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Author(s): Koay Phong Lee, Yap Sook Fwe and Berinderjeet Kaur
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SOME FINDINGS ON THE PERFORMANCE OF PRIMARY FOUR PUPILS FROM THE IPMA STUDY

Koay Phong Lee, Yap Sook Fwe & Berinderjeet Kaur
National Institute of Education
Nanyang Technological University
Singapore

Abstract

As part of the International Project on Mathematical Attainment (IPMA) study, 1027 primary four pupils from three primary schools in Singapore were tested for their mathematical performance at the end of their school year in 2002. This paper discusses their performance on some word problems and items related to number patterns and scales.

Introduction

Singapore joined the International Project on Mathematical Attainment (IPMA) in January 1999. This project is a longitudinal and international one (Burghes, 1998a). At present Brazil, China, Czech Republic, England, Finland, Greece, Hungary, Ireland, Japan, Poland, Russia, Singapore, South Africa, Ukraine, United States of America and Vietnam are participating in the project. The aim of this project is to monitor the mathematical progress of children from their first year of schooling throughout primary school. It hopes to identify and examine the various factors that affect their progress, with the ultimate aim of making recommendations at an international level for good practice in the teaching and learning of mathematics. More details about the project are available from the project’s website: http://www.intermep.org

Approximately 1000 pupils studying in three primary schools in Singapore are participating in the IPMA. The subjects are entire cohorts of pupils who were in Primary 1 in 1999. In 2002, the subjects were in Primary 4 and were administered Test 5 (Burghes, 1998b) towards the end of their school year. 1027 pupils (567 girls and 460 boys) took the Test. A thorough analysis of the performance of the pupils (Koay, Yap & Kaur; 2003) on the Test is available from the National Institute of Education’s (Singapore) library. This paper only reports on the performance of these pupils on word problems and items related to number patterns and scales in the Test.

Word Problems

There were altogether nine word problems in the Test, three were 1-step and six were 2-step word problems. These problems may be classified under Whole Numbers or Money & Measures according to the primary mathematics syllabus (Ministry of Education, 2000). Table 1 shows the test items, the context and the facility indices. The item numbers are as found in the Test.
Table 1: Performance on Word Problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>Topic</th>
<th>No. of step</th>
<th>Facility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. Mary buys two pencils costing 20¢ and 25¢. What is her change from 50¢?</td>
<td>Money</td>
<td>2-step</td>
<td>93.09</td>
</tr>
<tr>
<td>14. Tickets cost $4 each. How many can be bought for $15?</td>
<td>Money</td>
<td>1-step</td>
<td>83.06</td>
</tr>
<tr>
<td>15. 20 cards are shared out equally among 5 children. How many cards does each child have?</td>
<td>Whole number</td>
<td>1-step</td>
<td>96.69</td>
</tr>
<tr>
<td>17. Peter thinks of a number. He multiplies it by 3, takes away 2 and gets 25. What was his number?</td>
<td>Whole number</td>
<td>2-step</td>
<td>69.43</td>
</tr>
<tr>
<td>18. A woman has $100. She earns $50 more and spends $70. How much does she have now?</td>
<td>Money</td>
<td>2-step</td>
<td>89.00</td>
</tr>
<tr>
<td>26. What is the total cost of four books at $1.15 each?</td>
<td>Money</td>
<td>1-step</td>
<td>89.58</td>
</tr>
<tr>
<td>31. I think of a number. I double it and take away 17. The answer is 45. What was the number?</td>
<td>Whole number</td>
<td>2-step</td>
<td>59.10</td>
</tr>
<tr>
<td>48. Pencils cost 15¢ each.</td>
<td>Money</td>
<td>2-step</td>
<td>47.42</td>
</tr>
<tr>
<td>(a) How many can be bought for $2?</td>
<td></td>
<td></td>
<td>48.10</td>
</tr>
<tr>
<td>(b) How much change will there be?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49. 6 tickets cost $2.10. What is the cost of 13 tickets?</td>
<td>Money</td>
<td>2-step</td>
<td>52.87</td>
</tr>
</tbody>
</table>

As shown in Table 1, at least 89% of Primary 4 pupils were able to solve routine 1-step arithmetic word problems. About 83% of the pupils were able to deal with remainder meaningfully. For example, they knew that the remainder had to be discarded in Item 14 so that the answer would make sense. The 2-step word problems that they found difficult are:

- Problems involving ‘undoing’ of operations (Item 17 and Item 31),
- Problems involving dollars and cents (Item 48 and Item 49)

Problems involving ‘undoing’ of operations

Item 17 and Item 31 are problems that require ‘undoing’ of operations in the solution process. These may be considered as non-routine problems for Primary 4 pupils. To solve Item 17, pupils need to work backwards starting from 25, undoing subtraction before undoing the multiplication.

