HELPING CHILDREN TALK ABOUT SHAPES: A CASE STUDY WITH TEN CHILDREN IN THE LEARNING SUPPORT PROGRAMME

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This paper describes an activity which attempts to change the discourse of a mathematics classroom with the specific intent to help children who may have difficulties with mathematics and ways of communicating. Ten 8-year old children in the Learning Support Programme were engaged in an open-ended geometric task. In this paper a brief description of van Hiele’s theory of geometric reasoning and Vygotsky’s socio-cultural theory of cognitive growth are presented in the introduction. This is followed by a detailed description of the open-ended task which required children to use the four basic shapes - triangles, squares, rectangles and circles to create animals with a tail. The findings showed that while the children were able to use the four basic shapes to make animals with the given condition, they were challenged to explain why the animal they had selected was their favourite. Suggestions on how to improve the children’s communication skills are discussed.

Introduction

This paper reports on an activity carried out with a group of ten eight-year old (Primary Two) children who were selected to join the Learning Support Programme (LSP) offered by the school. The LSP teacher explained that the children were selected to participate in the LSP as they had difficulties with the English language and
mathematics, and had underperformed in the Primary One assessment the year before. I observed the children in one mathematics lesson and found that while they were on task with set work, they were very reserved during the entire activity. They did not talk about the mathematics that they were doing because these tasks did not require any form of communication on their part. One of the children was referred to the educational psychologist from Ministry of Education for observation. I interviewed this child and found that she was unsure of herself, and unable to identify the four geometric shapes and the basic colours. Because the children were reserved in their disposition, the teacher welcomed activities where the children would be given an opportunity to talk about the mathematics they were working on in a novel yet meaningful manner and in a non-threatening atmosphere. Having just completed the unit on shapes, the LSP teacher asked for an activity which provided opportunities for children to apply van Hiele’s geometric thought at Level 0. For reasons to be discussed later, the teacher selected the activity ‘My favourite animal’.

In this activity ‘My favourite animal’, the children were to use the four geometric shapes – triangles, rectangles, squares and circles to create figures that resembled animals and the children had to talk about the animals they had created. To encourage the children to use their imagination and reasoning, certain conditions were imposed on this open-ended task. Details of the task are presented in the study section.

van Hiele’s Levels of Geometric Thought

Learners of geometry progress through a sequence of five levels in geometric reasoning known as the van Hiele levels (Fuys, Geddes & Tischler, 1988). Children at the lowest level - Level 0 – operate visually. At this level of visual reasoning, children recognise shapes by their appearance. For example, children recognise a square
because it reminds them of square tiles. The attributes of the shapes are not recognised and described at this level. At Level 1 – the descriptive level – children no longer identify a given shape by its appearance. Children at this level can describe the physical attributes of the shape and identify the shape according to the given description but are also unable to order properties of shapes where squares are seen as sub-sets of rectangles. This ability to order shapes in a manner in which shapes are related to each other are carried out by children at the third level – Level 2 – Informal deductive level. Here children can explain that because squares and rectangles share certain attributes – they both have four right angles and two pairs of opposite sides parallel and equal, squares are therefore special types of rectangles. Children reasoning at Level 3 – Formal deduction, can prove theorems deductively and are able to show how different theorems within a system are related. Children at the last level – Level 4 – Rigour are able to establish theorems in different systems and are able to compare and contrast these systems. The last two levels are likely to be achieved with students in pre-universities or universities.

As these levels are hierarchical, progress through these levels is sequential. For students to reason at Level 1, students must be well grounded at Level 0 reasoning. Students should be able to visualise a square before they can begin to describe its attributes. Progress through the stages is contingent upon instruction, and not maturation.

