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# Information Processing Taxonomy (IPT): An Alternative Technique for Assessing Mathematical Problem-Solving

Fong Ho Kheong

## Abstract

**This paper describes the development of the Information Processing Taxonomy (IPT) Model which models children's thinking at different hierarchical levels. The main features of the Model consist of retrieving information from external memory and primary and secondary productions of Type A and B information. These features constitute the components of different levels of thinking which could be used to confirm and describe the children's cognitive processes. Examples are shown to illustrate how the Model could be used to construct test items at different levels on the basis of these features. The Guttman procedure was applied to test the consistency of the hierarchical nature of the Model. Results show that the items constructed using the Model were scalable.**

**Descriptors: Problem-solving, information processing, ratio and proportion, assessment**

## The Background

*Some models for assessing mathematical abilities*

One technique which mathematics educators have used for assessment in mathematical abilities is the classical **psychometric** approach. The approach was sought as a guide towards understanding intelligence by analysing children's ability to solve mathematical problems. Different sets of factors were proposed to account for the mental abilities of children (Barnett, 1975; Krutetskii, 1976; Thurstone, 1938). This results in applying step-wise regression analysis to predict pupils' intelligence or problem-solving ability in the sixties.

One widely used technique for evaluating the quality of a student's output is **Bloom's taxonomy** (Bloom et al., 1956). It was designed for teachers and evaluators to arrange questions in hierarchical levels of difficulty. Six levels of responses were outlined in Bloom's taxonomy: Knowledge, Comprehension, Application, Analysis, Synthesis and Evaluation. The taxonomy helps to guide the selection of items for a test and assesses the ability of the students according to the categories (Bloom's classifications) of questions they have designed.

Another technique of assessing the quality of students' responses to items is the **SOLO** (the Structure of the Observed Learning Outcome) **taxonomy** hypothesised by Biggs and Collis (1982). While Piaget's theory described the hypothetical cognitive structure in terms of different stages, Biggs and Collis outlined the hierarchical levels of students' responses to items. The five SOLO levels of responses are Prestructural, Unistructural, Multistructural, Relational and Extended Abstract. Each level of response seems to correspond to Piaget's developmental stage (Pre-operational, Early Concrete, Middle Concrete, Concrete Generalisation and Formal Operation).

### *Psychological theories relevant to the present technique of assessment*

**The information processing system**

While Piaget's theory of cognitive development and the psychometric theories have been theorised to explain the intellectual abilities of human beings, the information processing theory advocated by Newell, Shaw and Simon (1960), Newell and Simon (1972) and Simon (1980) is viewed as looking into human intelligence and capabilities from a different perspective.

According to the information processing theory (Newell and Simon, 1972), there are basically two main components in the information processing system (IPS) which characterise the nature of human memories, namely the short-term memory (STM) and the long-term memory (LTM). Other theorists expanded the IPS to include other components such as the working memory (WM) and external memory (EM) (Case, 1980; Mayer, 1983; Nason and Redden, 1984).

The **STM** is part of the information processing system in which coded information from the external source is operated and stored temporarily for immediate use. Besides receiving in-coming information from the external senses, the STM also receives information from the LTM for processing information. The information received at the STM becomes the current state of knowledge of the information processing system. The STM is often referred to as the working memory in which the information must be encoded in a form which is recognisable by the information stored in the LTM.

Mayer (1983) indicated that the **LTM** is an organised set of information in which each piece of information must be found by following a search path. While the information stored in the STM fades away with time, the LTM is able to store information with minimum loss. The extent to which concepts or processes can be utilised depends on the way the concepts and processes are structured in the LTM (Nason and Redden, 1984).

**Production systems**

Using the concept of programming, Newell and Simon model human learning and problem-solving processes in terms of production systems. A production system is a set of related productions. Each production specifies the conditions in which an action should occur. The set of productions is viewed as the body of knowledge which is represented in the LTM of the human brain in the form of condition-action rules. For example, when a person sees a red traffic light, the brakes are applied to stop the car. An action is said to be executed when the brakes are applied in reaction to the condition of seeing a red traffic light.

The production memory contains lists of condition-action rules. They may or may not be organised, but rules might be grouped together in terms of the goal they help to satisfy.