![Diagram showing undoing operations]

 Undo the operations

Although these pupils have mastered the computational skills (minimum F.I. for items on addition/subtraction of two 2-digit numbers and items on multiplication/division within...
the multiplication tables were 91.33% and 94.16% respectively), at least 31% of the pupils were unable to ‘undo’ the multiplication and subtraction correctly. Some pupils did not work backwards and ‘undo’ the operations as shown in the following example. The pupil replaced ‘it’ in the problem statement with the number 25 and then performed the operations in the sequence given, that is, multiply by 3 then subtract 2.

There were others who did not work backwards but attempted to ‘undo’ one of the operations as illustrated below. This pupil started with 25, multiplied it by 3 and then attempted to ‘undo’ take away 2 to get the answer 77.

Pupil performance dips for Item 31. About 48% of the pupils were not able to answer this item correctly. The lower facility index for Item 31 may be attributed to the following:

- the larger numbers involved and,
- the use of the word ‘double’ instead of ‘multiply’.

The use of larger numbers increases the chance of computational errors and the use of the word ‘double’ prevents pupils from ‘undoing’ the operation. Apparently, it is easier for pupils to use division to ‘undo’ multiplication than to halve to ‘undo’ double.

Performance in these two items illustrates the lack of competence of P4 pupils to work backwards and ‘undo’ operations to solve problems. In Item 31, pupils with low mathematical achievement (as measured by the total score in the Test) were found to have a tendency to stop after one operation, giving incorrect responses such as 28 (Pupil A) and 62 (Pupil B), while pupils with higher mathematical achievement attempted to perform two operations, at least one of which was incorrect. Consequently, answers such as 124 (Pupil C) and 14 (Pupil D) were found most frequently among the incorrect responses of these pupils (see Figure 1).
Figure 1: Samples of Pupil Responses to Item 31

It appears that solving such problems has not been part of the mathematics curriculum for many of the pupils in the cohort. Moreover, very few pupils were found to have worked forward to verify that their solution satisfied the given problem situations and made sense as illustrated by the work of Pupil E below (Figure 2).

Figure 2: Checking Solution (Pupil E)

Problems involving dollars and cents.

Item 48 and Item 49 require pupils to compute in dollars and cents. Only about 48% of the Primary 4 pupils were able to solve Item 48 correctly. This relatively poor performance may be due to the fact that the solution to Part (a) of Item 48 requires pupils to divide 200 by 15 or 2 by 0.15, both of these algorithms are not taught to pupils in Primary 4. Moreover, any incorrect answer in Part (a) would lead to an incorrect answer in Part (b). Low achievers resorted to guessing as shown by the great variety of incorrect responses. Pupils with high mathematical attainment tended to commit computational errors and common incorrect responses were 12 and 20¢ for Part (a) and Part (b) respectively. This reveals the lack of reflection after problem solving among the pupils.

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Had the pupils given a thought to their solutions, they would have realized that 20¢ is more than 15¢, hence their answers to Part (a) and Part (b) were incorrect.

About 53% of the pupils were able to answer Item 49 correctly. The solution required the understanding of rate, a concept that the pupils would only be taught the following year i.e. Primary 5. However, it appears that some pupils did have informal knowledge of rate as shown in Figure 3. Pupil F used the unitary approach while Pupil G used an alternate method that revealed a greater number sense. Pupil F divided $2.10 by 6 to find the unit price and then multiplied the quotient by 13 to find the answer. Pupil G found the unit price and then added the price to twice the price for 6 tickets. He did not have to multiply a decimal by a 2-digit number, a skill not found in Primary 4 mathematics syllabus (the skill is taught in Primary 5).

Even though there were a great variety of incorrect responses, the two common causes of error were

- incorrect execution of division and/or multiplication in the unitary approach leading to responses such as 4.45, 45.50 or 455;
- ignoring the 6 and multiplying 2.10 by 13 leading to responses such as 27.30 or 8.40 if multiplied incorrectly. For example

\[
\begin{align*}
\text{Unitary method (Pupil F)} & \quad \text{Alternate method (Pupil G)} \\
2.10 & \times 13 \\
6.30 & + 2.10 \\
8.40 & \\
\end{align*}
\]

Again, if pupils had checked their calculations and verified the accuracy and reasonableness of their answers, they would have discovered their error. For example, they can estimate the cost of 12 tickets as $4.20 hence infer that answers such as 45.50, 455, 27.30 and 8.40 are not reasonable.