**Teaching and Learning from a Vygotskian Perspective**

A central principle underpinning Vygotsky’s perspective of learning is that cognitive growth evolves out of children’s social interactions. Vygotsky favoured the view that children learn more sophisticated cognitive strategies from their interactions with more skilled thinkers (Vygotsky, 1962, 1978, Wertsch & Tulviste, 1992) and argued that language shapes thoughts in important ways. Children’s
thinking develops as they use language to interact with skilled partners on tasks that are within their means and they learn to incorporate what the skilled thinkers say to them into what they will say to themselves. There are three levels to such interactions – social speech, private speech and inner speech (Vygotsky, 1962). Research (Vygotsky, 1962; Berk, 1992; Berk & Landau, 1993) shows that in their interactions with a skilled partner, children are engaged in social speech. Here they learn from the skilled partner how to use language tools to solve problems. In their attempts to gain mastery over language, children mutter to themselves or are engaged in private speech. Once they are confident that they have control over such tools, these utterances become inner speech and their thinking are no longer audible to an outside audience. Through such interactions, children learn the problem solving tools of that particular social group. The work of Oliver Sacks with deaf children made a strong case that language shapes thoughts. Sacks (1989) in his book, *Seeing Voices* demonstrated that deaf children who were denied access to the American Sign Language or other language systems showed delayed cognitive development and underachievement compared to those who had access to language as a tool of thought. It is also possible that children who have had no exposure to the meaningful use of language in learning and talking about mathematics may develop inhibitions in their ability to do so.

The Vygotskian perspective of cognitive development on teaching and learning suggests that teaching in the form of guided interactions which addresses the needs of individual children in a classroom and where the teacher is the skilled partner in learning is important. This means that children who are not used to talking about mathematics in a mathematics classroom may benefit from activities where the teacher shows how this can be done. The children, who are thus guided and by modelling the teacher, may
then, be able to engage in similar activities where mathematics is part of the classroom discourse.

The Study

Researchers like van Hiele (1999) had successfully used play to help children learn geometry. Similarly the children of the study were presented with a task where they played with shapes. According to the primary mathematics curriculum, (Curriculum Planning & Development Division, 2000) the children who were in Primary Two were only required to identify visually the four basic geometric shapes – triangles, squares, rectangles and circles.

The task required the children, working in pairs, to use three triangles, two squares and one circle to make an animal with a tail. All the shapes provided had to be used in their construction and they were to make as many different animals as possible. This part of the activity provided the children with an opportunity to use their imagination to create different animals. Working in pairs encourages the children to talk to each other about their creations. For example, a child could say to his or her partner – "Let’s use the circle for the animal’s head and the square for its body". This sort of talk helps the children review their knowledge of the names of the shapes without having to do so by rote. The children were then asked to select their favourite animal and talk about the animal. Here the children made overt their internal speech as they explained to their peers their favourite animal. The LSP teacher chose this task because it was simple and did not require the children, who were weak in their mathematical knowledge, to apply complex mathematical concepts such as operating with numbers and the four operations. Further, the task was non-threatening since there were no correct or wrong answers. The focus of children’s talk was to check if they could identify shapes accurately and talk about their creations. Prior discussion with the LSP teacher suggested that because these children had consistently failed in their mathematics,
they might be reluctant to reveal their own thinking for fear of being ridiculed. As the task was relatively direct, the children who were reserved and anxious may be encouraged to talk about the shapes and how they used them to make animals. This task also gave the teacher an opportunity to observe and assess the children’s use of language and their confidence working with shapes in an open-ended task.

**The Participants**

The ten children who participated in this study had failed in their Primary One English Language and mathematics tests, scoring below 20 per cent for each test. They were placed in the LSP class instead of the normal English language and mathematics classes. In the LSP class they followed special lessons conducted by the teacher and because the LSP class was small these children received more individual attention than they normally would have, had they remained in the mainstream classes. One of the aims of the LSP was to help such children improve in their reading, writing and arithmetic so that they can rejoin the mainstream classes in Primary Three.

The following was the profile of one of the children whom I had the opportunity to interview for 1.5 hours two days before the delivery of the lesson. Although the educational psychologist was looking into Lida’s case, the LSP teacher asked if I could provide additional support for mathematics. This profile, although not representative of all the children in the group nevertheless explains why talking, in particular communicating mathematics, may not be a simple task for some children.

