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Metacognition in Mathematical Problem Solving

Philip Wong

Abstract

Metacognition is considered by most educationists as an element necessary for many cognitive tasks. In problem solving, it has been said that possessing knowledge alone is insufficient and problem solvers need to exhibit high level cognitive skills like “self-regulation skills” (also known as metacognitive strategies) for successful problem solving.

A study on students’ metacognitive strategies was carried out with over a thousand secondary and pre-university students from 12 schools. A questionnaire adapted from Biggs (1987) was administered to students at various levels (Secondary 2, Secondary 4, Pre-University 1), from different academic tracks (General, Science, Arts) and academic streams (Special, Express, and Normal). They were required to self-report on their metacognitive beliefs; their use of metacognitive strategies in mental tasks involving memory, problem solving and comprehension; and their attitudes towards the learning of various academic subjects. 20 items from the questionnaire which were related to problem solving were categorized into four stages, namely, orientation, organisation, execution and verification and data from these items were analysed.

Some findings that emerged were:
(a) Normal stream students exhibited a lower usage of metacognitive strategies as compared to students from the Express and Special streams.
(b) Metacognitive strategies used by Normal stream students tended to be of the “surface” type.
(c) There was no significant difference in the frequency of usage of metacognitive strategies between students from different academic tracks.
(d) During the problem solving process, students spent most time on evaluation of answers rather than on monitoring their understanding.
(e) Students from different levels (Secondary 2, Secondary 4 and Pre-University) exhibited similar frequency of usage of metacognitive strategies in problem solving.

The implications of these findings on future research and development projects as well as the teaching of metacognitive strategies are discussed in the paper.

Descriptors: Metacognition, Mathematics, problem solving, learning strategies

Problem solving is a complex task involving many types of knowledge and skills. Skills in planning, monitoring and revising strategies are as important as having a large body of knowledge. It is undeniable that problem solving requires specialized knowledge such as linguistic, factual, schematic, strategic and procedural knowledge (Mayer, 1987). A number of researchers have also included metacognitive knowledge as another important factor that differentiates between the good and the average problem solver (Gagne, 1985; Lester 1982).
Previous studies on metacognition have concentrated on tasks involving reading and memory work and little work has been done with metacognition in problem solving. In mathematics, there is much interest to make students aware of metacognition and to develop their metacognitive skills. Lately researchers have begun to look at metacognitive skills in problem solving and have started to develop theoretical frameworks (Garofalo & Lester, 1985; Lester, 1985, Schoenfeld, 1985).

Metacognition is generally considered as "knowing about knowing" or what Schmitt and Newby (1986) refer to as "a body of knowledge that reflects knowledge itself". In other words, metacognition involves knowing the cognitive processes associated with an instructional task, and being able to use and monitor appropriate cognitive processes during the task. Although metacognition has been loosely defined, most psychologists (eg Brown, 1978; Flavell, 1976) consider metacognition to consist of two separate but related aspects of knowledge about cognition and regulation of cognition.

Knowledge about cognition implies that a person is knowledgeable about variables that will affect one's instructional performance in a learning situation or during an instructional process. Identifying one's weaknesses or strengths in certain topics of mathematics, realizing that one is careless in computation and tends to make computational mistakes, recognizing that one is weak in processing spatial and visual information, and to be aware of the different effects of semantic and syntactic structures (eg, vocabulary, extraneous information and order of events) on the difficulty of word problems are some examples of knowledge of cognition.

Regulation of cognition involves the type of decision behaviours exhibited in order to plan, monitor and evaluate one's action. Sternberg (1983) labels these types of behaviours as executive skills and has even proposed certain training strategies for the development of these executive skills. Although these skills are trainable, it is also believed that the regulation process is controlled by one's cognitive knowledge (Kluwe, 1987). In mathematics problem solving, for example, a student who believes that he/she tends to make computation mistakes and thus slows down and proceeds cautiously during the computation part of problem solving and re-checking the answers, is said to be exhibiting this executive skill.

