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PROBLEM-POSING IN TEACHING UNIVERSITY ALGEBRA

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Abstract: Posing or raising appropriate problems is necessary and important for active and deep learning in mathematics. However, students rarely make effort to find thinking problems by themselves. Such an attitude and behavior often lead to passive learning and cause various difficulties and problems in mathematics teaching. The main objective of this paper is to explore the ways to develop students' ability to find and pose good mathematical problems and thus to promote more active learning in mathematics.

Main problems and causes

Curiosity is one of the most important inner motivation for learning, and curiosity often starts from inquiry and asking. In teaching higher mathematics, one of the main problem we often meet is that students are not active in asking questions beyond the solutions of tutorial problems. As a result, lecturers cannot get feedback promptly and students easily get bored and become passive and shallow learners. The following are some of the main causes for this phenomenon:

Don't know what to ask. This is especially the case when students have not gained enough basic knowledge, or they still do not well understand what have been taught, or have not had time to think about it. This explains why a good student generally asks more questions than a weaker student.

Don't know how to ask. Usually this is due to lack of mathematical communication skill and practice. The question itself might not be very clear and students do not know how to formulate it clearly. Furthermore, asking the right questions in the right way is a fundamental skill in mathematics inquiry, yet in itself it receives surprisingly little explicit attention in mathematics training. Students are taught little about the process of question-asking itself.

Not used to asking. For a long time, students might have been trained mainly to answer and solve the problems provided by teachers, they never thought it is necessary for them to find problems. Except those who finally do research in mathematics, most of them do not have much chance to see the importance of finding their own problems. There are also cultural background factors influencing students' habit. For example, oriental students generally are less willing to ask questions than their western counterparts.

Lack of motivation. Unlike tutorial or project work, posing problem is usually not listed as an official learning task, thus students do not find it necessary to spend much time to think about how to pose a problem.

Apprehension and worry. Some students think that their teachers dislike their asking, asking too much may get lower assessment.

Why and how?

The important roles played by posing and formulating questions in both learning and research have been realized by people from various areas.

“Science is a process of asking questions, in most cases precise, quantitative questions that allow distinctions to be drawn between alternative expansions of events.”

(Barnard, Gilbert and McGregor, 1993)

“Questions are magic. Questions have the power to turn confusion into clarity, resistance into acceptance, division into consensus, and the frustration of not knowing what to say into the satisfaction of having said it”.

(Deep and Sussmann, 1993)

“He who seeks for methods without having a definite problem in mind seeks for the most part in vain”

Hilbert, D.

“As long as a branch of science offers an abundance of problems, so long it is alive; a lack of problems foreshadows extinction or the cessation of independent development.”

Hilbert, D

“The formulation of a problem is often more essential than its solution, which may be a matter of mathematical or experimental skill. To raise new questions, new possibilities, to regard old questions from a new angle, requires creative imagination and marks real advance in science.”

Einstein, A.

Basically, the main roles of problem posing and formulating in promoting active learning can be summarized as follows:

Asking questions arouses curiosity and interests

Asking different questions leads to different action

Deep questions often lead to new discoveries

Finding a good question is often the key step for research in mathematics

Learning without asking is a passive learning

In order to develop students' problem-posing technique we need to teach them the basic skills and methods for doing this. The following are some basic strategies for finding mathematical problems.

Brown and Walter's method. The main strategy explored by Brown and Walter in their book entitled “The art of problem posing” is the so-called WHAT-IF-NOT scheme. It starts by observing an existing result, a equation or a sequence of numbers; then list some attributes involved and break down the attributes into more subtle ones. Then select some of the attributes and ask: what might they be if the attribute is changed? This procedure can be repeated as many times as possible.

Necessary and sufficient condition questions. In mathematics we often meet theorems of the form “If p is true then q is true”. Then one can ask: if q is true, must p be true? That is whether q is also sufficient for p to be true. For example Hilbert's bases theorem says that if R is Noetherian then so is $R[X]$, so one can ask: if $R[X]$ is Noetherian, must R be Noetherian?

Relating and linking different structures. For instance, one can ask: is every cyclic group abelian? Is every integral domain a field? Is every ideal a sub ring?

Generalizing existing results. Generalization is one of the most popular motivation in developing mathematical theory. By generalizing existing structures, notions and results one often obtains a brand new theory.

Implementation and feedback

In mathematics, finding a quality problem is generally more demanding and challenging than it sounds, it needs deep understanding and well digestion of all materials and content involved, as well as suitable training and practice. It is therefore necessary for teachers to create a learning environment for students to explore and raise their own quality problems, and hence to make learning more motivated and active. In this section we shall explain how we applied the problem-posing strategy in teaching mathematics modules at the National Institute of Education, Singapore.