**Number Patterns**

There were four items (with eight questions) on number sequence in the Test. Six of the number sequences involve additive relationships while the other two involve
multiplicative relationships. Table 2 shows the number sequences found in the Test and their respective facility indices.

Table 2: Performance on Number Sequence Items

<table>
<thead>
<tr>
<th>Item</th>
<th>Number sequence</th>
<th>Facility Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>7a</td>
<td>3, 6, 9, 12, _____</td>
<td>98.54</td>
</tr>
<tr>
<td></td>
<td>20, 18, 16, 14, _____</td>
<td>98.05</td>
</tr>
<tr>
<td></td>
<td>2, 6, 10, 14, _____</td>
<td>96.04</td>
</tr>
<tr>
<td>11a</td>
<td>31, 37, 43, _____, _____</td>
<td>90.75</td>
</tr>
<tr>
<td></td>
<td>_____, 12, 19, 26, _____</td>
<td>89.68</td>
</tr>
<tr>
<td></td>
<td>3, 9, 27, _______</td>
<td>54.43</td>
</tr>
<tr>
<td>20a</td>
<td>312, 316, 321, 327, _____, _____</td>
<td>72.54</td>
</tr>
<tr>
<td></td>
<td>Write down the rule.</td>
<td>38.56</td>
</tr>
<tr>
<td>24</td>
<td>A supermarket sells five different sized bottles of drink. The Super Giant size follows the same content pattern as the four smaller bottles. How much drink does the Super Giant size contain?</td>
<td>44.99</td>
</tr>
</tbody>
</table>

It appears that four task characteristics that may affect pupil performance are:
- size of the numbers
- position of the missing numbers
- the relationship between the numbers (additive or multiplicative)
- context

The larger the numbers used in the sequence, the more difficult it is for the pupils to find the missing numbers in the sequence. For the number sequences involving additive relationships (Item 7, Items 11a and b, Item 20), pupil performance dropped considerably when 3-digit numbers are used in the sequence. In addition, pupils have great difficulties writing down the rule used to find the missing numbers in the sequence as shown by the low facility index for Item 20 b.

Table 3 shows the facility indices of Item 11a and Item 11b over the years. Comparing the pupil performance in Items 11a and Item 11b over the grade levels, it seems that the effect of the position of the missing number in the sequence diminishes over the years.
The number sequences in Item 11c and Item 24 involve multiplicative relationships. Pupils performed rather badly in these two items. Only 54.43% and 44.99% of the Primary 4 pupils were able to find the missing numbers correctly in these two items. Pupils would have learnt how to multiply a number by two and three and yet they had great difficulty applying the multiplication skills to complete the sequence. Their performance worsened when a number sequence is placed in context as in Item 24. The language used in the problem might be a contributing factor. Pupils might not understand the phrase such as the same content pattern. The common incorrect responses for Item 24 were 1000 ml (i.e. 800 + 200) and 1500 ml (i.e. sum of all the given values).

**Scale**

Table 4 shows the two items in the Test that assess pupils’ ability to read and use scales.

**Table 4: Performance on Items Involving Scales**

<table>
<thead>
<tr>
<th>Item</th>
<th>Scale</th>
<th>Facility indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>To which number is the arrow pointing?</td>
<td>78.29</td>
</tr>
<tr>
<td>40</td>
<td>Monthly car sales in 1990 are shown below. Each car represents 50 000 cars.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Jan</td>
<td>How many cars were sold in February?</td>
</tr>
<tr>
<td></td>
<td>Feb</td>
<td>In what month were car sales lowest?</td>
</tr>
<tr>
<td></td>
<td>Mar</td>
<td>How many cars were sold in that month?</td>
</tr>
<tr>
<td></td>
<td>Apr</td>
<td></td>
</tr>
<tr>
<td></td>
<td>May</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Jun</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Jul</td>
<td></td>
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<td>Aug</td>
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<td></td>
<td>Sep</td>
<td></td>
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<tr>
<td></td>
<td>Oct</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nov</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dec</td>
<td></td>
</tr>
</tbody>
</table>
Even though pupil performance in Item 19 has steadily improved over the years, from 33% in the year 2000, 63.76% in 2001 to 78.27% in 2002, more than 21% of the pupils still could not interpret the value of each marking on the scale. Some of them ignored the fact that there are only 4 markings between 600 and 700 on the scale; they counted each marking as one and gave the answer as 602. Others interpreted each marking as representing 10 and arrived at the incorrect answer of 620. A quick check of their answers by counting on, for example in the latter case, 610, 620, 630, 640 and 650 would have led them to see that their strategies were inappropriate.

