Title	Examining kindergarten approximation skills as a predictor of children requiring learning support for mathematics
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Examining Kindergarten Approximation Skills as a Predictor of Children Requiring Learning Support for Mathematics

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Bills Payment

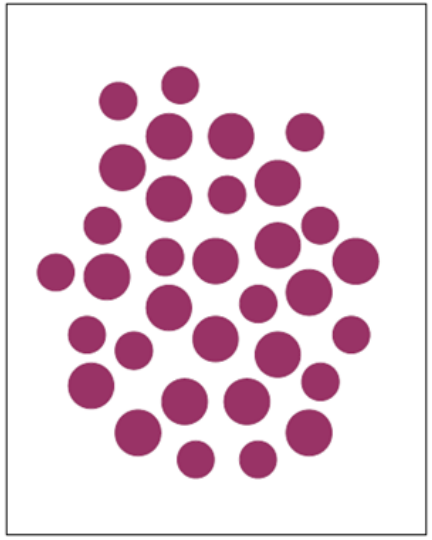
Which has more

Which has more

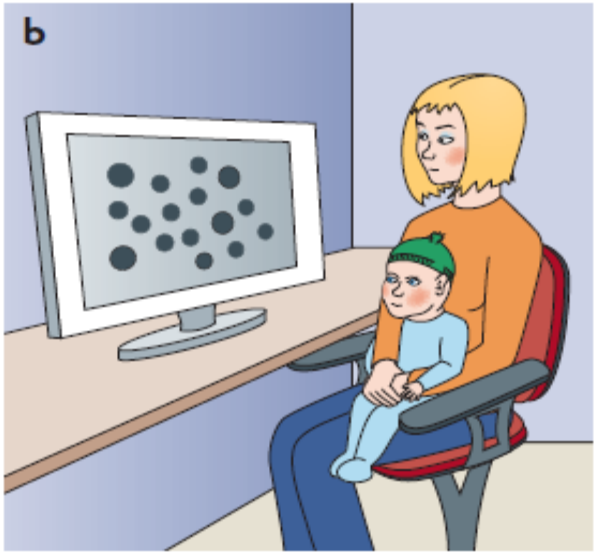
Which has more

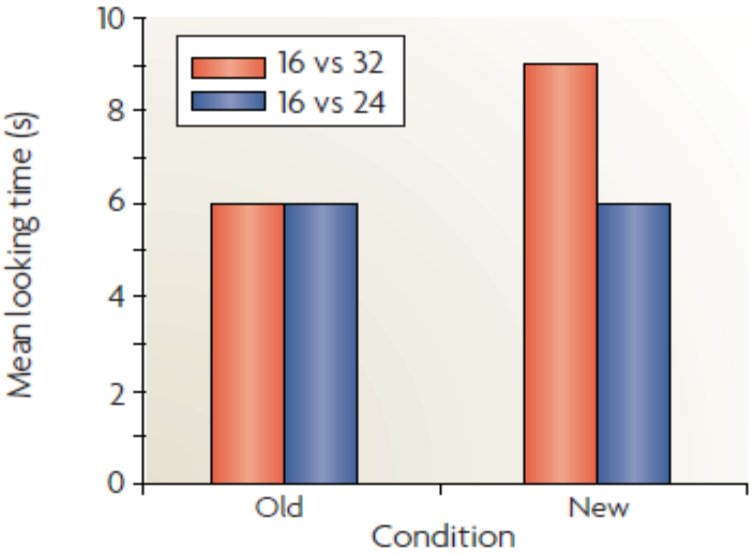
Which has more

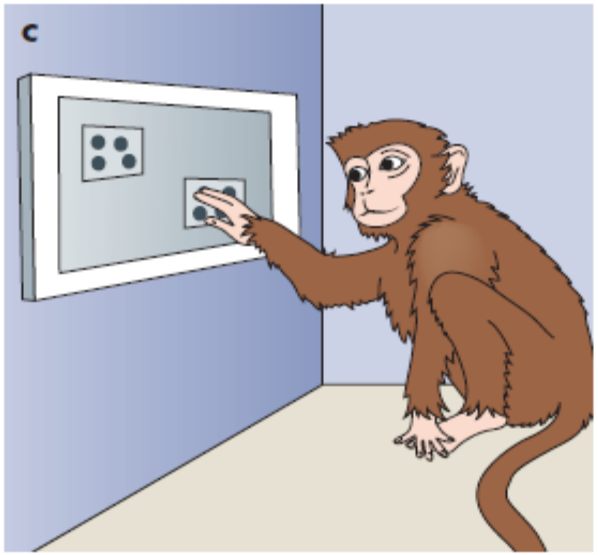
Which has more

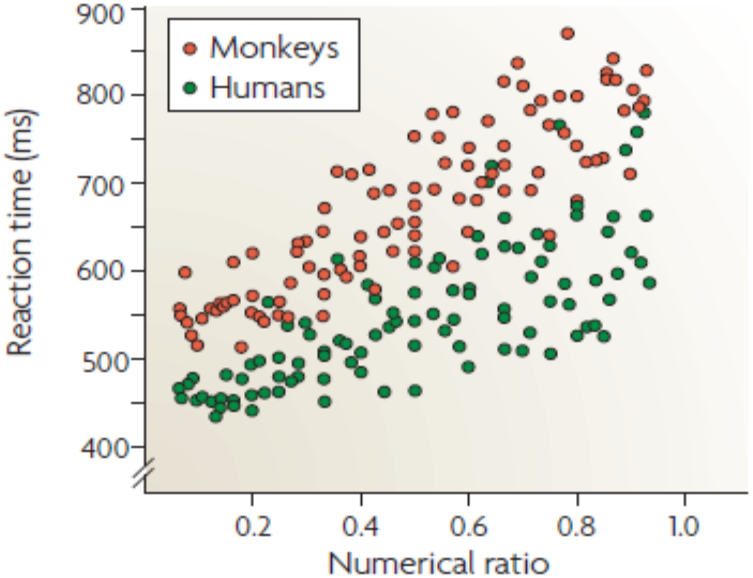


The Approximate Number System (ANS)









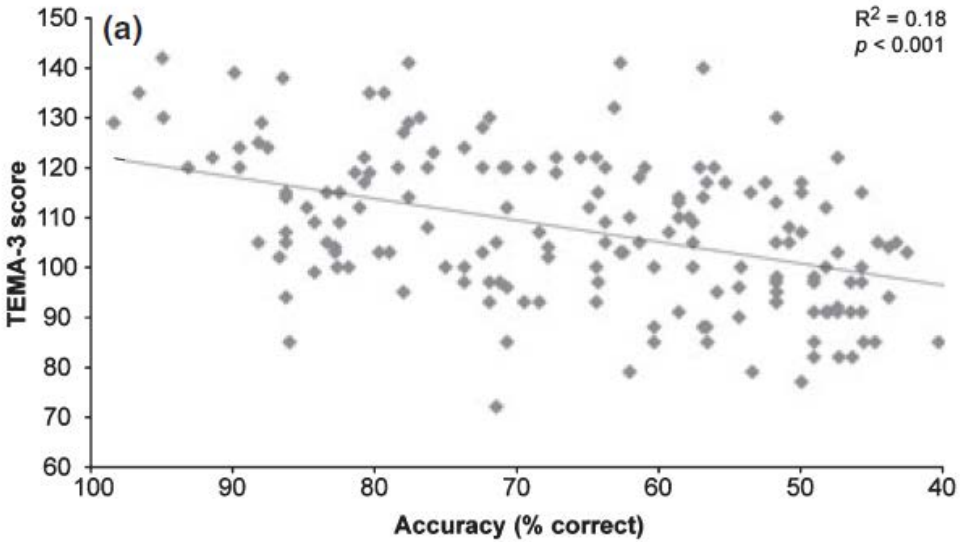
ANS and math achievement

Approximation skill 1: Acuity of non-symbolic representation measured by numerical discrimination task.