*The case of Lida.* Lida was one of two girls from a single parent home. Her mother washed plates for a coffee shop and was the sole breadwinner of the family. Lida was not aware of her father whom she had never met. While Lida could count orally from one
to twenty, her knowledge of the number facts was uncertain. When asked to use number discs to form numbers she made 01 and thought that meant 10. However when queried to say the number aloud, she realised her mistake and corrected the arrangement of the discs to form 10. She could not say how many fingers there were on one hand. When asked, she had to count all her fingers on one hand before giving a correct answer. She could not even identify, either in English or in her mother tongue, the four primary colours of red, blue, yellow and green. She was very interested in the toys that were used to engage her in talk. She explained that she did not play games at home because her mother said she (the mother) was too old to play games. Furthermore she was too tired after spending her day working.

The Lesson

The lesson was designed by the LSP teacher. She designed the lesson plan and refinements were made with the author via a series of emails. The final lesson was divided into five phases – (i) Review by the teacher, (ii) Review by the children, (iii) Scaffolding by the teacher, (iv) Children creating their animals, and (v) Children talking about their creations. These phases are described in detail as follows.

Phase 1: Review by the teacher. To prepare the children for the task, the teacher reviewed the four basic shapes learnt in Primary One and Primary Two. The teacher used a bag with four pockets, each labelled with the name of the four basic shapes – rectangles, squares, triangles and circles. The children were first asked to identify and then spell the name of the shape shown by the teacher. The shape was then placed in the correct pocket (see Figure 1). Once the teacher was confident that all the children were able to identify the shapes accurately she then proceeded to Phase 2.

Phase 2: Review by the children. Here the children were given shapes to sort. Each child was given a bag containing one example
of each of the four basic shapes. The shapes were all from a standard box set of pattern blocks. One bag had a missing shape while another had a different example of a circle. The children were asked to check if they had all the four shapes and to name the shape that was missing. One child was able to say that there was a missing triangle. Another child was puzzled by the circular coaster she found in her bag as it was very different from the circles held by her friends who had standard circles provided in Pattern Blocks. The other children were equally surprised when she held up her coaster. The teacher took this opportunity to review the children’s understanding of circles by asking, “Is this a circle?” The class agreed it was. Once the teacher was confident that the children could identify the shapes, the children then took turns to sort their shapes by placing them into the correct pockets (see Figure 1).

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*Figure 1. The children’s bag for the sorting activity.*

**Phase 3: Scaffolding by the teacher.** To ensure that the children understood that they had to create an animal with a tail using the specific number of shapes (three triangles, two squares, and one circle) the teacher created a figure of an animal according to the specified conditions. When asked, the children were able to explain why the figure fulfilled the conditions of the task. The children were then assigned to work in pairs. There were five groups each consisting of a pair of children, identified by the letters A to E.

Phase 1 to Phase 3 took about 30 minutes to complete.
Phase 4: Children creating their animals. The children were assigned their partners and their seats (see Figure 2 for seating arrangement). Three groups sat around one table whilst the remaining two groups sat at the next table. At the centre of each table were different containers, each containing cut-outs of rectangles, squares, circles and triangles of different sizes and colours. These shapes were sorted according to their size – big and small, and colours - pink, yellow, blue and green. The children glued their creations on the response sheet provided (see Appendix A).

![Figure 2. Seating arrangement.](image)

Table 1

Table 2

Containers holding the different shapes according to size and colour.

Children’s seat

The teacher monitored the children’s progress working at the task and handed out extra response sheets when requested. This phase took about 20 minutes to complete and towards the end of this phase the children were asked to show their creations to the rest of the class.

Phase 5: Children talking about their creations. Each group of children was then asked to talk about their creations and to say which was their favourite and why. This phase took another 20 minutes to complete.
A research assistant video-taped the 1.5 hour lesson while I observed the children and made notes of their behaviours. The video-taped lesson and the observation notes were the main sources of data for the study. While interviews with the children at the end of the activity would have provided information that may help explain some of their behaviours, they were not conducted as the children had to rejoin their class for the next lesson. We were encouraged not to cause the children to miss any of their other lessons. However my discussion with the LSP teacher at the end of the session provided useful insights to better understand some of the children’s work.

**Findings and Discussion**

Insights of the children’s working style and some of their difficulties with the open-ended task were based on the discussions with the LSP teacher. Figure 3 shows the different animals made by the five different groups; identified by the letters A to E. Each of the five groups created the lion while the hippopotamus was made by only one group. However all groups were able to create three animals.

