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Author(s)	Wong Kin Mun Desmond and Yeo Kai Kow Joseph
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## EXCEL AS A TOOL TO ENHANCE LEARNING OF PROBABILITY

Wong Kin Mun Desmond  
Ngee Ann Secondary School  
Singapore

Yeo Kai Kow Joseph  
National Institute of Education  
Nanyang Technological University  
Singapore

### Abstract.

Probability is often an immensely challenging topic for students at the secondary level. The main difficulty for these students often lies in their inability to relate the theories learnt in classroom to their everyday lives. It is for this reason that lessons on the topic of Probability, especially those at the introductory level, rely heavily on hands-on activities. As empirical probabilities will only converge to the results predicted by theory, implementation of hands-on activities is, inherently, ridden with problems and difficulties. The problem is so acute that ‘mistake-prone’, ‘repetitive’ and ‘time consuming’ are but a few phrases which students have associated with traditional hands-on activities.

This apparent lack of meaningful hands-on activities has motivated the authors to develop an IT-based discovery activity for the topic of Probability at the secondary level. Microsoft’s Excel has been embraced by the authors in automating the counting and calculations involved in obtaining experimental probabilities which would otherwise be cumbersome to obtain. To enhance the appeal of the discovery, the authors have used the game of Monopoly as the theme of analysis.

This paper provides pertinent details of the discovery activity. It also describes the benefits and possible barriers to successful implementation of the activity.

### Introduction

A direct consequence of the rise of the Information Technology era lies in the strong emphasis on information and data processing. With this emphasis, data handling and statistics have become essential and indispensable components in the mathematics curriculum throughout the world (Ministry of Education, 2001a; Ministry of Education, 2001b, 2003; NCTM, 2000; Holmes, 1980).

The Law of Large Numbers (Watkins, 1981) asserts that large samples are more likely to reflect the parent population while small samples often produce experimental probabilities that differ markedly from the parent distribution. The concept of sample is fundamental to data analysis and reasoning under uncertainty.

Recent curriculum reforms in school mathematics (Australian Educational Council and Curriculum, 1994; Department of Educational and Science and the Welsh Office, 1991) have advocated broadening the scope of probability in the middle school curriculum. In the United

States, the National Council of Teachers of Mathematics (2000) has called for students to carry out simulations of random phenomena, and to compare experimental results to the mathematically derived probabilities (referred to as theoretical probabilities henceforth). This avocation of simulations to foster probabilistic reasoning is based on students' difficulty in making a connection between the experimental probability (using relative frequencies to determine the likelihood of an event) and the theoretical probability of an event (Aspinwall & Tarr, 2001).

### **Rationale behind Using Excel in Teaching of the Topic of Probability**

The advantages of employing an activity based probability lessons have been well documented (Bradstreet, 1996; Garfield, 1995; Mrudulla, Richard, Ann, & Jeffrey, 1997). In Gnandesikan (1997), students were asked to comment on what they liked best and what they liked least about an activity based probability lesson, and some dislikes highlighted by the students were:

- a. It is easy to make mistakes.
- b. It is not useful.
- c. It is boring.
- d. It is repetitive, tedious and time consuming.
- e. It is too simple.

In the light of the abovementioned pitfall on the activity based Probability lessons, the authors believed that Microsoft Excel could be a very useful tool in enhancing learning in an activity based Probability lesson based on the following reasons:

1. The Singapore's Ministry of Education have purchased licenses for the installation and use of common Microsoft applications in all schools. Microsoft Excel is one of the Microsoft applications readily available in all schools. No additional software is required. When the Singapore's Ministry of Education's implemented the IT MasterPlan I to all schools, a large majority of teachers have acquired the knowledge of using the Excel. Thus, to carry out an activity based lesson on Probability which incorporate Microsoft Excel will not be a problem for most teachers.
2. The probability taught at the secondary level is at the introductory level. An essential component of introductory probability is to focus on the meaning behind the probabilities calculated by algorithms such as possibility diagrams and probability tree diagrams, ie. how these theoretical probability relate to the actual occurrences (experimental probabilities). This would thus encompass showing the students the convergence of the experimental probabilities to their corresponding theoretical counterparts as the sample size increases. As 'observable' convergence only occur when the sample size is sufficiently large, this presents difficulty to actual hands-on type of probability activities, as keeping track of outcomes physically gets 'exponentially' cumbersome, boring and mistake-prone as sample size increases. This is exemplified by the students' dislikes in Gnandesikan's (1997) study.