The production rules state that when any form of information in the working memory matches the "condition" statements of production, the production will evoke the "action" statement. Using the previous example of a traffic light, if the content of the working memory is "the traffic light is red" the activation rule will elicit the action "apply the brakes to stop the car".

## An Information Processing Taxonomy (IPT) Model for Assessing Mathematical Abilities

### *Critique of the existing models for assessing mathematical abilities*

Although both taxonomies (Bloom and SOLO) have proven to be useful, they contain some short-comings in the actual implementation. Biggs and Collis (1982) commented that Bloom's taxonomy is intended to guide the selection of items and to seek adequate coverage for a test, rather than to evaluate the quality of a student's response to a particular item. According to them it is difficult to apply the taxonomy meaningfully to open-ended responses. Furthermore, the taxonomy does not provide adequate guidelines to devise items to draw out levels of response much beyond comprehension (level). Bloom's levels are set in advance and the teachers apply

them to construct questions, but the levels do not arise in a natural manner. The SOLO taxonomy also has its drawbacks. In this model, three kinds of data are used: (1) irrelevant or inappropriate data, (2) related and given data in display, and (3) related and hypothetical, not given, data. Although the model has identified all the different types of data, it could not explain how Types 2 and 3 were generated and represented. In other words, the model is limited when it comes to explaining the cognitive processes which generate those related and not given data. Secondly, the model lumps together all the related data and given data as a single group of information. They are used and represented freely without acknowledging any distinction concerning the availability to retrieve them.

The Bloom and SOLO taxonomies are useful to a certain extent for the construction of items to elicit pupils' responses at different levels. However, the two taxonomies cannot provide a more rigorous explanation of the cognitive processes of the whole procedure of solving a problem leading to its solution. It seems that a more comprehensive model should be considered for future development in assessing mathematical abilities.

A technique reported in this paper could be used to describe the quality of students' abilities to solve mathematics problems on the basis of the information processing theory. Span and Overtoon-Corsmit (1986) pointed out that the processing of information takes place at different levels. Wagner and Sternberg (1984) viewed intelligence as deriving from the ways in which people mentally represent and process information. Span and Overtoon-Corsmit added that information processing may be applied to text comprehension or problem-solving. Nason and Redden (1984) stated that two factors which can influence human learning are the information retrieved from the long-term memory and the information that is currently being processed at a particular point in time. Simon (1987) described problem-solving processes through a sequence of states of knowledge. Each state is being transformed to another state by applying mental operators to it. These summarised statements of cognitive psychologists have implications on the present IPT Model for assessing mathematical abilities.

### *Description of the Hypothetical IPT Model*

The cognitive psychologists' views, the information processing theory and the production system provide the author with the basis to hypothesise a scheme for assessing students' mathematical problem-solving ability. The IPT Model consists of two general features: (a) a set of information processing features such as the external source (ES), STM and LTM, and (b) an operating system comprising perception through stimulus, retrieving information from ES, and operations based on primary and secondary productions. The Model is described in Table 1 in terms of the hierarchical levels and the different levels of processing information in solving mathematical problems.

The first column in the table gives the order of the hierarchical levels of problem difficulties; level one being the least difficult and level five the most difficult. The second column describes the characteristic features of the model and operating systems involved in problem-solving. The former includes external sources (information), short-term memory/working memory, long-term memory, and the Type A and Type B information. The latter comprises the production system and perception of information from external source. Each characteristic is described briefly below:

**External Source (Information):** This refers to the data or information which can be elicited from a question. Each question is viewed as consisting of pieces of information which could be in the form of facts, numerals, procedures, etc.