This self-regulation process is important for successful problem solving. Schoenfeld (1985) after an analysis of college students' protocol of their problem solving processes, concluded that at the self-control level, the lack of monitoring and assessing the situation could lead to failure in problem solving. Despite its importance, this cognitive procedure is not clearly demonstrated by young children and college students. Garofalo and Lester (1985) in their research found that young children did not routinely analyse information provided in the problem and did not monitor progress or validate the results. College students too, were not very efficient in regulating their problem solving behaviours. Schoenfeld (1985) found that the overall quality of college students' monitoring, assessing and executive decision-making in problem solving was relatively poor.

Metacognition in Problem Solving

Using Polya's (1957) heuristic problem-solving model as a foundation, Lester and associates (Lester, 1985; Garofalo & Lester, 1985) proposed a cognitive-metacognitive framework for performance in various mathematical tasks. The framework consists of four cognitive components of orientation, organization, execution and verification. The four components correspond to Polya's four phases of problem solving of understanding, planning, carrying out the plan, and looking back. However, Lester differentiates his framework from Polya's as he believes that his "model purports to describe the categories of the cognitive component in terms of points during problem solving where metacognitive actions might occur" (Lester, 1985; p 62). The four components can be briefly described as follows:
(1) ORIENTATION: At this stage, students need to assess and understand the problem. The skills exercised at this stage would be those of comprehension; analysis of information; assessment of familiarity of problem and task difficulty and the formation of internal representation.

(2) ORGANIZATION: This involves identifying goals, then planning for the whole task and sub-tasks in order to achieve the goals and sub-goals.

(3) EXECUTION: The monitoring of behaviours exhibited in the execution of the plans falls into this category. It includes monitoring computation actions, maintaining progress towards the goal and assessing trade-off decisions between factors influencing the success of the problem-solving process.

(4) VERIFICATION: This stage involves the monitoring and evaluation of the three components of orientation, organization and the execution of the whole problem-solving process.

Each component is controlled by metacognitive decisions made by the individual and the type of decisions will depend on his/her own knowledge of metacognition. For example, in the cognitive component of orientation, an individual may want to rephrase the text in order to help him/her understand the problem situation better or if the individual believes that he/she is better at processing visual information, he/she may reorganize and represent the text information visually. Thus, an individual with better metacognitive knowledge can use his/her executive decisions for better planning, execution, and monitoring of the problem solving process and, hopefully, achieve higher success in solving problems. The depth of one's metacognitive knowledge can influence the type of strategies one uses for monitoring and regulating cognition during problem solving. In the orientation component, for example, an individual may use different types of strategies: "surface" strategy such as re-reading the problem, or “deep” strategy such as recalling old materials to link new materials found in the problem or “achieving” strategy such as analysing and representing problem information in another format.

While there are extensive studies on metacognition carried out with experts and novices, with academically disabled students (Slife, Weiss & Bell, 1985) and with young children (eg, Myers & Paris, 1978), there are insufficient studies carried out with youths from different academic backgrounds. This is important as knowledge gained in this area could provide teachers with some guidelines on what to teach to students with different academic backgrounds. There are a number of unanswered questions on the effects of academic settings on students’ metacognition. For example, do students from different grade levels exhibit different amounts of cognitive knowledge? Do students from lower grade levels exhibit less frequent use of metacognitive skills such as monitoring, planning and verifying their answers when solving mathematics problems? What type of strategies do different grade-level students employ? Are the strategies surface type, deep, or achieving ones? Do students from different streams and different academic tracks exhibit different frequency of usage of metacognitive processes?

Objectives of This Study

This study investigates the metacognitive processes used by secondary school students in mathematics. Specifically, it seeks to answer the following questions:

(1) How frequently do students employ metacognitive strategies during mathematics problem solving?

(2) Do students from different academic settings (academic stream, academic tracks and grade levels) differ in their usage of metacognitive strategies?

(3) Do students from different academic settings use different types of strategies (surface, deep or achieving strategies)?
Method

Subjects

Over 2500 students from nine secondary and four pre-university junior colleges participated in the research on learning and teaching strategies. The subjects were selected using the stratified sampling method. Within each category of schools and pre-university colleges, the schools were randomly selected and within each school, the classes of students from each stream, level and academic track, were randomly chosen. Whole classes were used in the survey and in each class, one third of the students was randomly assigned to answer the Language form questionnaire on learning strategies, another third answered the Science and Mathematics form, and the rest answered the Social Studies form.