### A Discovery Activity Incorporating Excel

The Cambridge GCE ‘O’ Levels Examination syllabus for Mathematics (subject code 4017) specifies that the subject contents for the topic on Probability are as follows::

- a. Calculate the probability of a single event as either a fraction or a decimal (not a ratio).
- b. Calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate (in possibility diagrams outcomes will be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of the branches).

With the syllabus in mind, the authors have developed a lesson which serves as a consolidation-cum-extension lesson for this topic of Probability. The prerequisite knowledge of this lesson includes the all the basic concept of probability as stated above. The lesson is carried out in two main discovery activities. It involves the students working in pairs in a full sized computer laboratory where 40 computers are available.

The teacher will begin the lesson with a short introduction to the board game, Monopoly. During the briefing, the teacher will emphasis the following points to the students:

- 1 The movement of the playing pawns is dictated by a roll(s) of two dice. Every roll of a double (two dice giving the same outcome), up to the third roll, qualifies the player for another roll of the two dice within the same turn. Three consecutive rolls of doubles lands the player’s playing pawn in jail. After each roll of the two dice, a player moves his/her playing pawn as usual, according to the sum of the two dice, and is subject to any privileges or penalties pertaining to the space on which he/she lands.
- 2 Useful information pertaining to the “Chance” and “Community Chest” cards:
  - a On the playing board, there are three “Chance” spaces and three “Community Chest” spaces.
  - b In each deck of “Chance” and “Community Chest”, there is one and only one card that reads “Go To Jail”. There are sixteen cards each in the “Chance” deck and “Community Chest” deck.
  - c When a player lands on either of these spaces he/she takes the top card from the corresponding deck, follows the instructions and immediately returns the card face-down to the bottom of the deck.
- 3 A player has to go to Jail when:
  - a his/her playing pawn lands on the space marked "Go to Jail".
  - b he/she gets a “Chance” or “Community Chest” card marked "Go to Jail".
  - c he/she throws doubles three times in succession.

A salient point to note is that any mention and description of the “Get Out of Jail” cards, which make up one card each in each of the “Chance” and “Community Chest” decks, are intentionally omitted at this juncture. This is to simplify the calculations in activity 2, which will be described later. In the event where there are student(s) who raises the issue of the “Get Out of Jail” cards,

the teacher adopting this lesson created by the authors should address this by commending him/her/them on his/her/their excellent observation and make it a point to let the whole class know that they are to assume the absence of such cards in their calculations. The students will, in fact, be given a chance to ponder this issue at the end of the worksheet accompanying this lesson.

### Activity 1

The objective of the first activity is to:

- assess students' understanding of experimental and theoretical probabilities.
- assess students' understanding of the suitability of the two techniques for calculating theoretical probability (namely the possibility diagram and the probability tree diagram).
- assess students' ability to use the possibility diagram for calculating theoretical probability.
- show the students first hand, the convergence of the experimental probability to the corresponding theoretical probability as the sample size gets progressively larger.

In this first activity, the students, working in pairs, are to calculate both the theoretical and experimental probabilities of each of the possible sum in a single roll of two dice. They are also required to compare the experimental probabilities with the corresponding theoretical probabilities as the sample size increases.

In order to calculate the experimental probabilities for an increasing sample size, the authors have developed an EXCEL program that simulates the roll of two dice. Screen captures of the dice roll simulator program are provided in figure 1.

