<table>
<thead>
<tr>
<th>Title</th>
<th>Mathematical problem solving in regular classrooms</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

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Mathematical Problem Solving in Regular Classrooms

YEAP BAN HAR
& BERINDERJEET KAUR

The revised Mathematics Syllabus for Secondary Schools (MOE, 1990) has problem solving as the theme for teaching and learning. This is only rightly so considering the many benefits associated with problem solving.

Firstly, the problem solving approach provides opportunities for the development of higher order skills. Such an approach encourages the development of metacognitive skills. Other skills, which are useful even in the adult life of students, can be incorporated. Mathematics then prepares students not merely for examinations but for life.

Secondly, problems can be put into context. Students are then able to appreciate the real life significance of mathematical concepts. Ability to link abstract, unfamiliar mathematical concepts to concrete, familiar real life situations is important in helping students grasp such concepts. Besides, students who can appreciate real life applications of mathematical concepts are more likely to use these concepts in future.

Thirdly, the use of problems can motivate students in the learning of mathematics. Students enjoy solving problems based on their areas of interest. In contrast, list of questions commonly found in textbooks can be frightening and demoralizing.

Although few teachers would disagree with the benefits of problem solving, many see this approach as time consuming. How then can they squeeze problem solving into their already congested scheme of work? Another concern is whether this approach would run contrary to the drill-and-practice mode that is so essential in the learning of mathematical skills.

In this article we shall attempt to show how teachers can adopt the problem solving approach without having to increase instruction time, at the same time retaining the time-tested drill-and-practice mode.
Textbook questions can be modified to incorporate problem solving skills. Two examples are given to illustrate how this can be done.

**An Example**

A question taken from a local textbook (Lee, 1982; p13) is given below:

There is a **two digit number** such that the sum of its digits is 6 while its product is one third the original number. Find this number. (Hint: Let one of the digits be x).

This question can be modified into a problem. The modified question has been put into a context that involves several students in the class.

**At The Fun Fair**

Secondary 3E3 visited a Fun Fair. Some of the boys and girls played the game "Guess The Clowns' Ages". In this game participants are to guess the ages of two clowns, Spotty and Stripey. A participant pays $1 to obtain a card that contains a clue. If the participant can guess the ages correctly then he or she wins $3. If only one age is guessed correctly then the participant wins $2. Calvin, Sitirafidah, David, Lester and Harpreet played the game.

*Their cards are shown below:*
Your job is to help them guess the ages, hopefully correctly. One group member is to record all the steps the group takes in the process of solving the problem (on a piece of writing paper). You will be asked to present your discussion to the class later. In the event where you have not arrived at the final answer, you present whatever discussion your group has done.

If you have arrived at the answer, respond to these questions:

1. Are the answers reasonable? Why?
2. Do the answers agree with the five cards that your classmates got?

If you have finished, attempt to solve this related problem:

Is it worth your $1 playing this game? Give reasons for your answers. You may want to consider how many clues you need before you can obtain the answers.

There are many advantages of using questions in context. Such questions are more appealing to students. Re-enacting the scenario in class introduce the element of fun. Students enjoy the lesson and are more motivated to solve such questions. At the same time, students are able to see the practical application or real life significance of the concept used to solve that problem. Many students are turned off by the long list of questions that appear in textbooks. The use of a worksheet to present a modified problem is a more attractive alternative. It is also observed that many students who cannot solve the question in its original form can solve the modified question. Finally, contextualizing questions allow teachers to incorporate problem solving skills. Hence teachers do not require additional instruction time to teach problem solving.

Apart from the benefits of solving a meaningful problem described above, students are given the opportunity to work in groups and experience certain problem solving situations.

When students work in small groups they learn to cooperate and to brainstorm. Brainstorming is important in problem solving. Students share ideas and are required to verbalize their thoughts clearly to
convince other group members. This leads to a greater understanding of the problem at hand.

When solving the above problem the better students may use the algebraic method involving quadratic equations. However, the weaker students are more likely to adopt the guess-and-check method. This, used in conjunction with a systematic approach can lead to good answers. Students get to practise the heuristic - guess and check - and be convinced that this is a perfectly correct and acceptable problem solving tool.