- Individual differences in numerical discrimination limits emerge before formal mathematical learning.

- Children's numerical discrimination precision at age 3/4 years predicts their standardized mathematics scores at age 5 or 6 years.

- Individual differences in 4-year-old children's numerical discrimination precision predict mathematical ability 6 months later, even when controlling for individual differences in mathematical ability at the time of initial testing.

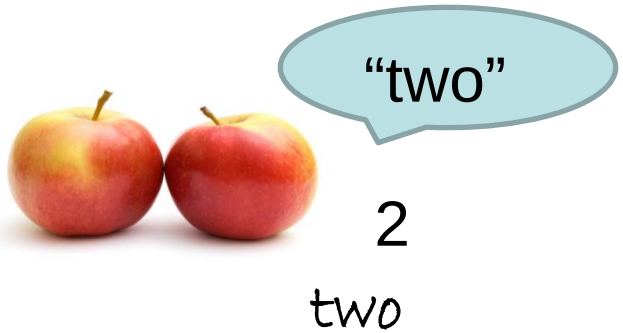


Libertus et al., (2011)

Mapping between symbolic and non-symbolic representations

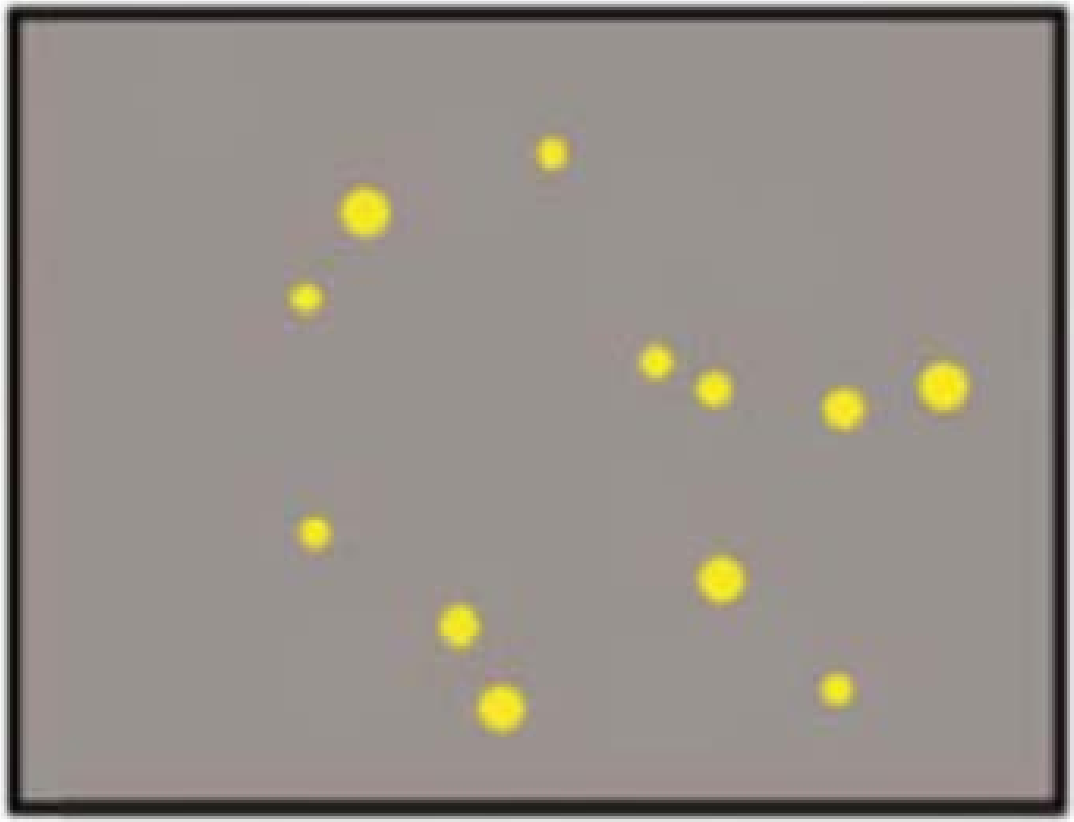
- Humans in many societies supplement their non-symbolic representations with a formal system of mathematics, which includes symbolic representations (e.g., number words and Arabic digits)

- By seeing many examples of a number, e.g., two eyes, two socks, two apples, labelled with the same number word and same symbol, child constructs precise concepts of small numerosities.



- Being able to transform numbers into the quantity representation is especially important, because we usually see or hear numbers as digits or words, but it is the quantity representation that makes us understand the "meaning" of a number and have a sense of how large it is.