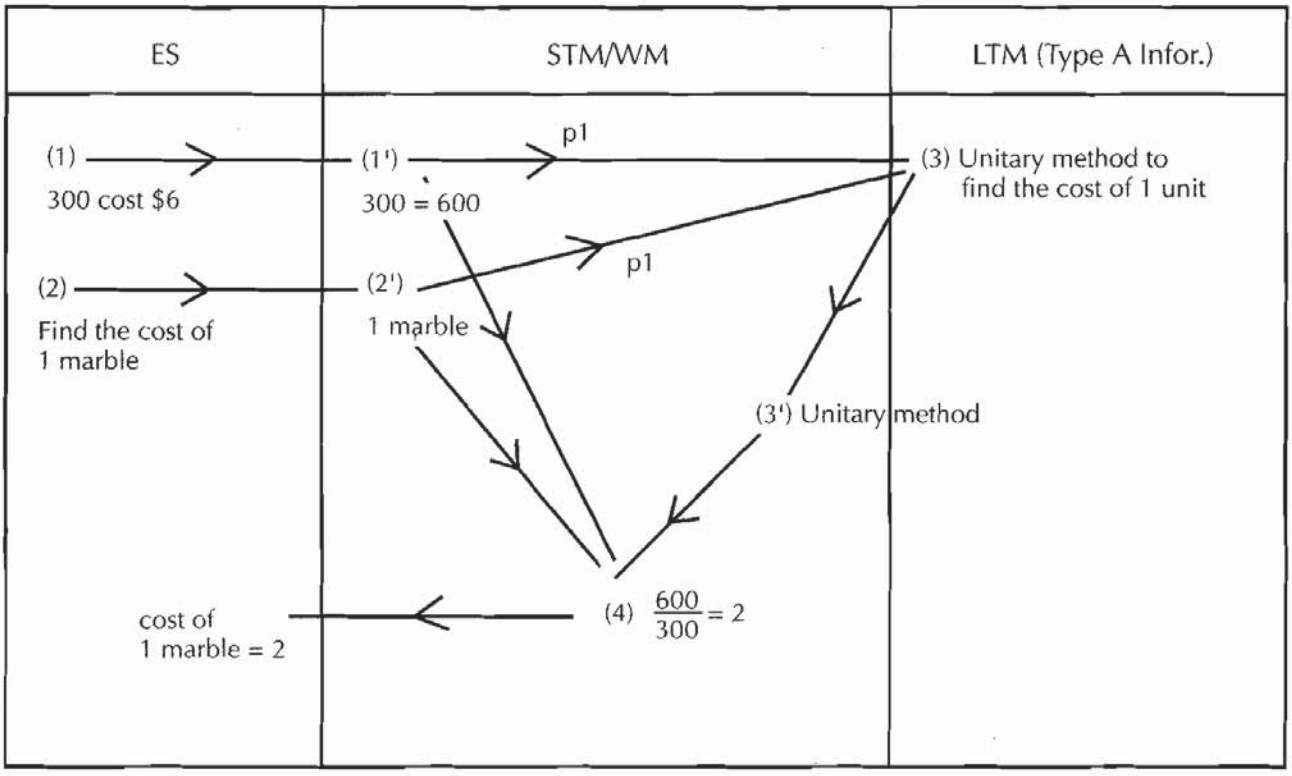
For example, if a question is given as, "The ratio of the three sides of a triangle is 1:2:2. If the perimeter of the triangle is 60 cm, find the length of the shortest side of the triangle.", then one piece of information is the ratio units "1:2:2". The question also comprises other information which could be in many different forms.

**Short-Term Memory/Working Memory (STM/WM):** All information, whether it comes

Table 1: Description of the Characteristic Features and Operating Systems of the Information-Processing Taxonomy (IPT) Model Which Consists of Five Hierarchical Levels

| Hierarchical Levels | Characteristic Features & Operating Systems Involved in Problem-Solving |
|---|---|
| One | ES, STM/WM; Perception through Stimulus, Retrieving information from ES. (WM = Working Memory) |
| Two | ES, STM/WM, LTM (Type A Information); Perception through Stimulus, Retrieving information from ES and Primary Production System. |
| Three | ES, STM/WM, LTM (Type A & B Information); Perception through Stimulus, Retrieving information from ES and Primary Production System. |
| Four | ES, STM/WM, LTM (Type A Information); Perception through Stimulus, Retrieving information from ES, Primary & Secondary Production System. |
| Five | ES, STM/WM, LTM (Type A & Type B Information); Perception through Stimulus, Retrieving information from ES, Primary & Secondary Production System |

Figure 1: Representation of Cognitive Processes for Solving Example 1



from the external sources or the long-term memory, is to be processed in the STM/WM. This part of the human brain has a limited capacity and is only able to hold certain chunks of information at one time. An example is given below to show the interaction of information at the STM.