The main activities involved in this program are:

Preparation. Explain to students what is a valid mathematical problem, what is a quality problem, how to find and formulate mathematical problems.

Posing problem. Each week, every student is asked to pose one problem based on the content of the previous lessons. The problem should not be chosen directly from any book, and should be formulated correctly, clearly and independently. The students are informed beforehand that their problems and performance in group discussion will be assessed and the results will form part of their continuous assessment.

Presentation. Inside each tutorial group, student present their problems. They will explain all the mathematical concepts and symbols involved in the problem, the background and expectation.

Group discussion. The main objectives of this part is to find solutions to the posed problem, with the help of group members. To make the discussion effective, each session is chaired by one student. Sometimes the posed problem is invalid, or ill formulated, then the lecturer needs to guide students to make necessary correction. When the discussion get stuck the teacher has to provide help to make it moving.

For example, the following is a problem posed by a student in the modern algebra class:

Is it true that the inverse of a group homomorphism is also a group homomorphism?

The ill-posed part in this problem is “the inverse of a homomorphism”. Generally, if a mapping is not a bijection it has no inverse. Thus the problem should be moderated to

Is it true that the inverse of a bijective group homomorphism is also a group homomorphism?

To encourage students to find quality problem teachers also need to praise those who have presented thinking and creative questions and explain why they are good problems. The following are some examples of such problems posed by students in the classes of previous years.

Can we construct a multiplicatively closed set S such that $S^{-1}R$ is isomorphic to R ?

What are the applications of Prime Avoidance Theorem?

Is $K[X]$ a local ring for any field K ?

Are there the notions maximal/prime sub modules? If they exist, are the following statements still true?

* N is prime iff M/N is a field; * N is prime iff M/N is an integral domain.

If M is an R -module, are there such things as Noetherian and Artinian modules?

Summary by lecturer. In order to ensure students make progress and learn from each session, it is crucial for lecturer to summarize each discussion promptly. By doing this students can have feedback immediately and thus improve their problem-posing ability.

We have carried out this program in the last two year in year 4 and year 5 mathematics modules. The feedback from students was quite favorable and encouraging. They felt that in order to find a satisfactory problem they have to review the lessons more carefully and broadly, they have to compare and explore the relations between different structures and concepts. And all these deepened their understanding and make the study more fun and more motivated. But some students also pointed out that posing problem once a week is a bit of too tense, they suggested more time be given to think about the new problems.

Other functions

Although the main objectives of problem-posing program is to promote active and deeper learning, it actually can be taken as a means for strengthening teaching in various ways.

First, it provides a new way for formative assessment. It is widely accepted that assessment is an important part of teaching and learning process. An effective assessment could enhance teaching and learning. The way students learn largely depends on how we assess them. There is now a trend to emphasize more on formative assessment. The problems posed by students and their contribution to the discussion could be evaluated to form part of the continuous assessment. The advantage of this assessment is that, unlike the project work or tutorial, they cannot copy others work, so it provides more reliable information. The following form has been designed to evaluate the problem posing activity.

	Maximal marks	Marks
1. Understanding	2	
2. Creative thinking	3	
3. Formulation	2	
4. Clarity	1.5	
5. Question handling	1.5	
Total	10	

Secondly, it serves as an effective way to motivate learning. It is clear that if a student is motivated by his strong and constant interests and the eager to know then he will learn more actively and more on his own initiative. Unfortunately, although some of the students seem naturally enthusiastic about learning, quite a number of them lack such enthusiasm, they need –or expect – their instructors to inspire, challenge, and stimulate them. As we have already pointed out, in this aspect, the problem-posing program definitely helps.

Thirdly, we can get feedback from students through their questions and participation in group discussion. The problem they asked reflects how much they have learnt and how well they have understood.

Concluding remarks

From what we have done, we believe that the problem posing strategy is an effective approach that really enhances mathematics teaching and encourages students to explore mathematics by

themselves and thus also strengthen their ability of problem solving. However something more need to be done to make this approach more functional.

First of all, more attention should be given to the solution of posed problems. It often occurred that students are satisfied with just posing the problems, and do not realize the importance of finding out the solutions, and they seldom take serious action to solve the problems. This was the main cause of producing trivial questions. Secondly, something has to be done to encourage more participation in discussion. Maybe, sometimes, we can reduce the scope of the problems, or start from one problem, so that everyone can prepare in advance. Thirdly, although we listed Brown and Walter's What-If-Not as an important problem posing strategy we still have not applied it in our practice; the main reason for this is that in the context of abstract and commutative algebra solving problems is much more difficult than in elementary mathematics, and thus what-if-not method demands much more time than what is available.

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