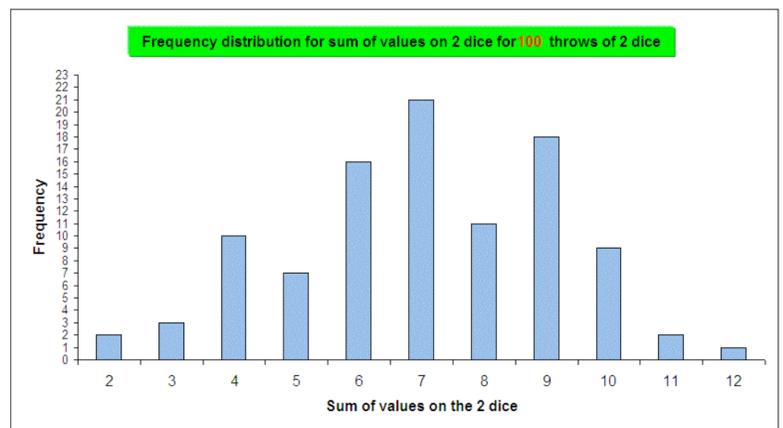
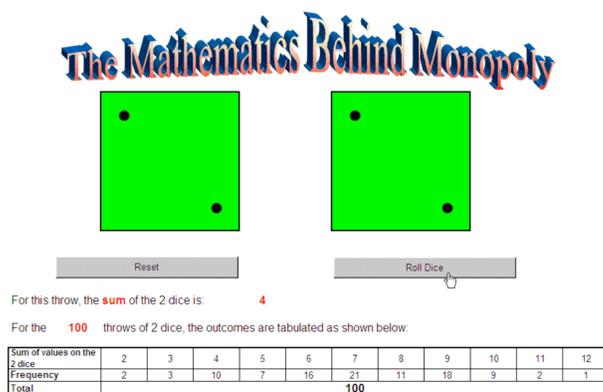


Figure 1: Image captures of the EXCEL program simulating using two dice.

Figure 1 shows the image captures of the EXCEL program simulating using two dice. Diagram on the left shows the simulator and the frequency distribution table after 100 rolls of the dice. Diagram on the right shows the corresponding histogram generated.

The students are to carry out the dice rolls for three cases (rolls of 10, 50 and 100) and record the results they obtained into a table provided in the worksheet. The students are then instructed to close this first program before proceeding on.

In the second Excel program, the students will be used to automate the calculations of the probabilities. At this juncture of the activity, the students will be required to provide a short description, in their worksheets, of how the experimental probabilities could be calculated from the results they have obtained thus far. This is followed by a question asking the students to select and justify a suitable technique for calculating the corresponding theoretical probabilities. They may be required to provide a brief explanation of how this technique could be used to calculate the corresponding theoretical probabilities. These questions serve a two-fold purpose. Firstly, it enables the teacher to assess the student's understanding of how experimental probabilities are calculated from raw data obtained from experiments as well as how much he/she knows with regards to the two techniques of calculating theoretical probabilities, namely the use of the probability tree diagram and the use of the possibility diagram. More importantly, the teacher can gain useful insight as to how much a student understand about the suitability of each of the techniques for calculating theoretical probabilities under different circumstances. Secondly, in order to answer these questions posted to them, the students have to recall and think through the processes of calculating both the experimental and theoretical probabilities. This gives the students a better understanding of the later part of this activity where the calculation of the experimental and theoretical probabilities are automatically calculated for the students based on their input.

Upon completion of the abovementioned questions, the students are provided a second EXCEL program within which the experimental probabilities are calculated, based on the data the students have generated using the first program, and compared with the corresponding theoretical probabilities which the students will have to use the program to generate.

