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Think of a number

Adapting the Birthday Problem for the classroom

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The Birthday Problem

In a class of about 40 students, a teacher asks if it is likely that two of the students have the same birthday. This is probably done early in the school year when the students do not yet know each other well, let alone know everyone else's birthday. Almost always, the answer from the students will be that it is highly unlikely that two of them have the same birthday. They reason intuitively that the chance would be in the region of $\frac{40}{365}$ which is about 1 in 9. The teacher then asks for each of their birthdays and to the students' surprise, it turns out, very often, that there are at least two students with the same birthday.

Welcome to the famous Birthday Problem! It is a staple in introductory probability texts (see, for example, Seymour, 1995, p. 232) and can be stated as follows:

> What is the probability that at least two persons from n randomly chosen persons have the same birthday?

The interesting aspect of this problem is that counter to intuition, the probability of at least two persons having the same birthday in a group of 40 persons is 0.89 which is about 8 in 9. The students are surprised, the teacher may explain how the calculation of the probability works, but then the activity usually ends there because the problem cannot be repeated with the same class.

In this article, we suggest an adaptation of the birthday problem which can be repeated in class until the full effect is obtained and which can be adjusted to suit varying group sizes.

Calculating the probability

In this section, we prepare ourselves by calculating the probability that at least two persons from 40 randomly chosen persons have the same birthday. To do this, the most efficient way is to first calculate the probability of the complementary event, which is that every person has a different birthday.

The first (random) person has 365 possibilities (days in a year). The second person has 364 possibilities different from the first person if both of them have different birthdays. Thus, the probability that the first two persons have different birthdays is

$$\frac{365}{365} \times \frac{364}{365} = \frac{364}{365}$$

The third person has 363 possibilities different from the first and second persons if all three of them have different birthdays. Thus, the probability that the first three persons have different birthdays is

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} = 0.991796$$

Let us do one more. The fourth person has 362 possibilities different from the first three persons if all four of them have different birthdays. Thus, the probability that the first four persons have different birthdays is

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} = 0.983644$$

We continue in this manner until we obtain the probability that 40 persons having different birthdays as

$$\frac{365}{365} \times \frac{364}{365} \times \ldots \times \frac{326}{365} = 0.108768$$

The final probability is a surprisingly low 0.108768. We seem to be multiplying numbers which are close to 1, i.e.,

$$\frac{364}{365}, \frac{363}{365}, \ldots \frac{326}{365}$$

and are surprised that the final product is much closer to 0. The reason is that the first few products stick fairly close to 1, but then move quickly down as more (and not necessarily many more) products are made. The graph in Figure 1 below shows the probability of non-coincidence as the group size increases, showing the trend towards 0.

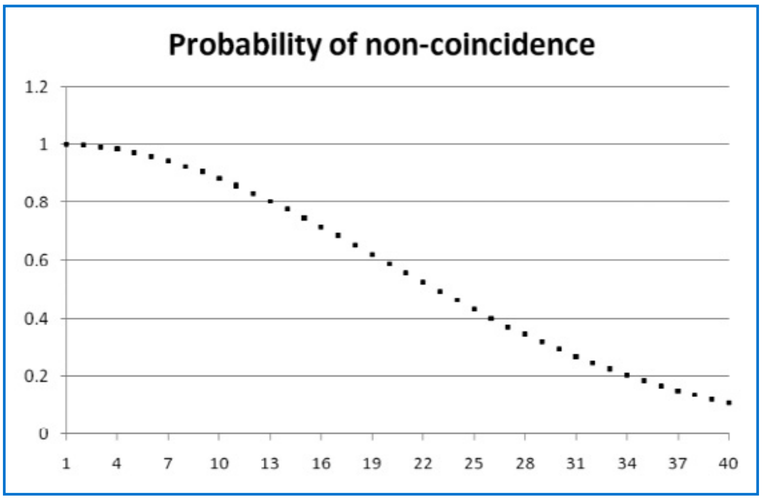

Figure 1

Finally, the probability that at least two persons from 40 randomly chosen persons have the same birthday

= 1 – probability that every person has a different birthday

= 1 – 0.108768

= 0.891232.

Note: In small classes of 15 or so, get the students to write down not only their own birthday, but also the birthday of one other person in their family. This way the 'group size' can be doubled, with a corresponding increase in the probability of a coincidence of birthdays. Strictly speaking, the calculation of this probability is not exactly as above because the students will certainly not write the birthday of a family member if it is exactly the same as hers!)

