THE USE OF SPREADSHEET IN NUMBER PATTERN INVESTIGATION

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Abstract: This paper describes an action research study looking at primary six pupils using spreadsheets to investigate the last digit of the results of natural numbers which are raised to any whole number exponent. The teacher worked with 2 primary six classes of 18 and 19 pupils in the Gifted Education Programme (GEP). The pupils worked in pairs, each pair was given a laptop. The study showed that pupils were able to analyse and make conjectures regarding the cyclic patterns of the numbers generated in the investigation. The teacher facilitated the discussion among the pupils. The paper discusses the implications of the use of spreadsheets in the teaching and learning of mathematics and mathematical investigations among high ability primary pupils.

Introduction

Activities which involve pupils in observing and analysing patterns in numbers with the objective to unravel unknown characteristics of numerical quantities often lead to much mathematical learning and understanding (Polya 1954, cited in Lampert, 1990). The use of computers in classrooms opens up new dimensions to such mathematical investigations. Spreadsheet programmes such as Excel and the Maths Cruncher enable pupils to generate data via the use of simple formulae. This frees the pupils from mundane computation activities, enabling them to concentrate on observation and analytical tasks. The pupils are thus provided with opportunities to discover for themselves number patterns and to establish new connections and develop knew knowledge in mathematics. This engages the pupils in higher levels of thinking.

This paper describes one such investigative activity. I worked with two Primary 6 classes in the Gifted Education Programme (GEP). These two classes, 6H and 6I had eighteen and nineteen pupils respectively. They were given an investigative task on exponential numbers. The objective of the investigation was to allow the pupils to discover for themselves the cyclic pattern of the last digit of the result of any natural number which was raised to any whole number exponent. . For example pupils were asked to investigate what would be the last digits of numbers such as 4^5, 4^6, 4^9, 5^5, 5^6, 5^9. They were then challenged to consider whether their conjectures could be extended to large numbers with similar last digits such as $24^{5,}$ 86^{34}, 19^{20}. The pupils must then make use of the cyclic pattern they discover to conjecture a generalised approach which they can use to determine the last digit of any exponential numbers.

Pupils' Background Knowledge

Pupils in the GEP were exposed to mathematical investigations since P4. They were exposed to enrichment activities in number patterns involving the squares and cubes of numbers, though they were not taught the concept of indices. Pupils were exposed to the Excel worksheet both in their mathematics classes and in their computer enrichment classes. They were able to harness the various functions of Excel to create tables and charts, enter formulae in cells and utilise the pull and drag technique to generate data.

Method

I conducted the first investigative lesson with class 6H. Reflecting on the learning and difficulties exhibited by the pupils in class 6H, I then modified the worksheets for the next class, 6I. The modifications allowed me to provide clearer instructions for the pupils so that their learning was not hampered by ambiguous instructions given in the worksheets.

I began the lesson by asking the pupils to work out mentally the values of the following numbers: 2^5, 4^4, 6^3, 5^3. This was to focus the children's mind on the forthcoming activity. While they were thinking about the answers for these numbers I distributed the worksheets to the pupils. After an explanation of the goals of the activity, the pupils were then paired up. Working in pairs enable the pupils to test their conjectures by talking to each other. Each pair was then given a laptop to work with. Upon setting up their laptop, they were free to begin their investigation. They were then asked to compare their mental solutions to the questions posed at the start of the lesson with the answers generated by the computer.

Learning exhibited by the pupils

Last digit of a number raised to a given exponent

	A	B	C	D	E	F	G
1	a	squares	cubes	powers of 4	powers of 5	powers of 6	powers of 9
2							

Most of the pupils encountered no difficulty in generating the exponential data needed for the investigation. A few pupils had problem getting their formulae working because they forgot to key in the '=' sign before the formula they keyed in for the cells in row 2. However, once they realised their mistake, they were also able to generate their data readily. Most of the pupils used the multiplication keys to generate the formula for the cells in row 2. For instance cell C2 was filled by the formula =a2*a2*a2. However, Jason and his partner were using the power key '^' to generate the same formula:=a2^3 and they taught the two groups of pupils beside them to do likewise. Most of the groups use the pull and drag technique to generate the data for each column after they create a formula. However, I noted that Liu Riu and Hui Xuan did something very different. They key in all the formulae for row 2, then highlight all the cells with formulae, and then pull and drag all the selected cells to generate all the data in one go. (This was something new which I learnt that day). However, they key in the wrong formula for cell E2, as a result, their value in cell E3: 64, was wrong.

Teacher (T): What is 2 to the power of 5?.

Pupil Liu Riu: 32

I pointed to cell E3 and smiled.

Pupil Liu Riu: Ooph!

Once alerted Liu Riu was able to correct the error.

All the pupils were able to predict the last digit for numbers ending in 1, 5 and 6 without the aid of the generated data. This was demonstrated by what Kharul said to his partner Si Yong:

Kharul: "It's easy. The last digit of 3120 must be 1, that for 751000 must be 5, and that for 8634 must be 6. Common sense, no need to refer to the data!"

I asked them to write down their explanations. They reasoned that they were interested only in the last digit of the result. Any number ending with the digit 1 when multiplied by itself any number of times would still give a number ending with a digit 1 because 1 multiplied by itself would give the number 1. Similarly this was also true for numbers ending in 5 and 6.