Seven hundred and seven students answered the Science and Mathematics form. Out of this, 37 sets of data were incomplete thus leaving a sample size of 670. The 670 students came from (a) three streams, namely, Special Assistance Programme (SAP), Normal Stream and Express stream; (b) three grade levels (Secondary 2, Secondary 4, and Pre-U 1); (c) three academic tracks (Arts, Science and General).

Instrument

The instrument used is the Study Skill Questionnaire (Chang, 1988; 1989). There were three forms, each pertaining to the study of specific subject areas, namely, Language, Science and Mathematics, and Social Studies. Within each form, there were three sections in the questionnaire with the first two sections being common to all the three forms. The first two sections contained items on learning strategies, attitude towards learning and their motives for learning and they were drawn from the Learning Process Questionnaire (Biggs, 1987). The third section contained items that were specific to the content area. For example, in the Science and Mathematics form, students were asked about the frequency of usage of metacognitive strategies in solving mathematical and science problems while in the Language form students were asked about their metacognitive strategies in reading comprehension and in listening.

This study reports only on the students’ returns in the Science and Mathematics form and on the section asking students about their metacognitive strategies in problem solving. There were 20 items related to strategies used in mathematical problem solving and for the purpose of this study, the items were classified into five sections. The first four sections followed the cognitive-metacognitive framework suggested by Garofalo and Lester (1985) with four items in each component. The fifth section of items measured students’ beliefs in strategies which would help them in problem solving.

(i) ORIENTATION COMPONENT: The items here concentrated on the process of reading and understanding of the problem (eg, I analyse and try to understand the information given and draw inferences).

(ii) ORGANIZATION COMPONENT: The items in this section concentrated on the approach and the planning for execution of procedures (eg, I turn an argument over in my mind a number of times before accepting it).

(iii) EXECUTION COMPONENT: The items tried to determine how students execute the plan during problem solving (eg, I find that drawing diagrams helps me to solve problems).

(iv) VERIFICATION COMPONENT: The items were directed at finding out how frequently various strategies were used to check answers and procedures (eg, I check over my test to avoid making mistakes).

(v) BELIEFS: The items determined the beliefs students have concerning mathematics problem solving (eg, I believe there is only one best way in solving a problem).

The questionnaire had been pilot tested, validated and used in a number of research studies (Chang, 1988; 1989).
Procedure

The questionnaire required students to rate each item on a 5-point Likert scale, with a score of 5 indicating a frequently-used metacognitive strategy while a score of 1 indicated a rarely-used or never-used strategy. The questionnaire was administered to the whole class by the class teacher. Most students were able to finish answering the questionnaire within a one-period lesson. The class teacher explained some phrasing of items to students who could not understand the item.

Results

There are five sets of subscores with a maximum of 20 points per set. There is a score for each of the four problem solving components (orientation, organization, execution and verification) and one score for students’ problem-solving beliefs. Analysis of variance (ANOVA) at 0.05 level of significance was carried out using the mean scores as the dependent variable. Three separate analyses were conducted with different independent variables, namely, stream; level; and academic track. The results of each analysis are described below.

Stream

The three streams of Express, Normal and Special are applicable to secondary schools only. Data from Pre-University students were not included in the analyses.

The means of all the problem phases were found to be statistically different. In all the four phases, Normal stream students scored lower than students from the Express and the SAP stream indicating that Normal stream students had reported less frequent use of metacognitive strategies than students from SAP and Express Stream students (Figure 1). A follow-up test using Duncan’s test showed that the means of SAP students and Express students were not significantly different.

The score in the verification component was highest compared to the three other phases. The means for the three phases of orientation, organization and execution were around 12.5 while the means for the verification component were around 15.5.

Based on classification by Biggs (1987), each item in the questionnaire was classified as either surface, deep or achieving strategy. On the analysis of individual items, it was found that Normal stream students used surface strategies more often than deep or achieving strategies. For example, they reported that they used surface strategies like “I need to attend to the instructions carefully in order to get the required results” (mean = 3.57) more frequently than to deep strategies like “I analyse and try to understand the information and draw inferences” (mean = 3.03).