**Example 1**
Jane had 300 marbles which cost $6.
Find the cost of one marble.

**Answer:**

$$\begin{aligned} 300 \text{ marbles} &= \$6 \\ &= 600 \text{ cents} \\ 1 \text{ marble} &= \frac{600}{300} \\ &= 2 \text{ cents} \end{aligned}$$

Figure 1 shows that two sources of information are operated at the STM level. (1'), 300 cost $6, and (2'), find the cost of 1 marble, are pieces of information retrieved from the ES. Information (3'), the unitary method, is retrieved from the LTM. They are operated at the STM to produce (4), 2 cents.

**Long-Term Memory:** Two types of information are considered to be stored in the long-term memory. Information which is related to the recent content area of study and information which is often rehearsed is classified as Type A information. Information of related topics is classified as Type B information. Type A information is assumed to be readily retrieved as compared to Type B information. Thus, the latter is placed at a higher hierarchical level than the former category. It is also assumed that information stored in the long-term memory fades away with time, especially when the information stored in the LTM is not frequently retrieved. Example 2 below shows the two different types of information which are stored in the LTM. They are retrieved and operated at the STM.

**Example 2**
The ratio of the three sides of a triangle is 1:2:2. If the perimeter of the triangle is 60 cm, find the length of the shortest side of the triangle.

**Answer:**

$$\begin{aligned} \text{Ratio of sides} &= 1:2:2 \\ \text{No. of parts of the sides} &= 1+2+2 \\ &= 5 \\ 5 \text{ parts} &= 60 \\ 1 \text{ part} &= \frac{60}{5} \\ &= 12 \text{ cm} \\ \text{Length of the shortest side} &= 12 \text{ cm} \end{aligned}$$

In this question, the topic being assessed is ratio and proportion. Hence any "skills" related to ratio and proportion are classified as Type A information. Notice that in Figure 2, converting ratio into parts and adding the parts and the unitary method are thus classified as Type A information. The statement "sum of 3 sides of triangle = perimeter" which is not a "skill" directly related to the topic ratio and proportion is classified as Type B information. These pieces of information described above are retrieved from the LTM.

**The Perception of Information from External Source:** Information from a question is first perceived and encoded in the STM. This is the first stage of the operating system in which the acquired information is processed in the STM.

For example, we suppose that a question statement is given as "The ratio of the three sides of a triangle is 1:2:2". If a responded statement is "No. of parts of the sides = 1+2+2 = 5" then it can be inferred that the problem-solver has retrieved the pieces of information 1:2:2 from the external source and encoded in the STM. Some form of operations happens in the STM (to be described in later paragraphs) and is later transformed and written as 1+2+2 = 5 as part of the problem-solver's written work. Using the IPT Model, this can be illustrated as shown in Figure 3.

**Production System:** A student's ability to solve problems depends on the extent to which he is able to elicit related information from the long-term memory. Questions which require students to elicit information from the long-term memory are thought to be more difficult than those

questions which require him to acquire information solely from an external source for processing in the short-term memory. Similarly, questions which require students to elicit Type B information are classified as higher order questions than those questions which require students to retrieve Type A information from the long-term memory or an external source. Hence, the other factor which determines a student's ability to solve a problem is the availability of knowledge stored in the LTM. The lack of related knowledge will greatly affect a student's ability to solve mathematical problems. To illustrate the way information is elicited from the LTM, Example 2 above is used for explanation.

Notice that there are three incidents (see Figure 2) in which information is elicited from the LTM:

(a) (1) —p1→ (4)
(b) (2) —p1→ (6)
(c) (3) —p1→ (8)

In (a), the condition statement (1), ratio of 3 sides = 1:2:2, evokes the action statement (4), converting ratio into parts and adding the parts. In (b), perimeter of triangle (2) evokes sum of three sides of triangle = perimeter, and in (c), find the length of the shortest side (3) evokes the