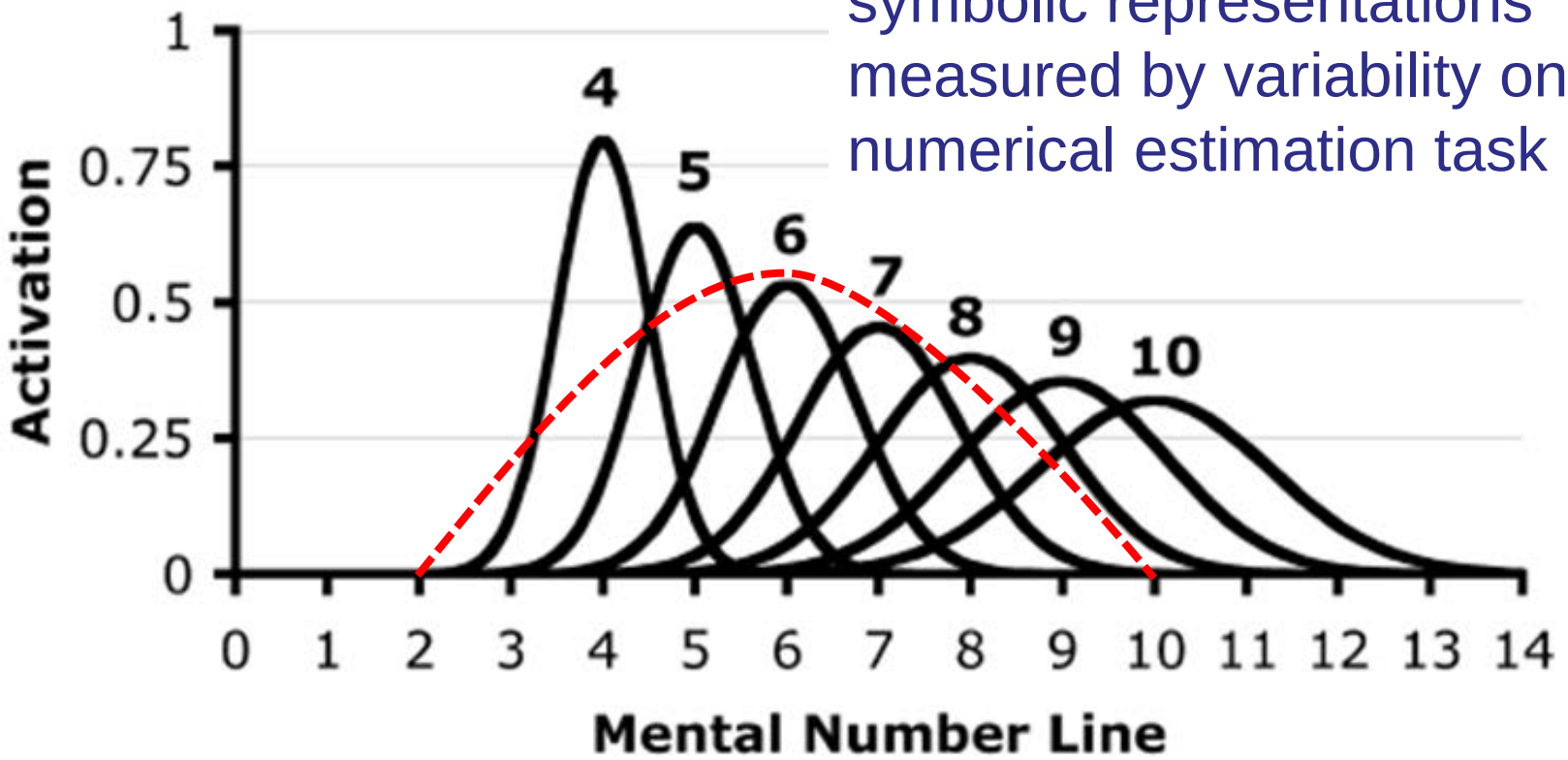
Magnitude estimation



Question: How many yellow dots?

Mapping to symbolic representations

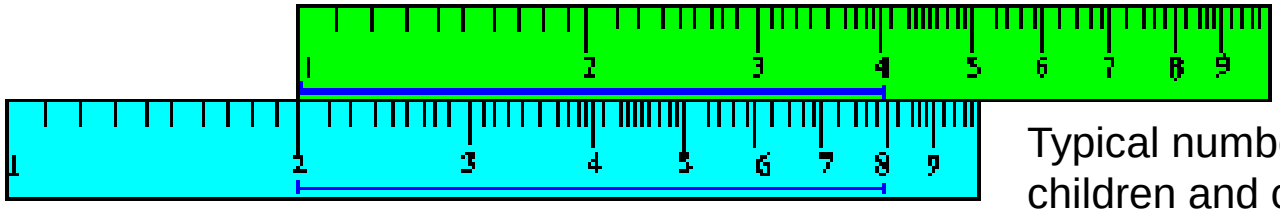
Approximation skill 2: Mapping between non-symbolic and symbolic representations measured by variability on numerical estimation task



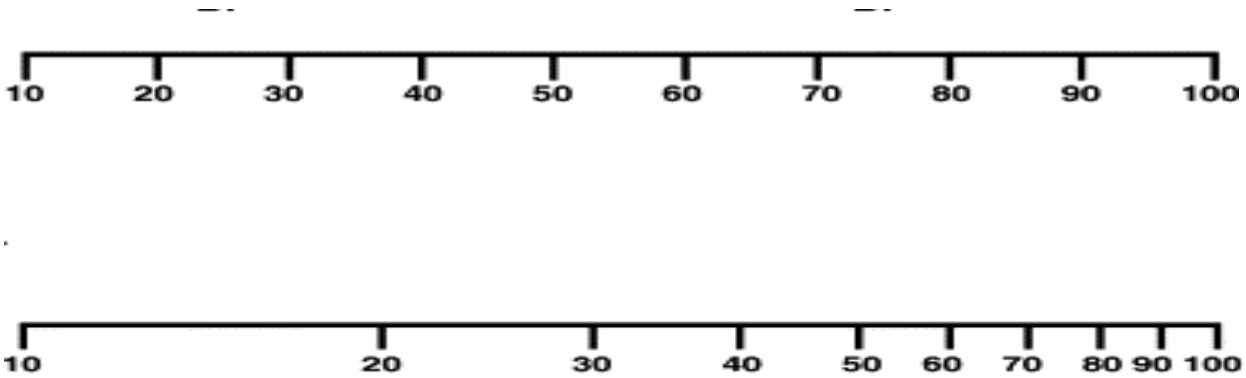
Children with greater variability in mapping show lower math ability *(Mazzocco et al, 2011).*

Thinking about numbers on a mental number-line

- Most individuals show an overlap between how they think about number and how they think about space.
- Helps us understand how numbers relate to one another (ordinality) and understand their relative size.

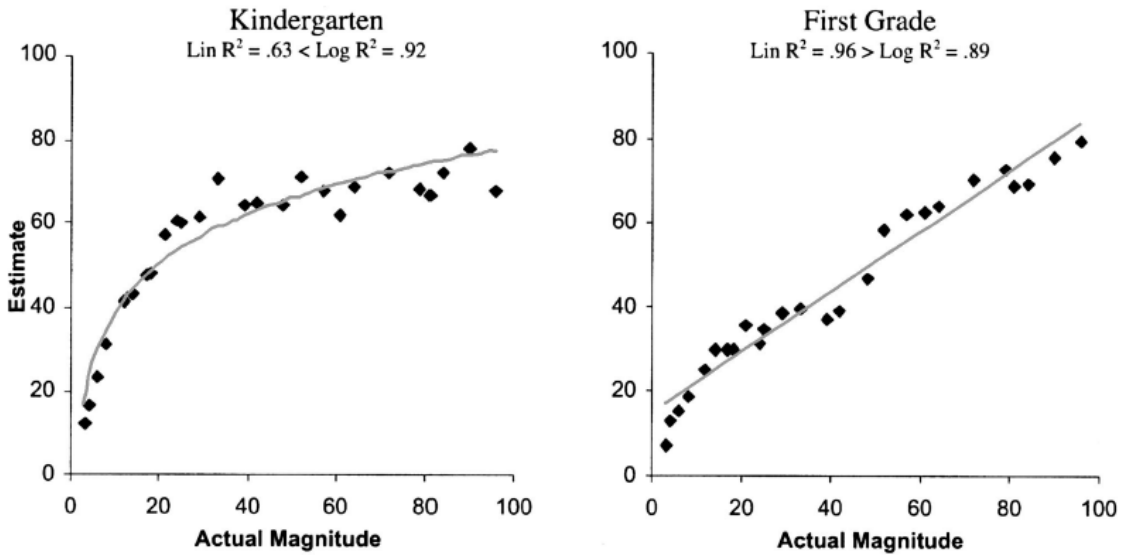


Typical number line estimation of young children and children with mathematical difficulties.

