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Representations of problem solving in mathematics textbooks in China, Singapore and US mathematics textbooks: A comparative study

Zhu Yan

Introduction
Over the last two decades, the role of textbooks in the teaching and learning of mathematics has received increasing attention from the international mathematics education community. In particular, researchers have considered textbooks as a key factor to explain the reasons for students’ different performance revealed in cross-national comparisons. However, compared to other research areas in mathematics education, studies focusing on textbooks are still inadequate. Many researchers have called for more research on textbooks (e.g., Fan & Kaeley, 2000; Graybeal, 1988; Love & Fimm, 1996; Sosniak & Stodolsky, 1993; Stodolsky, 1989).

This study analyzed mathematics textbooks in China, Singapore, and the United States, with particular attention to problem solving, which has occupied a central place in mathematics teaching and learning since the 1980s. The general research question of the study is “how problem solving is represented in three series of mathematics textbooks at the lower secondary level in China, Singapore, and the United States?” More specifically, the study investigated how problem types and problem solving strategies were represented in the mathematics textbooks. By doing so, the study is intended to provide a relatively comprehensive picture about how textbooks represent the ideas of problem solving in mathematics across the three educational systems and therefore explore possible ways to improve the representation of problem solving in mathematics textbooks.

Methods
This study defined a problem as “a situation that requires a decision and/or answer”; it was believed that such a definition was more meaningful as well as operational for textbook analysis study. Based on this definition, a general conceptual framework was established, including seven main classifications of problem types:

- Routine problems versus Non-routine problems;
- Traditional problems versus Non-traditional problems;
- Open-ended problems versus Closed-ended problems;
- Application problems versus Non-application problems;
- Single-step problems versus Multiple-step problems;
- Sufficient data problems, Excessive data problems, and Insufficient data problems; and
- Problems in pure mathematical form, Problems in verbal form, Problems in visual form, and Problems in combined form.

and two levels of problem solving strategies: general strategies, which were analyzed using Pólya’s four-stage problem solving model, and specific heuristics, including “acting it out”, “looking for a pattern”, “working backwards”, to just name a few.

A total of nine textbooks were examined in the study, five from China, two from Singapore, and two from the United States, listed as follows:


Content analysis was the main research method of the study. All problems in the textbooks and all solutions to the solved text problems were examined and coded based on the established framework. After that, the coding results were analyzed using both quantitative and qualitative methods.

Results and Discussions
The study found that the US textbooks provided many more problems than the Asian books in terms of the total number (China: 4850, Singapore: 991 4, US: 13286). All the textbooks presented problems in a variety of ways. Routine problems and traditional problems comprised the majority of problems in all the three series. In particular, the percentages of non-routine problems and non-traditional problems in the books were lower than 4%. Moreover, most of the problems in the textbooks were not contextualized in real world situations.

It was found that the US textbooks provided more types of non-traditional problems. In contrast, both Asian series did not have journal writing tasks and the Chinese series further did not have project problems. However, the problems in the US textbooks were less challenging than those in the Asian books, in terms of the number of steps involved in problem solutions.

More than 90% of problems in all the examined textbooks were close-ended problems. The result appears to be
consistent with those revealed on students’ performance on open-ended problems, as researchers found when solving open-ended problems, the majority of students only provided one answer (e.g., Cai, 1995).

In addition, almost all the problems in the textbooks were supplied with just enough information. Consistently, research on students’ performance in problem solving has showed that many students attempted to use all the information presented in problems in their solutions no matter the information was necessary or not for the solutions (e.g., Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980a).

On the representation of general strategies, the study found that the problem solutions presented in the two Asian series in most cases only covered “carrying out the plan”, whereas two-thirds of the solutions to the text problems in the US books modeled at least two problem solving stages. With respect to the particular solving stages, the study showed that the Singapore series modeled the stage of “looking back” less frequently than the other two series.

The study also found that all the series introduced a considerable number of specific problem solving heuristics (China: 11, Singapore: 16, US: 14). Nine of the heuristics were in common across the three series and two were only introduced in the Singapore series. However, after the introduction of the heuristics, all the series did not illustrate frequently the use of the heuristics in solving problems.

Implications and Recommendations

The study showed that there were considerable gaps between syllabus and textbooks. Therefore, it is important for policy makers, curriculum developers, textbook authors, and most importantly, teachers to realize the existing gaps and take necessary actions to overcome the gaps.

It is believed that more non-routine and non-traditional problems of various types could be added into the textbooks, as some researchers claimed such types of problems might contribute more to students’ understanding than traditional, rule-based problems (e.g., Daniel, 1999; Siemon, Virgona, & Cornielle, 2001).

US textbook developers could consider including more multiple-step problems to provide more challenges to students. In fact, the National Assessment of Educational Progress (NAEP) found that many US students actually tended to use one single operation to solve all types of problems (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980b).

In addition, more application problems need to be added into students’ textbooks. According to Schoenfeld (1992), “The mathematics learned in school has little or nothing to do with the real world a typical student understanding about the nature of math”. Moreover, adding more application problems could also be one way to increase the variety of problems.

Reference


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