors, an updating and a
combined inhibition/switching factor, and found only updating significantly predicted math achievement, both concurrently and longitudinally. Other studies that have found or assumed a separation of updating, inhibition and shifting also find that only updating predicts performance (multiplicative reasoning; Agostino, Johnson, & Pascual-Leone, 2010) or that the unique variance of shifting and inhibition to predicting mathematics is accounted for once differences in reading and IQ have been controlled for (Bull & Scerif, 2001).

Most studies with older children (> 9 years) have included measures from each of the three EF domains (Jenks, van Lieshout, & de Moor, 2012; Lee et al., 2009; Rose, Feldman, & Jankowski, 2011; St Clair-Thompson & Gathercole, 2006; van der Sluis, de Jong, & van der Leij, 2007). Two studies assumed a single factor EF latent (Roebers et al, 2012; Neuenschwander et al, 2012) and found that EF had a direct effect on mathematics achievement as assessed by curriculum based tests, teacher grades, and classroom learning related behaviors. Of the remaining studies, despite differences in the measures used for both EF and mathematics, all found updating explained significant variance in mathematical achievement. Only two studies employed CFA to explore the structure of EF; Van der Sluis et al. (2007) found support for an updating and a shifting latent variable, but measures of inhibition failed to load on a common factor. In contrast, Rose et al. (2011) found a four factor model best explained their data, which contained measures of processing speed in addition to those of EF.

Interestingly, a number of these studies found processing speed to be a more important explanatory construct than EF, with speed fully accounting for poorer EF performance (Van der Sluis, de Jong, & van der Leij, 2004; Rose et al., 2011). Van der Sluis et al. (2007) explained their findings in terms of the simplicity of their arithmetic tasks, arguing that the task placed limited demand on updating abilities, and that individual differences could be accounted for by
basic processing efficiency. This suggests that mathematics will only be demanding of EF if the task is experienced as complex or difficult by the problem solver. One caveat to this interpretation is that these studies used time-sensitive measures of arithmetic efficiency. However, associations to processing speed have been found in studies that did not include time-sensitive measures. Andersson (2010) found speed associated with different aspects of arithmetic (fact retrieval, word problems, place values, and calculation principles) and Clark et al. (2013) found processing speed to be indirectly associated with mathematics performance, mediated by EF. These findings are consistent with processing speed being an upstream variable that is directly, or indirectly, associated with performance. An alternative explanation is that processing speed and EF are highly correlated but functionally distinct factors; Lee et al (2013) found much of the variance in individual EF tasks to be attributable to speed, but that at the latent level, EF factors were still highly correlated with processing speed. Therefore, what is being captured in these studies is perhaps not the primacy of processing speed over EF, but a period during development when individual differences in speed and EF are highly correlated.

There is evidence of a relation between shifting and mathematical performance, but the findings are not particularly robust. For example, relationships are rendered non-significant once sample characteristics are accounted for (Jenks et al., 2012), or are only found with specific skills (arithmetic calculation) but not a multitude of other measures (Andersson, 2010). Perhaps the strongest evidence comes from two recent meta-analyses (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit., 2013; Yeniad, Malda, Mesman, van Ijzendoorn, & Pieper, 2013), both of which show a significant correlation between mathematics achievement and shifting. Effect sizes were higher for younger children and those with mathematical difficulties (Friso-van den Bos et al., 2013). Although Yeniad et al controlled for the influence of intelligence, the
influence of other EFs or processing speed on this relation between shifting and mathematics was not controlled, and the findings should be interpreted with caution. Furthermore, Friso-van den Bos et al found that the strongest correlation to mathematical performance came from verbal updating, whilst the strength of the correlations between inhibition and shifting to mathematical performance did not differ, with both showing lower correlations with mathematics compared to verbal and visual-spatial updating.

Summary, Issues, and Implications.

Perhaps the clearest outcome from this brief review is that studies involving children from early childhood to the mid-adolescence years consistently find a strong, often unique, relationship between updating and math achievement, but results regarding shifting and inhibition are less clear (see van der Ven et al, 2012 for similar conclusions). These findings hold across a range of mathematical tasks, including early developing numerical skills, arithmetic, word problems, and curriculum measures.

Most studies that found a significant role for inhibition or shifting had assumed rather than tested for separation between the three aspects of EF and/or had not controlled for other EF skills. The assumption of EF differentiation is unsafe for several reasons. First, though some EF tasks may rely on one aspect of EF more than another, they all draw on multiple underlying EF processes. Lan et al (2011) and Van der Ven et al (2012) argue that all EF tasks require updating skills; updating is necessary to maintain representations of the inhibition and shifting task requirements and of the sets between which shifting is necessary. Take, e.g., tasks used to measure inhibition such as Head-Toes-Knees-Shoulders (HTKS) and Stroop-like tasks. HTKS requires the child to touch their toes if the experimenter touches her head, and to touch their shoulders if the experimenter touches her knees. In the day-night Stroop, the child is told to
respond “night” to a picture of the sun, and “day” to a picture of the moon. Whilst these tasks clearly require inhibition of an automatic response to copy behaviors or say words matching the pictures, they also require maintenance of rules in memory, and the ability to switch between those rules; a bivariate correlation between such tasks and mathematics achievement could indicate a role for inhibitory, switch, or updating processes. Monette et al (2011) found that dependent measures from inhibitory tasks (errors on the day-night Stroop task) loaded together with an updating factor, implying this might reflect loss of task instruction from memory. Hence, it is possible that the relationship between inhibition and mathematics will only be evidenced when the EF inhibitory task has a minimal memory load and no requirement for rule switching (e.g., self-restraint tasks like NEPSY statue and gift delay, as used by Espy et al., 2004).