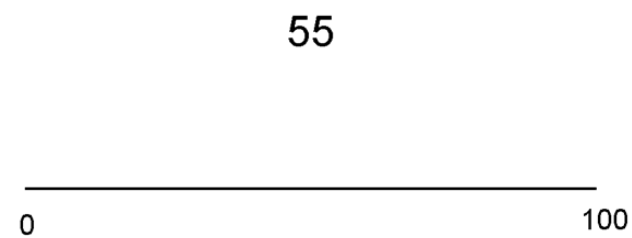


38 + 25 = 83

Number-line estimation



Approximation skill 3: ordinal and linear estimation of number on a mental number line



Booth & Siegler (2008). More accurate number-line approximation correlated with math achievement and estimation accuracy in untrained arithmetic problem (ages 7-8)

Correlations Among Number Line Estimation, Addition, Short-Term Memory, and Math Achievement Test Performance: Pretest Data

Measure	Number line estimation (R^2_{Lin})	Math achievement (WRAT)	Addition (PAE)
Math achievement (WRAT)	.57**		
Addition (PAE)	.41**	.48**	
Short-term memory	.07	.21*	.21*

Note. df = 103. PAE = percent absolute error; WRAT = Wide Range Achievement Test.
*$p < .05$. **$p < .01$.

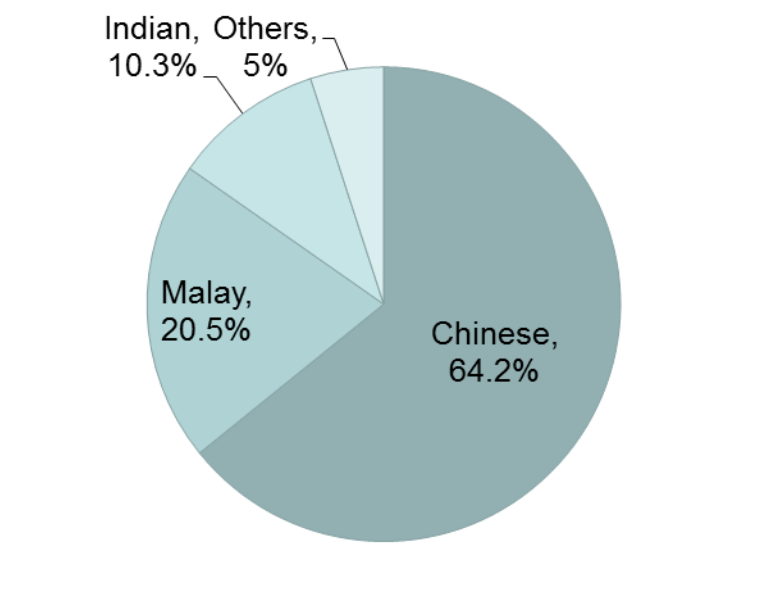
Research Question 1

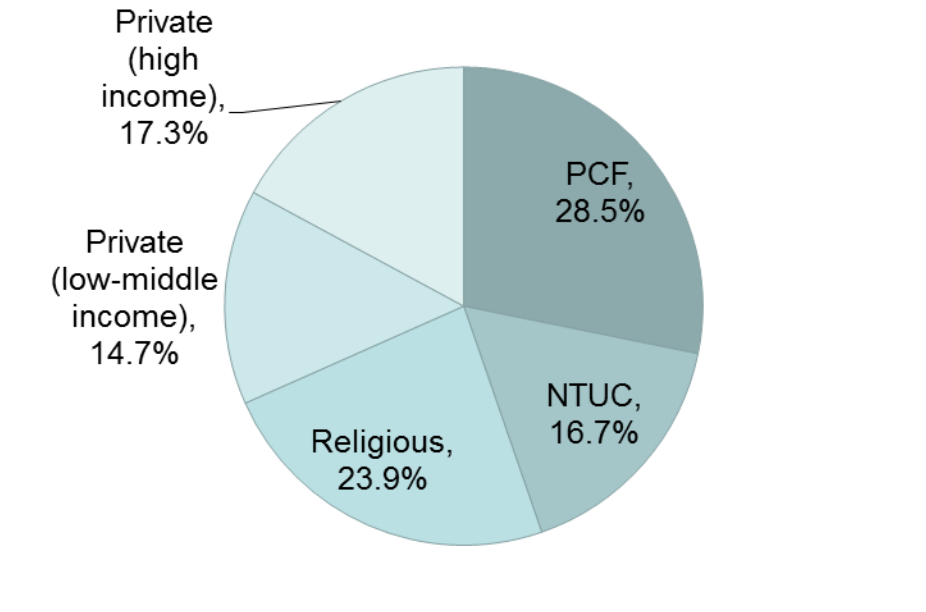
Do approximation skills measured in K2 successfully predict math achievement?

- Concurrent math achievement at K2?
- Longitudinal math achievement at P1?
- Sensitive predictor of need for LSM?

Participants

- K2:
  - 347 (169 girls) children (5:5 – 6:10, mean age = 6 years, 2 months) from 21 preschools





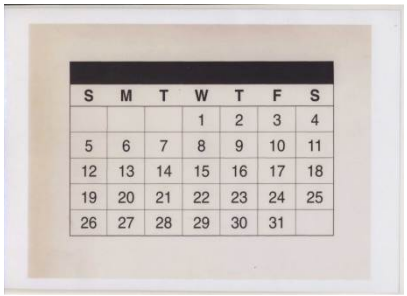
- P1:
  - Follow up of 230 children (mean age = 6 years, 8 months) dispersed across 104 primary schools.

Methods – standardised achievement tests

- Mathematical ability
  - WIAT Numerical Operations (K2 and P1) – number recognition and writing, counting, number order, single and multi-digit arithmetic.
  - WIAT Mathematical Reasoning (K2 and P1)

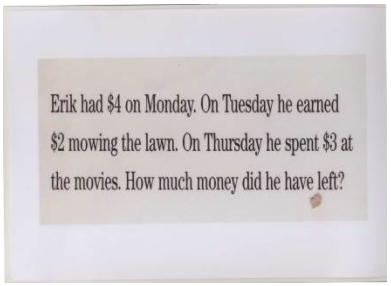
Which fishbowl has more fish?

If 2 ducks flew away, how many would be left?



S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

On what day of the week is the 14th?

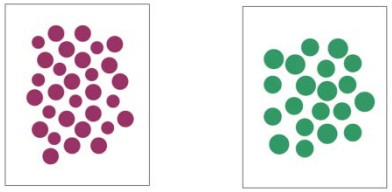


Erik had $4 on Monday. On Tuesday he earned $2 mowing the lawn. On Thursday he spent $3 at the movies. How much money did he have left?

  - WIAT addition and subtraction fluency (P1 only)
- Nonverbal Reasoning
  - Raven's Progressive Matrices
- Vocabulary
  - Peabody Picture Vocabulary Test

Approximation Skills

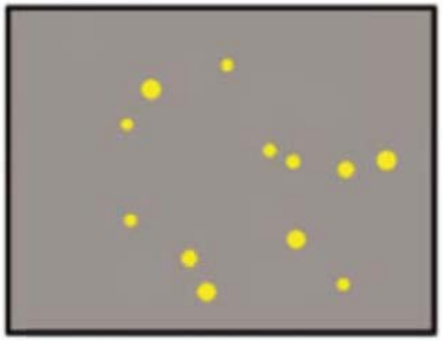
Numerical discrimination



Question: Which has more

Acuity of the non-symbolic representation

Numerical estimation

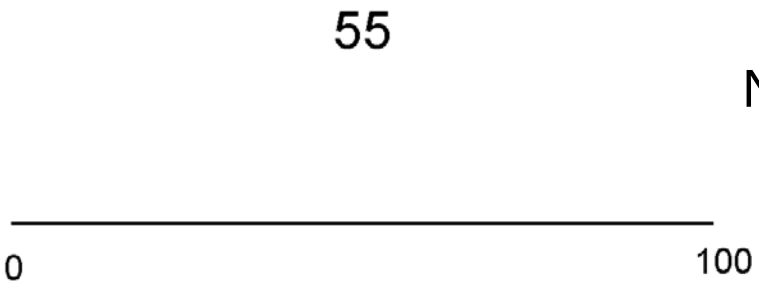


Question: How many yellow dots?