However the pupils took quite a while (about 15 minutes) to discover the cyclic patterns in the last digits of exponential of numbers where the last digits were not 1, 5 or 6. Nurul and Joanna were very excited when they discover a pattern.

Joanna: The last digit for any exponential number which ends with 3 repeats in a cycle: 3, 9, 7, 1!

Other pupils soon discovered the cyclic patterns for the other numbers. However many of the pupils were not careful enough in utilising the cyclic pattern to derive their solutions for the challengers (see worksheets in Appendix). From the data generated the pupils noted that the result of any number ending with the digit 7 raised to exponents of 1, 2, 3, 4 ended in a cyclic pattern of 7, 9, 3, 1 respectively. So for 97^{43}, the last digit must be 3, since 43 + 4 = 10 remainder 3. But some groups wrote down 9, while other groups wrote down 1. However, they were able to figure out their mistake and write down the right answer when they were asked to check their solutions.

Law of indices

For the column G2, the pupils were only to use the multiplication key to generate the solutions. Many of the pupils came out with the following formula:

= **A2*A2*A2*A2*A2*A2*A2*A2*A2.**

I asked the pupils to write down as many formulae as possible which will give the value for 1^9 for cell G2. Many of the pupils were unsure of what I was looking at. To provide suitable scaffolding for them, I asked them to consider ways of getting the formula for (i) column C from the entries of columns A and B (ii) column F from the entries in the previous columns. As a result of these links, the pupils were able to provide many alternative formulae for cell G2. The various formulae suggested by the pupils for cell G2 were recorded in the left column of the table below.

=A2*A2*A2*A2*A2*A2*A2*A2*A2	a x a x a x a x a x a x a x a x a
=B2*B2*B2*B2*A1	a^2 x a^2 x a^2 x a^2 x a
=C2*C2*C2	a^3 x a^3 x a^3
=D2*E2	a^4 x a^5
=B2*B2*E2	a^2 x a^2 x a^5
=C2*F2	a^3 x a^6

The pupils were then encouraged to re-interpret their formulae using multiplication symbol and index notation (as in the right column of the table above). After the expressions on the right column were elicited from the pupils and put down on the board, I asked the class:

Teacher: Now, look at these expressions carefully, what do you notice about powers of these numbers?

Pupil Yi Xiang: They all add up to 9.

I then wrote on the board the following questions for the pupils to consider.

a^{12} x a^9 =

$a^{89} \times a^{11} =$

The class was able to say what the solutions were to the above questions. Thus, this activity enabled the pupils to develop an intuitive understanding of the law of indices: $a^m \times a^n = a^{m+n}$.

Computer Based Lesson versus Traditional Chalkboard Based lesson

What differentiates the above mathematical investigation from the traditional chalkboard based investigation mathematics lesson? The most obvious difference is that the pupils have to learn how to use the spreadsheet. Once they have acquired the necessary skills then they are exposed to the power in terms of speed offered by the computer. Pupils are relieved of the mundane drudgery of working out the values of the different numbers. The computer can work out the values in the blink of an eye and then the pupils are free to make their observations, comparisons, deductions and generalisations.

However the above activity shows that pupils in such mathematics classrooms still commit the same mistakes as in the traditional classrooms. Rather than spending the freed up time to look back and review their solutions, pupils are too easily satisfied with their solutions. They tend not to review their solutions until the teacher, as the authority in the mathematics classroom indicates that something is not quite right with their solution. For example pupils Liu Riu and Hui Xuan had very different and much more efficient way of generating the data for the investigative activity. They keyed in all the formulae for row A2, highlighted all the formulae and generated the data all in one go. However they had keyed in the wrong formula in cell E2 but failed to realise it. Only when they were prompted to look at their value for cell E2, did they realise that something was amiss.

Pupils might be too willing to accept the answers provided by the computer and hence, focused on the product rather than on relational understanding (Skemp, 1978). However the teacher, as facilitator can safeguard this from happening by directing the pupils to articulate and communicate their findings with the rest of the class, thus making their thinking and understanding overt to the pupils themselves and to the rest of the class.

In addition to classroom discussion the pupils could be encouraged to write or keep a journal of their findings. In their writing they have to focus on articulating their understanding of concepts learned as well as provide examples of how their understanding could be used to solve other related mathematics problems.

Conclusion

In the above investigative task conducted with GEP pupils one could argue that the computer could be dispensed off. Hand held calculators could be just as effective as the spreadsheet. Or even the traditional talk and chalk method could be employed successfully and effectively. However encouraging pupils to use spreadsheet allows them to acquire essential IT skills developmentally in an interesting maths based context. These skills could be transferred to other mathematical and practical situations. Through this activity, the pupils were also made to realise that the effectiveness of computer as a learning or working tool hinges on the competence of the user. Using the computer in mathematics does not exempt them from having to think. Rather pupils face the fact that mistakes are made by humans and humans must be vigilant to detect and address them. Regardless of the sophistication of the tools made available to solve maths problems, pupils must still execute good mathematical thinking.

References

Lampert, M. (1990). 'When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching.' *American Educational Research Journal*, *27*(1), 29-63.

Skemp, R. R. (1978). 'Relational Understanding and Instrumental Understanding'. *Arithmetic Teacher*, *26*(3), 9-15.