This leads to a second issue of task impurity highlighted in previous studies. The amount of observed variance that is attributable to EF in any one task tends to be small (Lee et al., 2013; Willoughby et al., 2012). The risk of relying on a single measure to estimate the relation between EF and math performance is that any resulting relation may be spurious and caused by incidental or task specific similarities across tasks. This emphasizes the importance of using multiple measures of each EF skill, combined with confirmatory analytical techniques to verify that each task is measuring the EF skill it is supposed to measure, and to statistically aggregate performance across a variety of tasks to obtain a measure of the true latent ability level. Finally, in studies conducted with younger children, CFA suggests that EF is undifferentiated, and that the rate of differentiation is slow; updating emerges as a separate factor by the time children enter into formal schooling, but inhibition and switch remain undifferentiated until the mid-adolescent years (Lee et al., 2013). Therefore, we may only observe independent contributions
from updating, inhibition, and shifting once differentiation is more complete, i.e., late adolescence.

One additional measurement concern is whether inhibition and shifting tasks provide adequate measures of individual differences in EF. WM span or updating tasks gradually increase in difficulty, take participants to the edge of their abilities, and terminate only after a pre-determined number of failures. As a result, they provide a measure of maximum sustained capacity. In contrast, difficulty levels of inhibition and shift tasks are typically set much easier, the rationale being that most tasks focus on reaction time differences, harvested from trials that have been answered accurately (see Friso-van den Bos et al., 2013 for a similar conclusion). Making tasks easier ensures that there are sufficient accurate trials, but may also render the tasks insufficiently demanding of EF. A challenge for the field is to develop inhibitory and switching tasks that are more demanding of EF.

Given the consistent findings regarding associations with updating, one possible explanation is that updating capacity is a limiting factor on the operations of inhibition and shifting, or a common resource that contributes to or underpins both inhibition and shifting. For this reason, we only observe associations between inhibition, shifting, and mathematics performance when the two EFs are assessed independently of updating. There are a couple of ways in which updating can operate as a limiting factor. Miyake and Friedman (2012) proposed that performances on EF tasks are influenced by two types of EF abilities: (a) a common EF ability that is synonymous with inhibitory ability, and (b) abilities that are specific to updating or switching. These EF abilities are deemed statistically distinguishable and mutually exclusive. This differs from their earlier framework (Miyake et al., 2000) which specifies joint influences from different EF abilities that are conceptually and statistically related. Our present review
suggests that in relation to mathematics, updating plays a dominant role and subsumes the
influences of inhibition and switching abilities, and that the speed or accuracy of completing
mathematical tasks is constrained by an individual’s updating capacity. However, the findings do
not speak directly to the structural relations between the various EFs. The observed relations
between EFs and mathematics performance can be accommodated within either framework.
Under Miyake and Friedman’s (2012) framework, our present finding amounts to strong
relations between the updating-specific ability and mathematics performance, but null relations
between the switching-specific and the common EF factors. Under the older framework, the
inter-relations and the specification of the various EFs are different, but it is still only updating
that contributes directly to mathematics performance.

EF is clearly important for the development of math and other academic skills even prior
to formal schooling, and poor EF may have a cumulative and delayed effect. Children with
poorer EF may have difficulties acquiring skills both in the course of typical, naturally occurring
development, and through instruction in the classroom, making errors in many learning activities
due to difficulties remembering and carrying out instructions, inhibiting irrelevant information
and staying focused on task, and monitoring progress and switching to more appropriate task
strategies. Training of EF skills has been the target of recent intervention (e.g., inhibition, see
Thorell, Lindqvist, Bergman, Bohlin, & Klingberg, 2009; switching, see Karbach & Kray, 2009;
see Kray & Ferdinand, 2013 for a brief overview). Studies have examined the benefits of
adaptive WM/updating training in terms of transfer to non-trained WM tasks and skills such as
math and reading. Following WM training, sustained improvements (up to 12 months) are
consistently reported in tasks that closely resemble the trained activities (e.g., Dunning, Holmes,
& Gathercole, 2013; Holmes, Gathercole, & Dunning, 2009). The evidence of far transfer to
cognitively similar but structurally different non-trained task is mixed (e.g., Jaeggi, Buschkuehl, Jonides, & Shah, 2011), and the majority of studies report no benefit of WM training to math (or literacy) achievement (see Melby-Lervag & Hulme, 2013 for a meta-analysis). It is possible that outcome measures employed to date lack sufficient sensitivity to detect subtle changes in learning, or that training needs to be targeted at more ecologically valid memory-demanding situations that more closely resemble the requirements of classroom tasks and environments. Further specification of EF, including the qualitative and quantitative changes across development, coupled with continued development of valid and reliable EF tasks, may benefit attempts to remediate problems that not only limit children’s mathematical development, but also other functional outcomes.