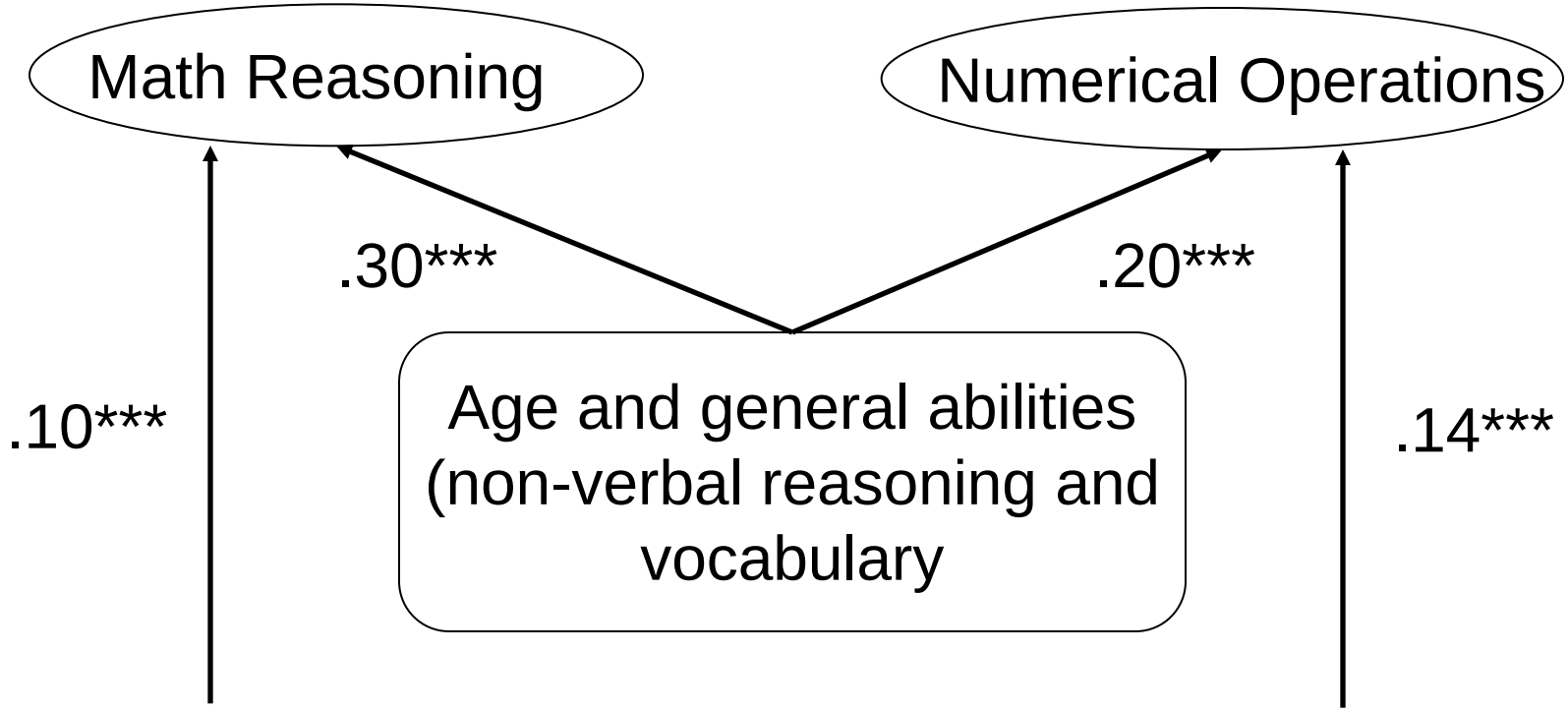
Accuracy of mapping between the non-symbolic and verbal number systems

Accuracy of ordinal and linear understanding of number

Number-line estimation



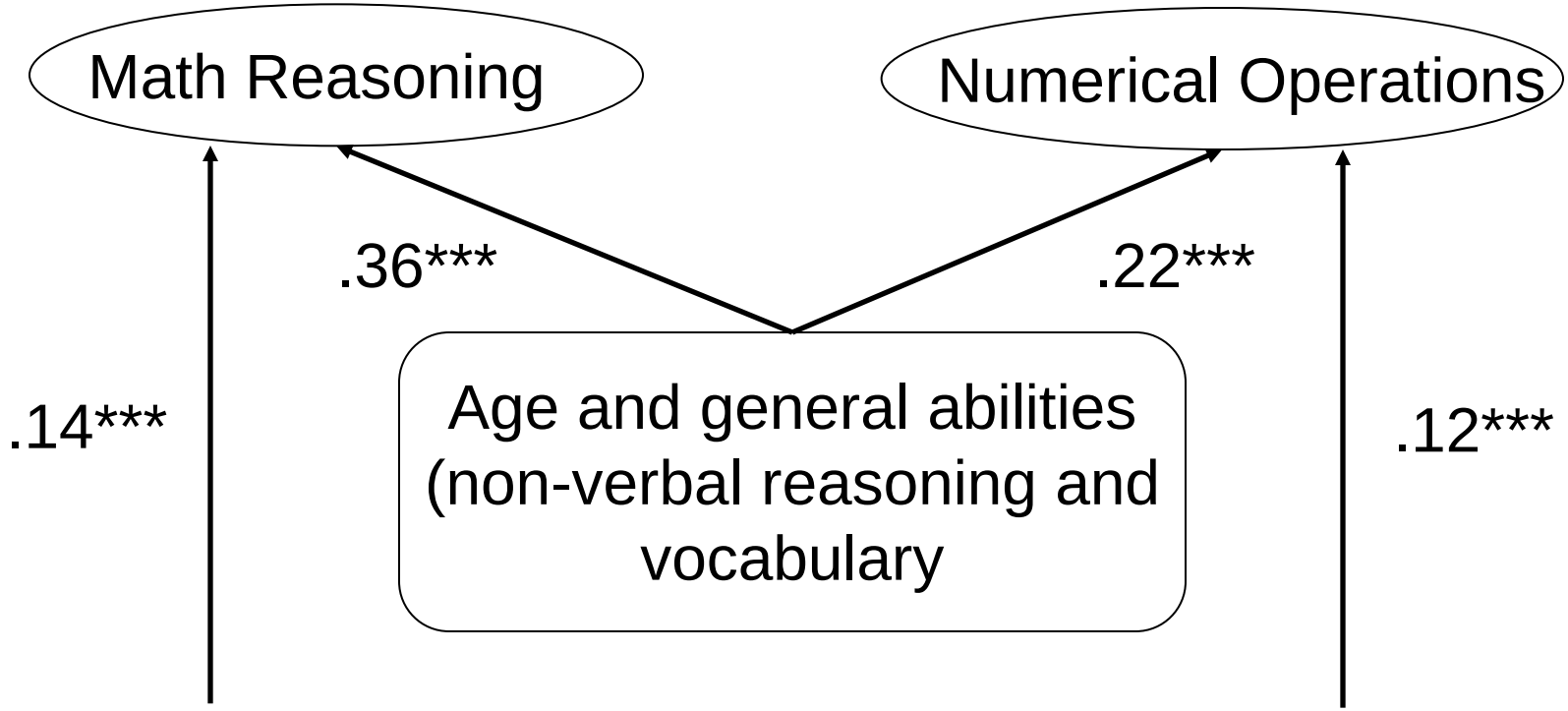
Predicting math achievement in K2



Approximation skills:
Number-line estimation (-.29***)
Numerical discrimination (.10*)