Figure 2: Representation of Cognitive Processes for Solving Example 2

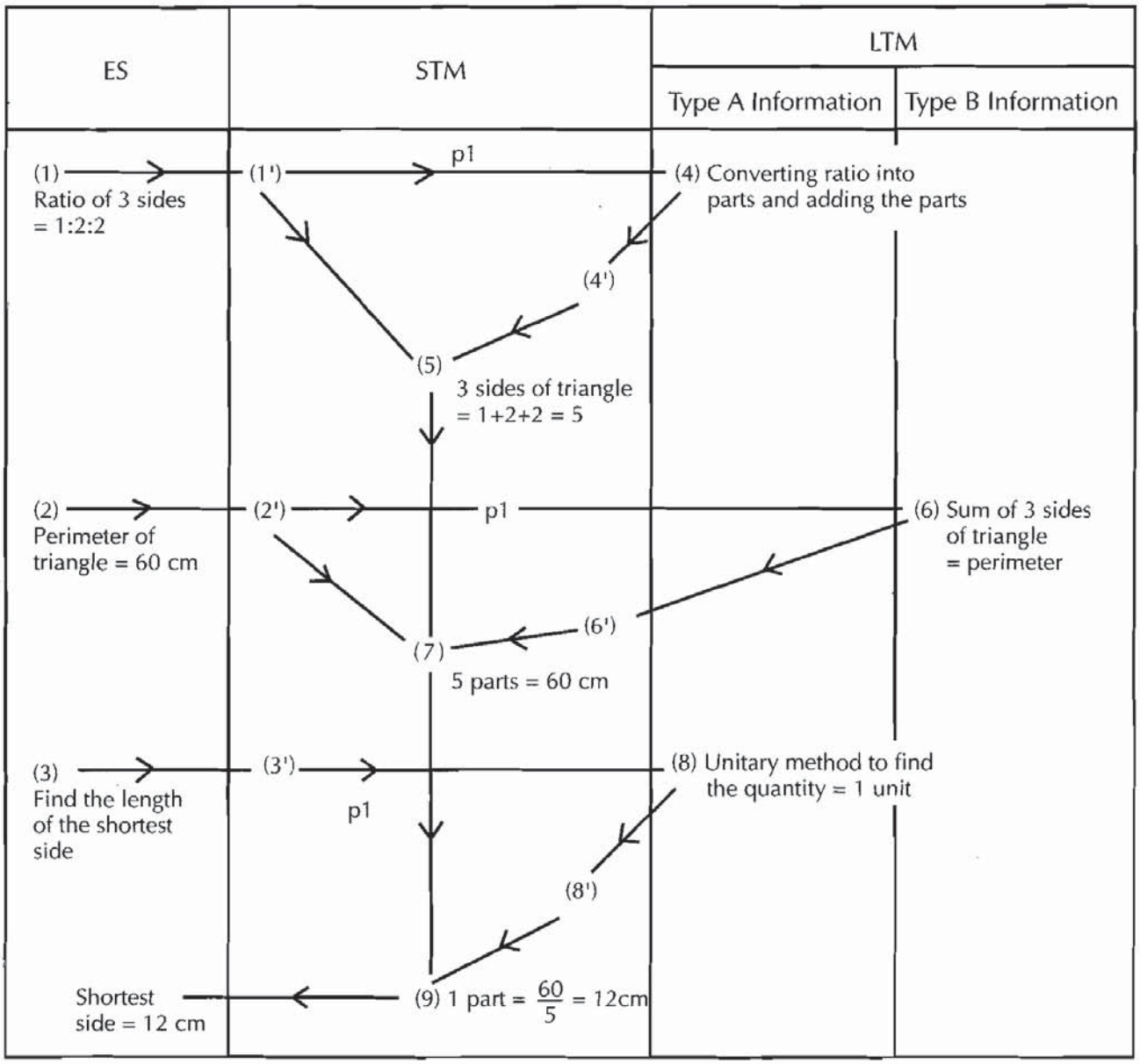
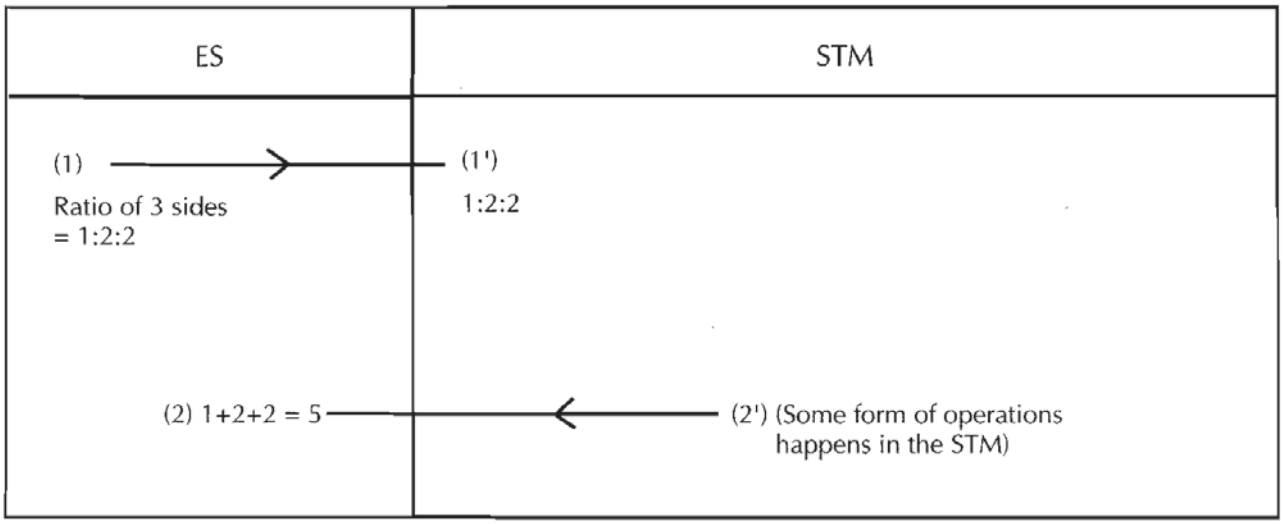