In this second Excel program, the students are to transfer the data in their worksheet into columns 2 thru 7 of the table on the left in Figure 2. Each cell in column 8 displays the sum of the corresponding values in cells in columns 2 thru 4; likewise, each cell in column 9 represents the sum of the corresponding values in columns 5 thru 7 while cells in column 10 total the values in corresponding cells in columns 8 and 9. The totals will be displayed in columns 8 thru 10 once the student has successfully keyed in their experimental data. The values from this table are used for the calculation of experimental probabilities which will be displayed in a comparison table such as that shown on the right in Figure 2.

(Experimental) Frequency Distribution Of Sum of Two Dice

No. of rolls Sum of 2 dice	Student A			Student B			Total Student A	Total Student B	Grand Total
	10	50	100	10	50	100			
2	0	0	3	0	0	1	3	1	4
3	0	1	1	1	2	7	2	10	12
4	1	6	8	2	6	8	15	16	31
5	2	8	15	2	6	11	25	19	44
6	0	8	16	1	8	15	24	24	48
7	2	8	10	2	10	19	20	31	51
8	2	7	18	0	8	16	27	24	51
9	1	7	15	1	5	14	23	20	43
10	2	5	10	1	2	3	17	6	23
11	0	0	3	0	2	5	3	7	10
12	0	0	1	0	1	1	1	2	3

Comparison Between Experimental and Theoretical Probabilities

No. of rolls Sum of 2 dice	Experimental Probability									Theoretical Probability	
	Student A				Student B				Both		
	10	50	100	160	10	50	100	160			320
2	0.00	0.00	0.03	0.02	0.00	0.00	0.01	0.01	0.01	0.01	
3	0.00	0.02	0.01	0.01	0.10	0.04	0.07	0.06	0.04		
4	0.10	0.12	0.08	0.09	0.20	0.12	0.08	0.10	0.10		
5	0.20	0.16	0.15	0.16	0.20	0.12	0.11	0.12	0.14		
6	0.00	0.16	0.16	0.15	0.10	0.16	0.15	0.15	0.15		
7	0.20	0.16	0.10	0.13	0.20	0.20	0.19	0.19	0.16		
8	0.20	0.14	0.18	0.17	0.00	0.16	0.16	0.15	0.16		
9	0.10	0.14	0.15	0.14	0.10	0.10	0.14	0.13	0.13		
10	0.20	0.10	0.10	0.11	0.10	0.04	0.03	0.04	0.07		
11	0.00	0.00	0.03	0.02	0.00	0.04	0.05	0.04	0.03		
12	0.00	0.00	0.01	0.01	0.00	0.02	0.01	0.01	0.01		
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		

Figure 2: Image captures showing the frequency distribution table

Figure 2 shows the image captures showing the frequency distribution table where students are required to input experimental data they have obtained from the first Excel program (left) and the corresponding entries generated in the table where a comparison will be made between the experimental probabilities and the corresponding theoretical probabilities (right).

For the calculation of theoretical probabilities, the students are required to provide the values in a blank possibility diagram as shown on the left in Figure 3. Upon input of correct values into the possibility diagram, the theoretical probabilities will be generated in decimal form in the last column in the comparison table, as shown in diagram on the right in figure 4. As activity 2 will involve the students calculating the probability of his/her pawn landing in “Jail”, the theoretical probabilities are also available in fractional form in a separate table shown in figure 4. A pertinent point the authors wish to highlight is that each roll of double (eg. (2,2), (5,5), etc.), up to the third successive double, will lead to another right to roll of dice within the same turn. This is the reason behind the provision of the theoretical probabilities for doubles and non-doubles for the even numbered sum of two dice in this table. The preceding explanations will be more apparent with the commencement of description on activity 2.