Approximation skills:
Number-line estimation (-.34***)
Numerical discrimination (.13*)

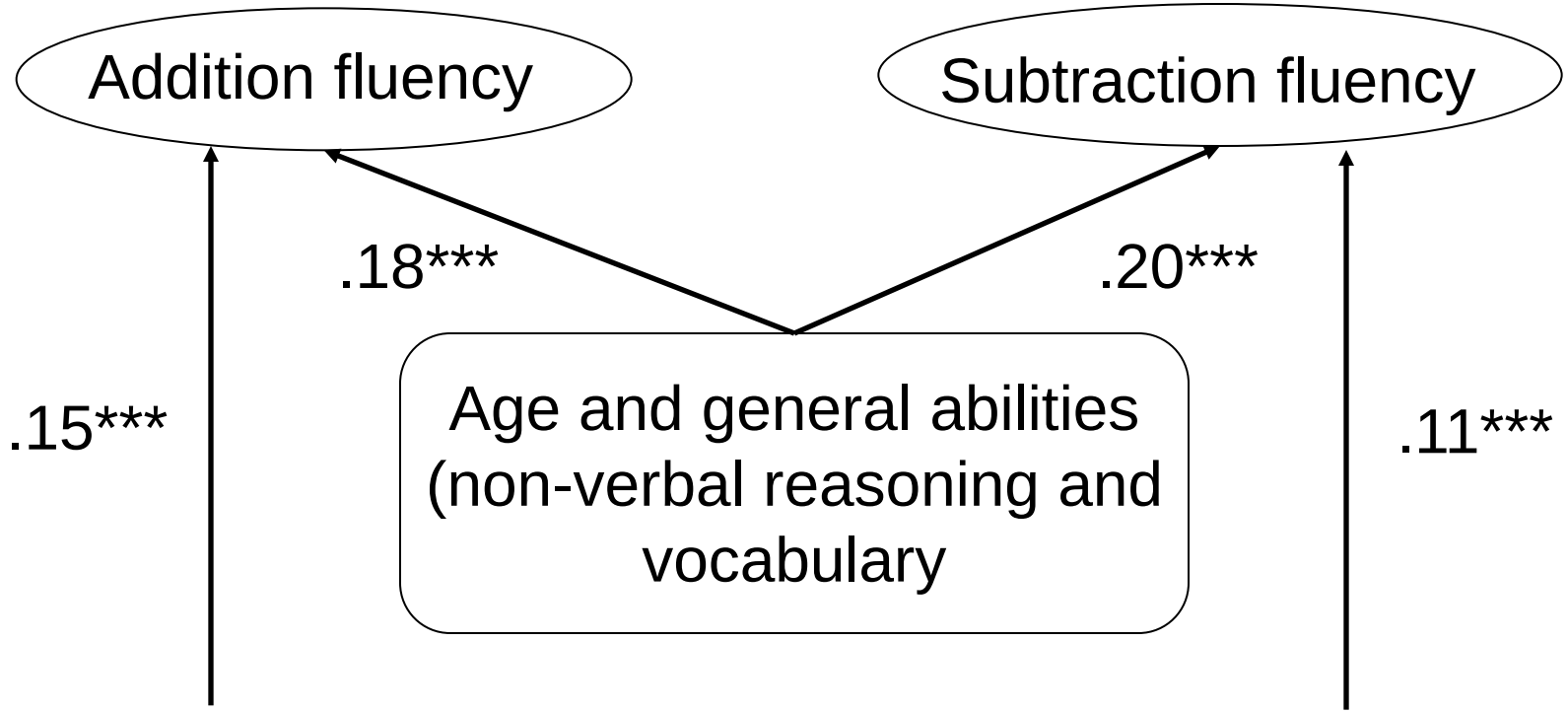
Predicting math achievement in P1



Approximation skills:
Number-line estimation (-.33***)
Numerical discrimination (.16**)

Approximation skills:
Number-line estimation (-.26***)
Numerical discrimination (.18**)

Predicting math achievement in P1



Approximation skills:
Number-line estimation (-.29***)
Numerical discrimination (.21**)

Approximation skills:
Number-line estimation (-.20**)
Numerical discrimination (.22**)

Do approximation skills predict which children end up in LSM?

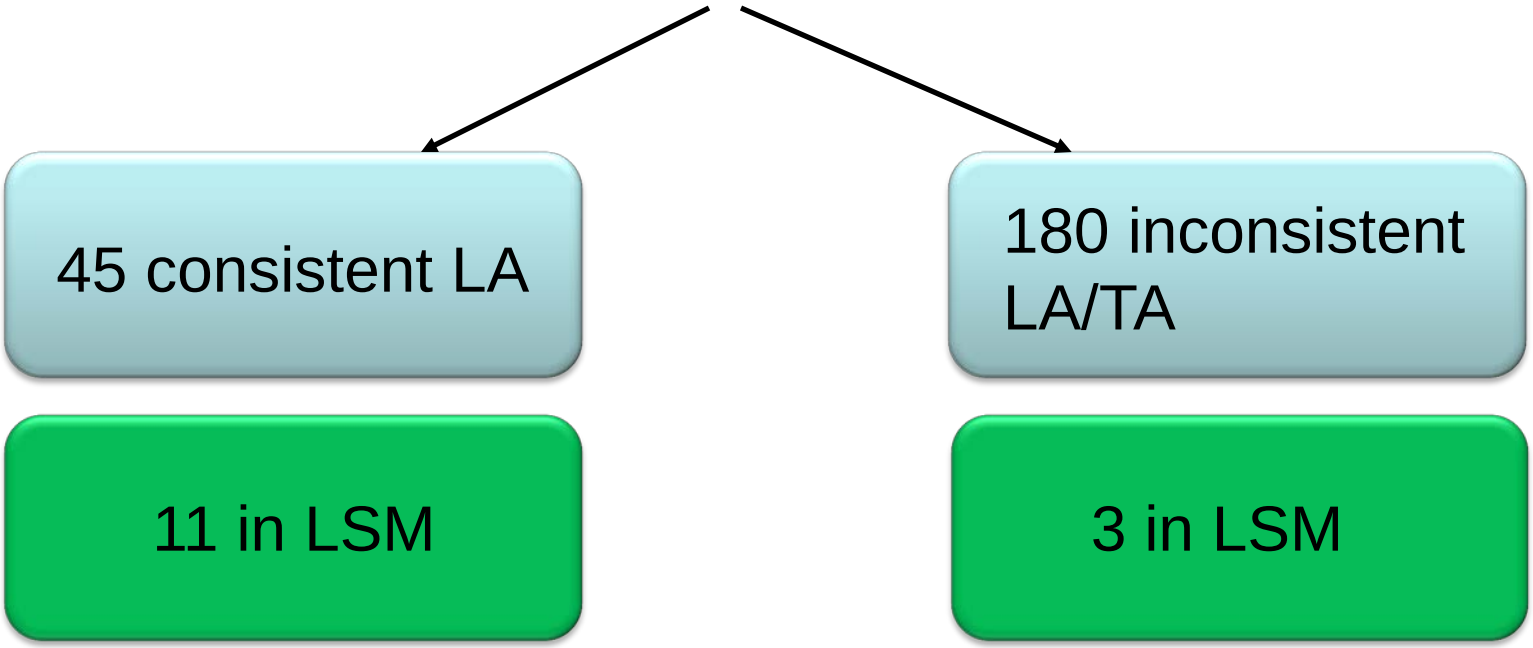
- 14 children (of 228 re-tested at P1) enrolled in LSM (6.1%)
  - Small number makes it difficult to do any robust analyses
  - is the school entry test for LSM catching all children with math difficulties?