As pupils share with the class their solutions they are exposed to the idea of different ways of solving a problem and also at times to the most efficient way.

Another Example

Questions (Lee, 1982; p61-62) such as those shown below provide students with ample practice in the determination of equations of straight lines.

1. Find the equation of the straight line joining each of the following pairs of points.
   (i) (1, 3), (2, 5)   (ii) (2, 4), (-2, 3)   (iii) (-2, -4), (1, -7)
2. Find the equation of the following straight lines, given the gradient and the coordinates of one point in each case.
   (i) 3 (1, 1)   (ii) 2, (2, 3)   (iii) \(\frac{1}{3}\), (0, 0)
3. Find the equation of the straight line passing through each of the following pairs of points.
   (i) (0, 4), (5, 0)   (ii) (0, -5), (-5, 0)   (iii) (0, 1), (-3, 0)
4. In each of the following diagrams, find the gradient and the y-intercept of the line where possible, and write down the equation of each line.

(i)  

Instead of the above questions, students may be presented with **The MRT Problem**.

**The MRT Problem**

1. Find the gradient of the MRT line that joins N3 and N6. Hence find its equation.

2. The line that joins C2 and E3 is parallel to the line described in Question 1. Find its equation.
3. Bedok South Avenue 2 is perpendicular to the East Line that joins E3 and E8 and it passes Bedok Stadium. What are the things you need to know to find the equation of Bedok South Avenue 2? Find the equation of Bedok South Avenue 2. [Teacher supplies the coordinates of Bedok Stadium as (5, 2) when asked.]

4. State the equation of the North line from N6 to Yishun, N9.

5. Somerset is approximately the same distance from N3 and C2. Find the coordinates of Somerset.

6. Find the equations of the lines that join
   a) W9 and W12,
   b) W3 and W9,
   c) B1 and B3.

7. Imagine that you work for the MRT Corporation. You are given the task to add two more lines to the existing MRT system.
   a) State the factors that you have to consider when making this decision.
   b) Find the equations of these two lines. Coordinates of other towns can be obtained from your teacher.
   c) Give reasons for your choice.

Students ultimately are still doing drill-and-practice on determination of equations of straight lines. These drill-and-practice questions are, however, disguised in the form of a situation familiar to students.

The modified questions have many advantages over the typical textbook questions. Firstly, the tedium associated with drill-and-practice is reduced as each question provides a different scenario. Secondly, students are required to translate verbal information into mathematical forms. Thirdly, unlike most textbook exercises, questions of the same nature are not grouped together. Hence students need to select the appropriate strategy to solve a sum. Otherwise, students are merely practising a skill. Fourthly, questions such as Question (7) provides
students with the chance to contribute to the problem. This gives students a sense of involvement and achievement. Finally, problem solving skills can be included.

Even as students are being drilled to perfect the skill of finding equations of straight lines, the teacher is teaching problem solving.

As the questions are varied each time the student is required to select an appropriate strategy.

Questions such as Question (3) require students to think about the problem before solving it. They are encouraged to identify necessary information. Besides, the teacher as a facilitator should ask probing questions as students progress through problems. Students will, after some time, model after the teacher and ask such probing questions.

Some questions [Questions (3) & (7)] do not provide students with sufficient information. Such questions help students identify relevant or redundant information which is typical in real life problem solving.

Questions such as Question (7) introduce elements of subjectivity. Such questions help remove a common misconception among students that qualitative ideas are not important in mathematics. Again, students are required to pause and think before proceeding with the problem. In this way students do not mechanically plough through the questions as they are likely to do for textbook (Lee, 1982) type of questions.

The primary aim of this article is to suggest how textbook questions can be modified to allow teachers to teach problem solving in the regular classroom. This is done without having to increase teaching time. Neither do we need to forgo drill-and-practice, which in our opinion is essential to the learning of mathematical skills. It is thus hoped that teachers would teach problem solving to students through modified textbook questions.

Students need to be trained to be good problem solvers. We cannot predict what domain specific knowledge they may need in their adult and working life but are certain that they will be solving problems each day.
Reference