Comparison Between Experimental and Theoretical Probabilities

Sum of 2 dice	No. of rolls		Experimental Probability								Theoretical Probability
	Student A				Student B				Both		
	10	50	100	160	10	50	100	160			
2	0.00	0.00	0.03	0.02	0.00	0.00	0.01	0.01	0.01	0.01	0.03
3	0.00	0.02	0.01	0.01	0.10	0.04	0.07	0.06	0.04	0.04	0.06
4	0.10	0.12	0.08	0.09	0.20	0.12	0.08	0.10	0.10	0.10	0.08
5	0.20	0.16	0.15	0.16	0.20	0.12	0.11	0.12	0.14	0.14	0.11
6	0.00	0.16	0.16	0.15	0.10	0.16	0.15	0.15	0.15	0.15	0.14
7	0.20	0.16	0.10	0.13	0.20	0.20	0.19	0.19	0.16	0.16	0.17
8	0.20	0.14	0.18	0.17	0.00	0.16	0.16	0.15	0.16	0.16	0.14
9	0.10	0.14	0.15	0.14	0.10	0.10	0.14	0.13	0.13	0.13	0.11
10	0.20	0.10	0.10	0.11	0.10	0.04	0.03	0.04	0.07	0.07	0.08
11	0.00	0.00	0.03	0.02	0.00	0.04	0.05	0.04	0.03	0.03	0.06
12	0.00	0.00	0.01	0.01	0.00	0.02	0.01	0.01	0.01	0.01	0.03
<b>Total</b>	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Figure 3: Image capture of the blank possibility diagram

Figure 3 shows the image capture of the blank possibility diagram where students are to enter the correct sum for the two dice (left) before the theoretical probabilities in the last column of the table contrasting the experimental probabilities with the corresponding theoretical probabilities are displayed (right).

Probability Distribution

Sum	2		3	4		5	6		7
	Doubles	Non Doubles		Doubles	Non Doubles		Doubles	Non Doubles	
Probability	$\frac{1}{36}$		$\frac{1}{18}$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{1}{6}$
	$\frac{1}{36}$			$\frac{1}{12}$			$\frac{5}{36}$		

Sum	8		9	10		11	12	
	Doubles	Non Doubles		Doubles	Non Doubles		Doubles	Non Doubles
Probability	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{36}$	
	$\frac{5}{36}$			$\frac{1}{12}$			$\frac{1}{36}$	

Total Probability: 1

Figure 4: Image capture of the theoretical probability distribution table

Figure 4: Image capture of the theoretical probability distribution table calculated from inputs in the possibility diagram (figure 3, left). In this table, the theoretical probabilities are given in fractional form, and provisions are made for doubles and non-doubles in even numbered sum of two dice.

The students are then required to transfer the values in the table shown on the right in figure 3 in this second program to a table in their worksheet shown in figure 5. They are then requested to use this table to observe the convergence of experimental probabilities to the corresponding theoretical probabilities as sample size increases.

This side-by-side comparison between the experimental probabilities for increasing number of dice rolls with their corresponding theoretical probabilities mark the end of activity 1. As this activity serve as a backbone to investigation 2, students are requested to save their data and proceed to activity 2 using the same program.

No. of rolls Sum of values on 2 dice	Experimental Probability					Theoretical Probability
	10	50	100	160	320	
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
Total						

Figure 5: Table in the students' worksheets

Figure 5 shows the table in the students' worksheets for filling in the results generated in the frequency distribution table like the one shown on the right in Figure 3.

## Activity 2

The objective of second activity is to:

- assess students' understanding of the suitability of the two techniques for calculating theoretical probability (namely the possibility diagram and the probability tree diagram).
- provide students with an opportunity to devise their own strategies in solving an 'out-of-textbook' question on the topic of probability.
- emphasise the importance of assumptions and their effect on the complexity of a problem.

In this second activity, the students, working in the same pair they were in for the first activity, are to devise a strategy to calculate the (theoretical) probability of a player's pawn landing up in Jail from a pre-assigned position. They are encouraged to use the data obtained from activity 1 and utilize the tools provided in the second Excel program.



Figure 6: Image capture of the ‘Monopoly Position Tracker’

Figure 6 shows the image capture of the ‘Monopoly Position Tracker’ showing Virginia Avenue as being 11 positions away from the Baltic Avenue.