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| Lion | Monkey | Fish | Bird | Tiger | Giraffe | Hippopotamus |

**Figure 3.** Types of animals made by each of the five groups.
The Children’s Working Style

Keeping track of the set condition. The video-taped lesson showed that each of the five pairs of children began their task by selecting the required number of shapes. Only after the selection process was completed did the children begin creating the animal. The children were thus free to concentrate on creating the animals without having to keep track of the number of each shape. Consequently it is inferred that the children understood the conditions of the task. Further they could identify and select the correct shapes provided in the containers placed at the centre of each table.

Discussions before the task. Although the children were working in pairs, they did not discuss with each other about the animal they wanted to create. Rather they began by first sticking a circle on the response sheet. Analysis of children’s creations showed that all the animals had a circle for the animals’ heads. This was not unusual as children often encounter representations of living objects with circular heads, for example cartoon characters both on televisions and printed medium. It was observed that the talk between children was aimed at encouraging and reassuring each other that it was acceptable to use a given shape in particular position. “Like this also can,” was the phrase often heard among the children. It was further observed that the teacher noticed the lack of communication between members of each pair of children as she tried to encourage children to be more communicative by suggesting that they tell their partners what they were doing, “You have to tell your friend what you are doing.”

One possible reason for the children’s lack of discussion is that this task challenged the children’s belief about doing mathematics. The LSP teacher conjectured that as the task was novel to the children, they were not aware that they could actually talk about the ‘solution’ to a problem even before they presented their answer
on the response sheet. Further when the teacher showed the children how to begin the task, she did not talk about her ideas prior to showing the children sample solutions.

Groups working independently from each other. The children did not copy each others’ work. In fact the first group of children on completion of their work christened their animal a tiger and promptly shared their ‘tiger’ with the children around the table. Their ‘tiger’ was the only tiger that was created. Fish was the next animal to be named but only two groups of children created the fish. All the five groups made a lion, but their creations were different from each other. This suggests that although the children may be influenced by the creations of the other children, each group was keen to offer different variations of similar types of animals. Figure 4 shows the five different ‘lions’ created by the children.

Figure 4. Different shapes of lions created by the five groups of children.
The children’s confidence working at the task. Once they had competed and named their creation they began to create new figures of other animals. The children were eager to produce more than one animal. A common refrain often uttered by the children during this activity was “Can we do one more?”. This suggests that the children were not limited by the notion that they were able to make only one animal.

The children’s difficulties with the open-ended task. Shapes of different sizes and colours were provided for the children to use in their construction. Analysis of the children’s creations showed that two groups used one colour for one animal and changed the colours for the next animal. It was noted that the constructions of these children were in one orientation. If they created an animal with a horizontal orientation, then the remaining two animals were similarly oriented (see Figures 5 & 6). The remaining three groups of children used different coloured shapes for the same animal resulting in multi-coloured animals and the different animals had different orientations, horizontal as well as vertical and all the figures were different (see Figure 7).

![Figure 5. Animals with the same orientation and each animal had the same colour.](image-url)
Figure 6. Animals with the same orientation and each animal had the same colour.

Figure 7. Multi-coloured creations and with different orientations.

Children Talking About Their Creations

When each group of children were asked to share their creations with the class and offer reasons for their choice, the children had no problems identifying their favourite animal although they were limited in their explanations. Some of the verbatim reasons offered by the children were:

- “I like the monkey because it is nice and cute.”
- “The tiger is very nice.”
- “The bird can fly very high and it has feathers. It is colourful (although the bird was pink). It is beautiful.”
• “The fish has a soft skin. The fish can swim in a tank.”
• “I like the bird because it is beautiful. I like it because it is pink in colour.”

Even when encouraged, the children were unable to say more than their one-line explanations. It is possible that these children were unfamiliar with activities where they were required to express their thoughts. Moreover the lack in their overt talk may indicate a lack in their inner speech. If such social activities were not part of the culture of their normal mathematics classroom, the language tools needed for such activities might not be well formed similar to the deficiency experienced by the deaf children in Sacks’ (1989, op. cit.) study. The teacher concurred that activities where mathematics formed part of children’s talk were rarely used in the normal mathematics lessons.