Raw scores	No LSM (mean)	No LSM (range)	LSM (mean)	LSM (range)
Math Reasoning K2	18.44	6-26	13.61	8-21
Numerical Ops K2	9.81	4-19.4*	6.50	4-11
Math Reasoning P1	23.56	8-42	15.64	8-25
Numerical Ops P1	11.82	6-21.73*	8.78	6-11
Addition fluency P1	15.15	0-36	7.07	1-13
Subtraction fluency P1	10.18	0-28	3.14	0-9

* Extreme values > 3SD from mean replaced with value at 3SD.

Predicting consistent low achievement at P1

- For each math achievement measure at P1, children scoring in the bottom 25th %ile were classified as low achieving (LA).
- Children who showed consistent LA (at least 3 of the 4 tests) were classified as consistent LA.
- Children who showed inconsistent LA (only on 1 or 2 tests) or TA were classified as inconsistent LA/TA



Predicting consistent low achievement at P1

Numerical discrimination and number-line estimation

	Predicted: consistent LA	Predicted: inconsistent LA/TA
Actual: Consistent LA	29	9
Actual: Inconsistent LA / TA	41	134

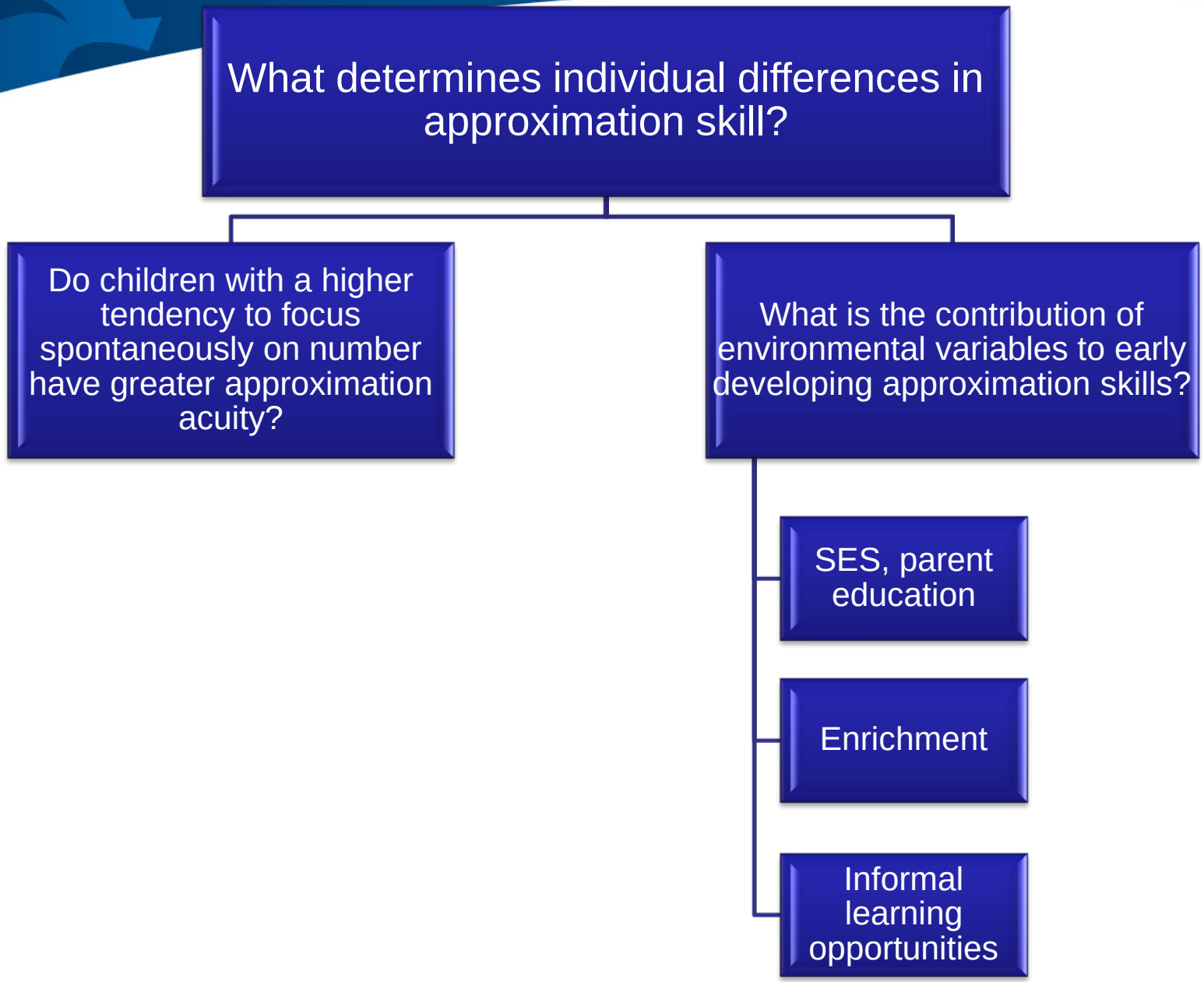
Sensitivity = .76
Specificity = .77
False Positive = .58
False Negative = .06

Number-line estimation only

	Predicted: consistent LA	Predicted: inconsistent LA/TA
Actual: Consistent LA	35	9
Actual: Inconsistent LA / TA	40	143

Sensitivity = .79
Specificity = .78
False Positive = .53
False Negative = .06

Research Questions 2 and 3

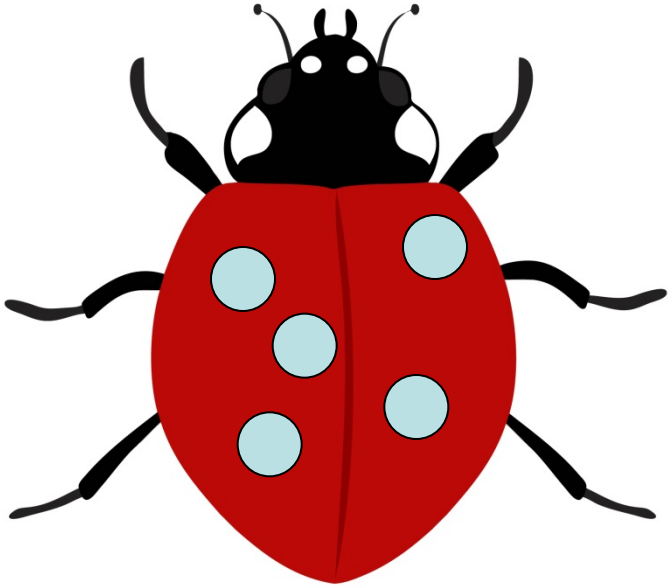


Spontaneous Focus on Number (SFON)