Prior to the commencement of their activity proper, two vital programs /tools which the authors have developed are brought to the attention of the students. The first tool, as the name ‘Monopoly Position Tracker’ aptly suggests, provides students with the destination position based on students’ input of their start position and the number of positions away. The second tool which the authors have developed specially for students’ use in activity two is the ‘Probability Multiplier and Adder’. As the name suggests, this tool takes in two probabilities in their fractional form, providing the multiplication and/or addition of these two probabilities in fractional form in its/their simplest reduced form.

**Probability Multiplier And Adder**

$$\frac{1}{7} \times \frac{2}{5} = \frac{2}{35}$$

$$\frac{1}{5} + \frac{3}{11} = \frac{26}{55}$$

Figure 7: Image capture of the ‘Probability Multiplier and Adder’ in action.

After exploring the use and workings of these two tools, the students, in pairs, are each given a starting position on the Monopoly board and asked to determine the probabilities of ending up in jail for a player whose pawn landed on that given position in his/her previous turn.

The authors would like to reiterate that for this calculation, the students are to neglect the presence of the ‘Get Out of Jail’ cards present in each of the ‘Chance’ and ‘Community Chest’ cards sets. With this condition in mind, a player can land his/her playing pawn in ‘Jail’ when the player’s roll(s) of dice for this turn either

- i. lands his/her playing pawn directly on the space marked "Go to Jail", or
- ii. lands his/her playing pawn on either the space marked ‘Chance’ or ‘Community Chest’ and the player subsequently draws a card marked "Go to Jail" from either of this deck of cards, or
- iii. are such that doubles (eg. (2,2), (5,5), etc.) occur three times in succession.

A sample probability tree diagram for the first roll of the dice is shown in Figure 8.

Upon completion of this main part of the activity, the students are then asked to reflect upon what they have done. It is at this juncture that they are presented with facts about the “Get Out of Jail” cards present in each of the “Chance” and “Community Chest” card decks, specifically with regards to the fact that a player can hold on to this card for as long as he/she desires, the card(s) being returned to the respective deck(s) only after the player has used the card(s) in exchange for the release of his/her pawn from “Jail”. The students are then asked how the inclusion of this fact will affect the probabilities they have just calculated. No calculations are needed from the students for this question.