Figure 1: Means of each component by stream (n = 550)
**Level**

The means of each component are shown in Figure 2. Statistically, there was no difference in the frequency of usage of metacognitive strategies between students from different levels, viz. Secondary 2, Secondary 4 and Pre-U 2. Again, the means for the verification component (averaging 16.0) were higher than the means of the other phases (averaging 12.5).

**Academic Track**

The means of the four phases are shown in Figure 3 and the means were found to be statistically not different for all the three tracks. The means for the verification component were higher than the means for the other three components (averaging 13.0).

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**Figure 2: Means of each component by Level (n = 670)**

**Figure 3: Means of each component by Academic Track (n = 640)**
Beliefs

Students' problem solving beliefs were investigated through four items (17, 18, 19, and 20). Figures 4, 5, and 6 show the means of the four items for stream, level and academic track, respectively. Students from the Normal stream, from Secondary Two and from General and Arts academic tracks believed that certain surface strategies were appropriate for developing problem solving skills as shown in Item 20 where they indicated that they memorized model answers more often than the other students.

Similarly, in Item 19, more students from Secondary 2, Normal stream, academic track of Arts and General, believed that there is only one best way to solve a problem. On the other hand, the Express and SAP students, students from the General and the Arts stream, and Secondary 4 and Pre-University 2 students believed that certain deep strategies (e.g., they needed a lot of drill and practice; that it is important to be able to solve problems set in past-year examinations) are important to their problem solving abilities. This is indicated by the higher ratings in Items 17 and 18.

Figure 4: Means of Metacognitive Beliefs by Stream

Figure 5: Means of Metacognitive beliefs by Academic Track
Figure 6: Means of Metacognitive Beliefs by Level

Discussion

The results show that the mean scores for the four components are above the half-way mark of 10 indicating that students are conscious of metacognition and that they used strategies for monitoring and regulating the processes necessary for problem solving. Most students indicated that they practiced some of these metacognitive activities at least half the time when they are solving problems. Although the four components are equally important for problem solving, the students practice the verification process more frequently than the other components.

This is indicated by the high scores for all the four items in the verification component. However, the four items of the questionnaire asked students on only one aspect of the verification component. The students were asked about the accuracy of their workings and the accuracy of their answers. This aspect of verification is emphasized during classroom instruction and hence, the high scores. Unfortunately, the questionnaire is unable to give us an insight about what really happened in their monitoring process during problem solving. A better method would be to use protocol analysis, interviews and observations as additional sources of data collection.

Although the results generally showed that students do practice metacognitive activities, certain groups of the student population were not very fluent in their usage. For example, the Normal stream students scored lower when compared to the SAP and Express stream students. This could be due to the selection process when students were streamed into SAP, Express or Normal. The Normal stream students follow a five-year secondary school education programme compared to the Express and the SAP stream students who follow a four-year secondary school programme. The students entered the various streams based on the achievement scores in their Primary School Leaving Examination. Thus academic ability could have an effect on the frequency of usage of metacognitive strategies and other researchers have reported similar findings (Chang, 1989; Peterson, 1988; Slife et al, 1985). Also, the methods used in the teaching of students from different ability groups could have caused the differences in the frequency of usage of metacognitive strategies. It has been reported that high-ability students preferred less structured instruction; instructions that were inductive rather than deductive; methods which required them to self-learn (Snow & Lohman, 1984) and these teaching methods employed by teachers could have encouraged the development of
metacognitive skills. On the other hand, lower-ability students preferred instruction to be highly structured, deductive rather than inductive and more focused on content and these methods are not conducive to the development of metacognitive skills. It appears that academic ability influences metacognition in three ways: first, the lack of academic ability impedes students' knowledge of strategies; second, the lack of knowledge of metacognition leads to poor academic performance and third, the teaching styles used can discourage or encourage the development of metacognitive skills. Unfortunately, findings from this research cannot differentiate between them.