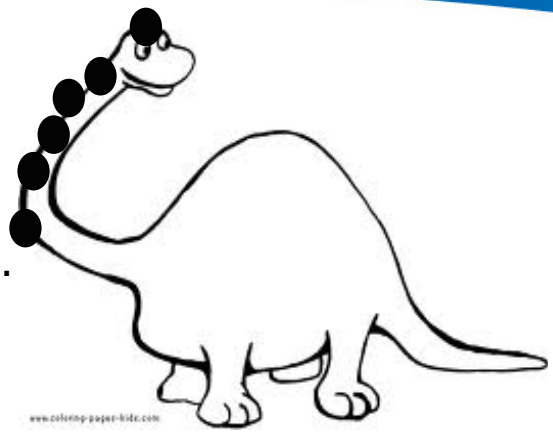
- A process of spontaneously (i.e., in a self-initiated way not prompted by others) focusing attention on the aspect of the exact number of a set of items or incidents.

- Indicates the amount of a child's spontaneous practice in using exact enumeration in her or his natural surroundings.

- SFON is positively related to the development of enumeration, number sequence skills and later arithmetic (*Hannula & Lehtinen, 2005; Hannula et al., 2007; Hannula et al., 2010).*



SFON Tasks – model task and post box task



E.g., Model Task

- Picture of dinosaur in front of E and one in front of the child.
- The aim is for the child to copy the model and make it look exactly like that of E.
- E stamps (e.g.) 6 nodes, then gives the child the stamp to make their picture look the same.
- Procedure repeated 2 more times with different number of different shaped nodes (3 trials x 2 tasks).

- E must not refer to numerosity in any way.
- SFON behaviours - Utterance including number words (“I’ll do 6 stamps”), use of fingers to express numbers, counting acts, other comments referring to quantities (“Oh I miscounted them”), interpretation of the goal of the task as quantitative (“I gave exact accurate number of them”)

- Children coded as “no SFON behaviors”, “inconsistent” and “consistent”

Spontaneous focus on number as an indicator of individual differences

	None (K2 N = 118) (P1 N = 80)	Inconsistent (K2 N = 125) (P1 N = 86)	Consistent (K2 N = 104) (P1 N = 64)	p
Mathematical Reasoning K2	16.91	18.38	18.04	= .03 N<I
Numerical Operations K2	9.07	10.14	9.92	= .02 N<CI
Mathematical Reasoning P1	21.83	23.44	23.91	= .06
Numerical Operations P1	11.32	11.94	11.52	= .42
Addition fluency P1	14.54	15.21	13.75	= .49
Subtraction Fluency P1	9.20	10.33	9.44	= .47
Numerical discrimination	76.02	77.39	78.54	= .38
Numerical estimation	.34	.35	.35	= .96
Number-line estimation (error)	20.07	18.10	16.87	= .01 N>C

Mediation analysis suggests effect of SFON on math are indirect via number-line estimation

Note: Groups do not differ in age, nonverbal reasoning, vocabulary, or SES (household income)

SES and math

- Many children from low-income backgrounds enter school with less knowledge than peers from middle-income backgrounds, and the achievement gap in mathematical knowledge progressively widens throughout their school career (National Mathematics Advisory Panel, 2008).

- Kindergarten children from middle-income homes are reported to engage in more math-related activities than their low income peers *(Jordan, Kaplan, Olah & Locuniak, 2006),* and the amount of informal instruction children receive at home on number concepts is a good predictor of the number skills shown by children as they enter school.

Low-income children are four times more likely than their middle-income counterparts to start school at a low level and to show flat growth between kindergarten and first grade in key areas of number competence.

SES relation to math outcomes and approximation skills

Higher parental income positively related to:

- Higher general abilities (vocabulary, non-verbal reasoning)
- Better math outcomes in all measures at K2 and P1
- More accurate numerical discrimination
- More accurate number-line estimation

- Just knowing that SES relates to outcomes tells us nothing about the underlying processes of that relationship
- Understanding the routes through which higher SES promotes better math outcomes may provide a focus for intervention for low SES children

Enrichment

Home numeracy environment

Enrichment

Parents who enrolled their children in math enrichment had a significantly higher income (on average approx. $350 more per household member per month)

	No math enrichment N = 173	Math enrichment N = 50
Math Reasoning K2	17.65	18.63
Numerical Operations K2	9.49	10.91
Math Reasoning P1	23.18	23.83
Numerical Operations P1	11.49	12.35
Addition Fluency P1	14.27	17.84
Subtraction Fluency P1	8.70	13.50
Numerical discrimination	76.63	81.87
Numerical estimation	.332	.335
Number-line estimation	17.50	18.32

Home Numeracy Environment

	Mean (SD)	Factor 1: Attitudes	Factor 2: Activities
I am confident about helping my child with math	4.13 (0.79)	.78	
I enjoyed math when I was at school	3.83 (1.00)	.74	
My child enjoys math at school	3.91 (0.71)	.74	
I am interested in finding out what math topics my child is learning	4.28 (0.65)	.69	
I am happy with my child's progress in math over the last year	3.79 (0.87)	.59	
I spend time teaching / helping my child with math	3.20 (0.85)		.52
I refer to number in everyday tasks	3.56 (1.73)		.59
My child plays games that involve counting or throwing a dice	2.63 (1.81)		.90
My child plays games that involve comparing quantities.	2.51 (1.72)		.91
Factors account for 57.45% variance overall		*α = .77*	*α = .76*

Home numeracy environment

Partial correlation of home numeracy environment (attitudes and activities) to math and approximation outcomes (controlling for income)

	Attitudes	Activities
Math Reasoning K2	.22**	.17*
Numerical Operations K2	.20**	ns
Math Reasoning P1	.18*	ns
Numerical Operations P1	.19*	ns
Addition Fluency P1	.16	ns
Subtraction Fluency P1	.17*	ns
Numerical discrimination	ns	ns
Numerical estimation	ns	ns
Number-line estimation	ns	-.17*

Correlation between factors = .03

Predicting consistent low achievement at P1 from K2 number-line estimation

	Predicted: consistent LA	Predicted: inconsistent LA/TA
Actual: Consistent LA	35 (NL error = 27.93)	9 false negatives (NL error = 15.98). LA in math despite good NL estimation. Compared to children in predicted consistent LA group, these children came from higher income homes but scored lower in terms of home numeracy environment (fewer activities and less positive parent attitudes)
Actual: Inconsistent LA / TA	40 false positives (NL error = 27.13). TA in math despite poor NL estimation. More likely to be female than males. Fluency skills as good as other TA children, but performing more poorly on other math outcomes	143 (NL error = 13.19)

Conclusions

- Children with poorer math ability showed less acuity in their non-symbolic representation of number and were less accurate in number-line estimation.

- When we think about using these tasks to predict later need for learning support for math, we need to think about how we define and measure low achievement:
  - Practical implications for screening at start of P1

- If need for LSM is defined as consistent low achievement (bottom 25%ile of the cohort), number-line estimation may be a quick and easy screening tool:
  - Correctly classified 79.5% of children who showed consistent LA at P1
  - BUT high false positive rate (21.8% of TA children classified as LA)

Conclusions