**Encouraging Children to be More Productive in Their Thinking - Modifications to the Current Task**

The objectives of this open-ended activity were to help children identify visually, geometric shapes as well as to talk about the shapes they used in their creation. This study showed that while children were able to reason at Level 0 - name the shapes and use them to make different animals; they found the oral task very demanding. The children’s difficulties could be a result of their lack of experience with such tasks in their daily classroom activities. If these children did not have experiences where language was used meaningfully, then it is very unlikely that these children will have opportunities to practice what they hear from the skilled others and internalise the social speech into private and then to inner speech. Hence the lack in their overt speech may suggest a deficiency in their inner speech and also a lack in examples of social speech. The ability to talk about mathematics and to use mathematics in communication are important outcomes of mathematics teaching (Cockcroft, 1982).
Such communication skills need to be developed as early as possible in the children’s learning of mathematics. Perhaps where English is not the first language of children, structured activities may be advantageous in facilitating the children’s learning of mathematics.

*Suggested modifications to the original task.* The LSP teacher and I held a discussion at the end of the session to seek ways of improving the task with the specific aim of helping such children. Two modifications were offered. The modified task was later tried out with 30 teachers of children with learning disabilities. Such tasks were novel to these teachers. However it was observed that the creations made by these teachers were more complicated and their talk was more substantive, using comparisons across creations involving numbers and the shapes.

*Reducing the number of constraints.* Remembering too many constraints could inhibit the children’s productivity and creativity. The condition that children had to use a fix number of shapes to create their animals was modified so that children could use any number of shapes instead. Children could also be encouraged to use numbers in their talk if the response sheet provided for this task was modified to include opportunities for them to list the number of each shape used (see Appendix B). This modification then links the geometric activity with abstract concepts of numbers thus encouraging the children to use numbers in their talk.

*Children providing exemplars.* Rather than the teacher showing the children how to create an animal that satisfies the conditions set out in the task, the teacher could allow the children to create an animal and then ask the children to talk about their initial creations. Different groups of children could then be asked to describe the process of creating the animal. The use of specific questions and prompts can perhaps guide the children to think further about their creations. Such questions could include (a) How did you decide what animal to make? (b) How many of each shape did you use?
(c) Does your animal have a tail? (d) Do you like the animal you made? (e) Can you tell us why?

Through this process of questioning, the rest of the children can learn from their peers the need to have prior discussions before making the animals and how to talk about their creations. Further as other children would be showing their examples, the children can learn how the conditions of the task could be met in different ways. The children learn to reflect upon their feelings towards their creations and how their peers use language to express their thoughts and feelings. After completing their first animal figures, the children can then proceed to create more animals. Only after they have constructed many figures, the teacher will gather the children to talk more about their creations and also their favourite animal. Later children could be asked to state the similarities and differences of the different animals. This helps children to use the ‘compare and contrast’ thinking skill as listed in the primary mathematics curriculum (Curriculum Planning & Development Division, 2000). Such situations provide the children with opportunities to learn more without the teacher having to teach more.

**Conclusion**

The outcome of this activity showed that children who have never engaged with novel mathematical experiences displayed ways of working at the task that were lacking in variety. The artefacts they produced were also limited. What children learn about mathematics, how they learn and how they think are shaped by their classroom experiences. Because children’s dispositions towards mathematics are shaped by their classroom experiences, particular attention should be given to the pedagogy as well as the tasks involved. Hence it is vital that teachers provide children of all learning abilities open-ended yet directed activities where children learn to engage with mathematics and the different forms of mathematical discourse.
References


## APPENDIX A

Response sheet on which children presented their work.

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<th>Name: ___________________________</th>
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<tr>
<td>Class: __________________________</td>
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<td>My animal is: ___________________</td>
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APPENDIX A

Response sheet on which children presented their work.

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<th>Name: ______________________________________________</th>
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<tr>
<td>Class: _____________________________________________</td>
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My animal is: _______________________________________

_________ triangles   __________ circles

_________ squares   __________ rectangles

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