Age and years of schooling are some other factors that could influence one's knowledge and application of metacognitive strategies. Awareness of metacognitive strategies starts at a very early age. Various studies on metamemory have shown that children as young as five years old are aware of strategies for recall (see Flavell & Wellman, 1977). It is also observed that older children are better at using various strategies for recall than young children (e.g., Brown, 1978). This is also true in reading comprehension. Myers and Paris (1978) found out that 12-year-old students were more aware of the effects of various variables, such as their knowledge of content and their interests in the stories, on their comprehension than 8-year-old students. Biggs (1987) in his study, noticed that young students may have the awareness of the needs of monitoring and regulating their cognitive processes but may not have sufficient executive control over them. However, in this study there was no statistical difference in scores between students of different ages as students from Secondary 2, Secondary 4 and Pre-U 2 indicated similar frequency of usage in the four components. While the level of usage remains the same across students from different levels and academic tracks, the types of strategies used differed. The younger students, the Arts and General track students and Normal stream students tended to use more surface strategies. However, the use of surface strategy should decline as the level changed to the higher level. Similar observations were also noted by Biggs (1987). This study does support the fact that students become increasingly more aware of metacognition with increasing years of schooling.

Research Implications

Research on metacognition should be a multifaceted task using a variety of research methods, instruments and a sample with different academic backgrounds. This study is the initial phase of the research project on effectiveness of learning strategies and is based on students' self report in a questionnaire. However, the use of self-report questionnaire has its limitations. Ideally, this report should be supported with evidence from interviews and verbal protocol. These procedures would give us a better insight of the type of problem-solving and metacognitive activities used. Unfortunately, there are limitations to this procedure. This research method is time consuming and only a handful of students can be interviewed. In Singapore where students are not very vocal, gathering data using this method is problematic and students have found this method to be unnatural, mentally demanding and difficult as they find it difficult to verbally express themselves (Wong, 1990).

Teaching Implications

This study shows that students need guided instruction in the use of metacognitive strategies for problem solving. Besides, the emphasis on verification of solutions, the students reported less frequent use of monitoring strategies in planning, executing and orientation. Teachers should therefore consider incorporating strategies to help students develop metacognitive skills in these three areas. Generally, teachers do not introduce metacognition as a topic in a lesson but instead subsume the concept of metacognition within the lesson content. Thus, the students metacognition is not taught explicitly and the concept of metacognition could be lost amongst the more important subject matter. Instead, there should be conscious and direct effort by the teachers to introduce the
concept of metacognition during the lesson. Students need to be informed of what metacognition is, how it works, when it works and be provided with some examples. At present, at the teacher training level, trainee teachers are exposed to a number of lectures on this aspect. They are encouraged to use various methods to achieve this. They could incorporate some of the teaching methods, activities, and approaches suggested by Callahan & Garofalo (1987), Long (1986) and Devine (1981).

Drawing from this study and other reports, lower ability students do not use strategies for metacognition as frequently as high ability students and this deficiency could lead to poor performance. They, therefore, need extra training in order to enable them to operate at the same level as the higher ability students. Various projects on teaching students thinking skills, reading skills and learning skills have been very successful. For example, Peterson and associates (cited in Peterson, 1988) conducted an extensive project which helped fourth grade students to develop thinking skills in mathematics, and noted that low ability students benefited more from the training. She said “the thinking skills training may have provided the low ability students with the thinking skills or cognitive strategies that they did not have, but that higher ability students did have already.... Acquisition of these strategies then permitted them to learn as effectively as the higher ability students within the class” (p. 10). Attempts have been made by schools and the Ministry of Education to introduce some of these projects to students. Some examples include: the publication of a handbook, Learning Skills in Content Area, for secondary school teachers to be used for conducting workshops on learning skills, the introduction of DeBono’s CORT programme to 25 secondary schools, and training programmes to student-teachers on effective teaching and learning skills.

Conclusion

This study reports on the frequency of usage of metacognitive activities and the type of strategies used by students from different academic settings in mathematical problem solving. It is generally observed that students are aware of metacognition although students from the Normal stream seem to use strategies on metacognition less frequently. There is a declining use of surface strategies and increasing use of deep and achieving strategies as the level changed to the higher level. The results of this study warrant a need to introduce the teaching of metacognition to all students especially to low ability students.

REFERENCES