Item 40, comprising of three parts, concerns a pictograph with scale. More than 97% of the Primary 4 pupils were able to identify the month with the lowest car sale by directly reading the graph. Performance of the pupils deteriorated when computation was required. About 83% of the pupils were able to interpret correctly the complete icons as required in Part (a) while only about 67% of the pupils were able to interpret correctly incomplete icons as required in Part (c). Besides computational errors, the most common incorrect responses for Part (a) and (c) are 4 and $\frac{11}{2}$ respectively, particularly amongst the pupils with low mathematical attainment. These pupils counted the number of icons found in the graph and ignored the scale given. Some pupils ‘invented’ ways to cope with the incomplete unit. The figure below illustrates how Pupil H coped with an incomplete unit. He first drew the missing half of the icon to make one whole unit and obtained two complete units for December sales. Multiplying 50 000 by 2, he got 100 000. However, he committed encoding error and wrote 10 000 in the response box.

Fig.4: Pupil H’s responses
Conclusions and Implications

This article provides a snapshot of the mathematical attainment of Primary 4 pupils in solving arithmetic word problems, completing number sequence and interpreting scales as revealed by their performance in the Test developed for IPMA.

Generally, the majority of Primary 4 pupils are able to solve routine 1 and 2-step arithmetic word problems. Pupils did not perform as well in word problems involving skills outside their mathematics textbooks and workbooks. Although working backwards is one of the problem solving heuristics recommended in the syllabus and mathematical thinking tasks involving the undoing of not more than two operations can be found in the teacher’s guide (e.g., TG 3A, 1999, p.64), not many Primary 4 pupils are able to undo two operations to determine the original number. Further examination of the textbooks, workbooks and teacher’s guides reveals the omission of the tasks similar to Item 17 and Item 31 that require the undoing of multiplication / division.

The results of the Test also indicate that many Primary 4 pupils were unable to solve 2-step word problems involving dollars and cents. This can be attributed to the fact that multiplication of a number with 2-decimal places by a 2-digit number is not in the Primary 4 mathematics syllabus. Many pupils who attempted to use this algorithm committed computational errors. However, some pupils were able to use proportional reasoning and avoid the algorithm. These pupils apparently have better number sense.

The results of the Test also reveal that to look for a pattern to complete a number sequence was not a difficult task for most Primary 4 pupils if the pattern is additive. Many pupils were unable to cope with sequences that are multiplicative. Such sequences were unfamiliar to the Primary 4 pupils. In addition, many of the pupils had great difficulty in explaining the rule used in a sequence. Their ability in communicating their reasoning seemed to lag behind the development of mathematical concepts and skills. Mathematical communication is an important dimension of classroom learning as well as assessment of mathematical understanding. Hence, teachers should place greater emphasis in reasoning, explaining and sense making than what is currently being practiced in class.

Performance in items involving scales indicates that there were many Primary 4 pupils who still have not mastered the skills of reading and using scales. Reading scales is introduced in the reading of pictographs in Primary 2, measurement in compound units in Primary 3 and the reading of bar graphs in Primary 4. Pictographs involving incomplete icons are excluded in Primary 2 syllabus and not introduced in Primary 3 or Primary 4 syllabus. In addition, in the reading of scales in Primary 3, only scales when 1 large unit is divided into 10 equal parts or 20 equal parts are involved. That may explain why pupils had difficulty interpreting the given scales in the Test.

An examination of the answers and written working of the pupils reveal the connection between mathematical knowledge, solution strategies and self-monitoring. The majority of pupils have mastered mathematical concepts and skills covered in the primary mathematics syllabus for Primary 4 but have not developed metacognitive skills, one of
the five components found in the framework as shown in Figure 5 (Ministry of Education, 2000) of the mathematics curriculum in Singapore. When performing a mathematical task, many pupils neither review their progress towards the answer nor try to check their calculations while they work. They seldom attempt to verify the accuracy and reasonableness of their answers. Very often, they are oblivious to the fact that they are stucked and failed to recognize their nonsensical answers.

To promote metacognitive thinking among pupils, teachers need to investigate the strategies pupils used to tackle problems, including non-routine problems, so that they can help pupils develop metacognitive skills that would enable them to monitor their progress at different stages of the solution process. To improve, if not maintain the mathematical attainment of primary school pupils in Singapore, teachers need to consciously consider ALL the components found in the framework when planning instructions. Over-emphasis on one or more components and negligent in one or the other will not improve the learning opportunities of ALL pupils.

References


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