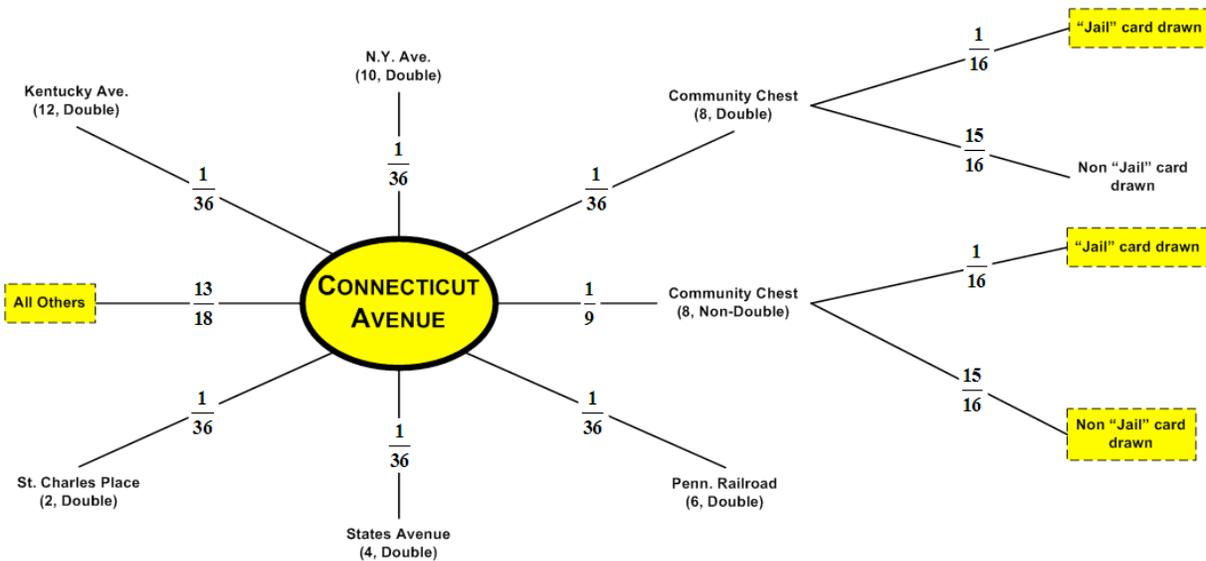


Figure 8: Sample Probability Tree Diagram

Figure 8: Sample Probability Tree Diagram for first throw of the dice of a turn for a player whose starting position is “Connecticut Avenue”. Outcomes that are boxed up are those which will terminate at this throw. Outcomes not boxed up are outcomes that will result in at least another roll of the dice within the same turn.

### End-of-Lesson Discussions

At the lesson closure, the teacher may conduct a discussion session with the students on the learning points as well as any difficulties and misconceptions the teacher may have observed from the students’ work during the lesson. Important points for discussion with the students would be:

1. The conditions under which convergence of the experimental probabilities to the corresponding theoretical probabilities take place.
2. The suitability as well as the advantages and disadvantages of the possibility diagram and probability tree diagram for calculation of probabilities.
3. The effect of the difference in assumptions regarding the “Get Out Of Jail” cards on the calculation of the theoretical probabilities of a player’s pawn landing up in “Jail”. The

teacher may emphasise that initially, an assumption of sampling with replacement is made, while in actuality, sample without replacement is possible.

### **Discussions and Conclusions**

Research has shown that the Law of Large Numbers is not intuitive to students of all ages (Fischbein & Schnarch, 1997; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Shaughnessy, 1992) – prior to instruction, only one of five case study students exhibited an awareness of the relationship between the number of trials and the probability of an event. The lesson developed by the authors will address this difficulty by showing the students first-hand, how the theoretical probabilities they calculate using the techniques they have learnt (use of possibility and/or probability tree diagrams) relate to how an actual event will occur. Although traditional hands-on activities do attempt to address this issue, a large sample is needed for actual convergence of experimental probabilities to values obtained from theory. These activities either fail to achieve their objective of convincing the students the relation between empirical and theoretical probabilities where small samples are used, or the activities become overly repetitive and hence boring and mistake-prone. An Excel program is therefore used to address this issue of showing the convergence of the experimental to the corresponding theoretical probabilities without the hassle of tracking the outcomes of a large number of rolls of the two dice.

Through the interesting theme of the board game Monopoly, the authors have also offered the teachers a tool to assess students' understanding of the fundamentals behind the use of possibility diagrams and probability tree diagrams, especially with regards to the different situations where the different methods are suitable.

A fundamental tenet of socioconstructivism is the view that language and agreement plays a role in establishing and justifying mathematical concepts (Ernest, 1996). The Excel simulation program and the accompanying worksheet on which the students are tasked to work upon in pairs should elicit students' probabilistic thinking. Through the whole-class discussion which follows, this thinking should, hopefully, be enhanced, refined and improved.

Notwithstanding the possible benefits students can derive from activity two of this lesson, the authors do foresee that it may prove to be too daunting to the average and weaker students. The authors would thus recommend that teachers use their discretion where activity two is concerned; they can either simplify it for the low ability students or simply reserve it for the more motivated, high ability students.

Although Aspinwall and Tarr (2001) reported growth in student's understanding of The Law of Large Numbers arising from simulation-rich instruction and concomitant mental activity, the authors have yet to research into the actual benefits of the lessons developed. This will constitute the second phase of the authors' work on this topic.

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