Adapting the problem

The Birthday Problem can only be used once for each group of persons because the birthdays of each person is fixed and once revealed, there is no longer any 'chance' involved. The adaptation, which circumvents this restriction, is as follows.

1. Ask the class: "If each of you thinks of a number (integer) between 1 and 365 inclusive, what is the chance that two of you will think of the same number? Is it a high chance or a low chance?" Expect the students to say a low chance.
2. Challenge the students by saying that you think otherwise.
3. Ask each student in the class to think of a number (integer) between 1 and 365 inclusive and not to reveal the number until told to do so.
4. When everyone is ready, tell the class that when you point to the student, he is to reveal his number. The other students will listen carefully to the stated number and if it matches any one's number, she is to raise her hand. The reason for this is that when two numbers match, the event has occurred and there is no need to check for the other numbers. There will then be time to try again with different numbers.
5. Continue for a few times until the point that the probability is indeed high is made.
6. You may now discuss either or both of the following.
   (a) How probability is often counter-intuitive. This can lead to discussions on why people gamble when actually they lose most of the time.
   (b) Calculate the probability of two persons thinking of the same number.

Certainly, the greatest number can be changed according to the number of persons in the group. For a group of 20, one can tease by starting with a maximum of 100, and after 'winning' a few times, allow for an increase to 150. The probability of the teacher 'winning' is still a healthy 0.73431.

An Excel program for quick computation

Recently, I tried a maximum of 150 for a class of 17 students and found myself 'losing' more than I ever did in previous demonstrations. It is quite well known that the probability that two persons have the same birthday in a group of 23 persons is about $\frac{1}{2}$. Since that translates to a maximum of 365 for a group of 23, I thought I was quite safe (which to me was at least a probability of 0.7) with a maximum of 150 for a class of 17. The probability turned out to be about 0.6.

To avoid the embarrassment but more importantly, to ensure that learning about probability properly occurs, I suggest setting up a simple program using a spreadsheet application, such as Excel, as described below for a quick check before any demonstration:

1. Write the maximum number in cell E1.
2. Column A will be used to record the number of persons. Write 1 in cell

A1 and 2 in cell A2. Highlight cells A1 and A2. Now move the cursor over the highlighted cells; the cursor will become a white cross. Move the white cross to the bottom right corner of the highlighted area until it becomes a black cross. Left-click, hold and drag down the column as far as you want, and then release. You will now have a series of consecutive numbers in Column A.

3. Column B will be used to record the probability of the corresponding number of persons in Column A all thinking of different numbers. Write 1 in cell B1. Write "=B1*(E$1-A1)/E$1" in cell B2. Highlight cell B2. Now move the cursor to the bottom right corner of the highlighted area until it becomes a black cross. Left-click, hold and drag down the column until it matches Column A, and then release. You will now have a series of probabilities corresponding to Column A.

4. Column C will be used to record the probability that at least two of the corresponding number of persons in Column A will think of the same number. Write "=1-B1" in cell C1. Highlight cell C1. Now move the cursor to the bottom right corner of the highlighted area until it becomes a black cross. Left-click, hold and drag down the column until it matches Column A, and then release. You will now have the required series of probabilities.

Figure 2 shows a screen-capture of the Excel program for maximum number 150.

B2 fx =B1*(E$1-A1)/E$1

	A	B	C	D	E	F
1	1	1	0		150	
2	2	0.993333	0.006667			
3	3	0.980089	0.019911			
4	4	0.960487	0.039513			
5	5	0.934874	0.065126			
6	6	0.903712	0.096288			
7	7	0.867563	0.132437			
8	8	0.827077	0.172923			
9	9	0.782966	0.217034			
10	10	0.735988	0.264012			
11	11	0.686922	0.313078			
12	12	0.636548	0.363452			
13	13	0.585624	0.414376			
14	14	0.53487	0.46513			
15	15	0.484949	0.515051			
16	16	0.436454	0.563546			
17	17	0.389899	0.610101			
18	18	0.34571	0.65429			
19	19	0.304225	0.695775			
20	20	0.26569	0.73431			

Figure 2

References

Seymour, L. (1995). *Schaum's outline of theory and problems in finite mathematics*. McGraw Hill.