Figure 3: Retrieval of Information from External Source




unitary method to find the quantity of 1 unit. In all the three cases, a condition statement is required to evoke an action statement which is stored in the LTM. The condition and action statements are thought to be pairs of statements. When the condition statement is matched against the encoded information from ES, the action statement is evoked. Statement (b) is hypothesised to be relatively more difficult to evoke than (a) and (c) and the action statement is classified as a Type B information. The other point to be stressed here is that all the condition statements come from the external source.

**Secondary Production of Information:** This refers to the activation of the production of information from the secondary source (other than the external source). This requires the problem-solver to relate the actions of the earlier production with the processes of the next stage of production. Questions which demand secondary production of information are placed at a higher order in the hierarchy than those which require only primary production in problem-solving. Example 3 below illustrates the use of secondary production of information in solving the problem.

**Example 3**

The ratio of the weight of Fei-fei to Tin-tin is 5:2. One year later, Tin-tin's weight has increased by 14 kg whereas Fei-fei's weight remains the same. Now Fei-fei's weight to that of Tin-tin's becomes 4:3. What is Fei-fei's weight?

**Answer:**

Now, F:T = 5:2 = 4x5 : 4x2 = 20:8

1 yr later, F:T = 4:3 = 5x4 : 5x3 = 20:15

Difference in Tin-tin's weight = 15 – 8 = 7 parts

Difference in Tin-tin's weight = 14 kg

$$7 \text{ parts} = 14$$

$$1 \text{ part} = \frac{14}{7} = 2 \text{ kg}$$

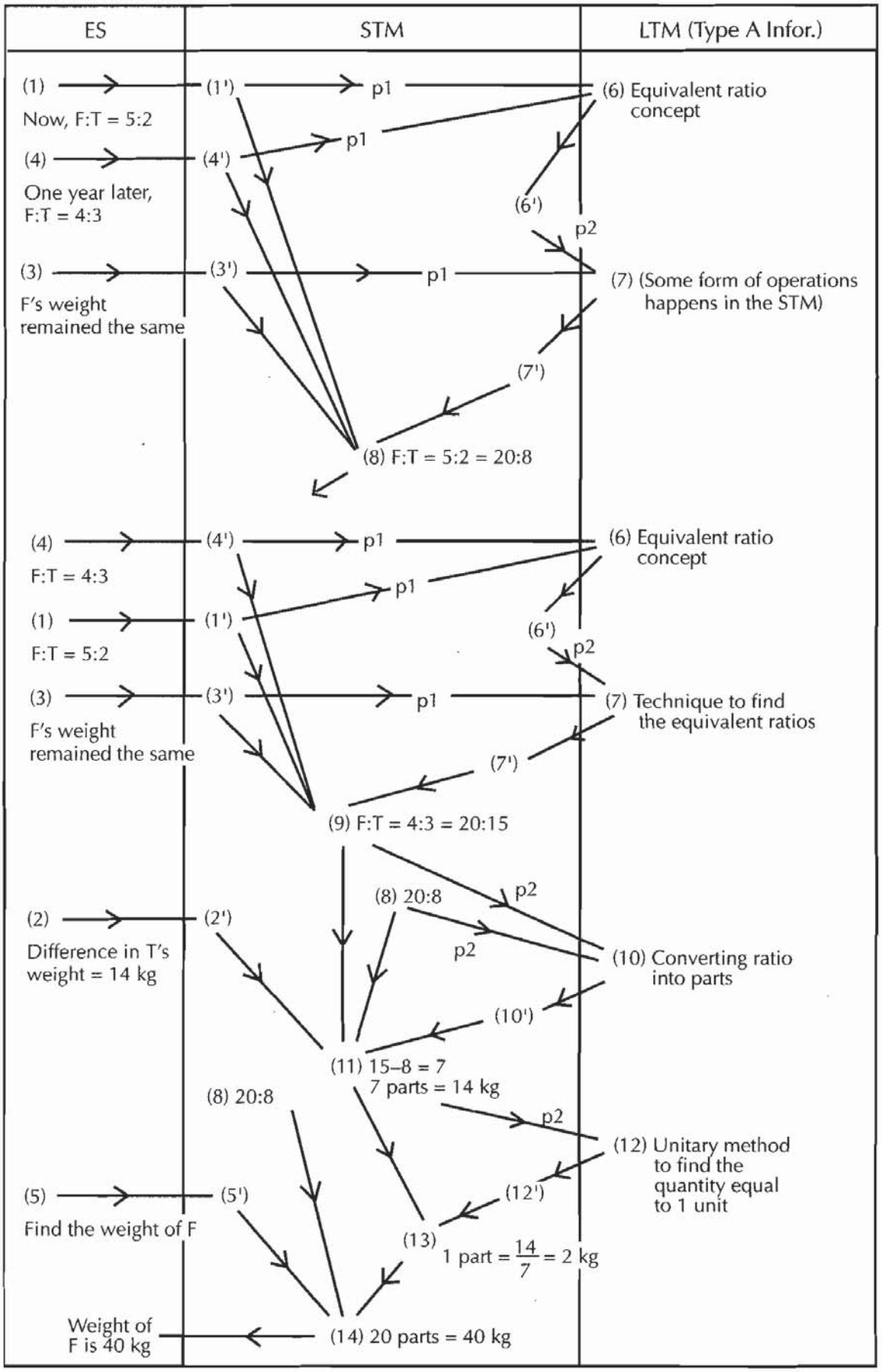
$$\text{Fei-fei's weight} = 20 \text{ parts} = 20\text{x}2 = 40 \text{ kg}$$

In Example 3, both the primary and secondary production of information were observed to elicit information from the secondary sources (Figure 4). The primary production is the equivalent ratio concept, whereas the secondary production is the technique to find the equivalent ratio, converting ratios into parts and the unitary method.

(1) —p1→ (6), equivalent ratio concept
(6') —p2→ (7), technique to find the equivalent ratios

Secondary production of information occurs when (6), equivalent ratio concept which comes from the STM evokes (7), technique to find the

Figure 4: Representation of Cognitive Processes for Solving Example 3



equivalent ratios. The main difference between the secondary production of information and the primary production of information is that the condition statement of the production pairs comes from the STM instead of the ES.

## Verification of the Hierarchical Nature of the IPT Model

An instrument, the Solving Word Problem (SWP), was constructed on the basis of the IPT Model for the purpose of verifying its hierarchical nature. The study was conducted in two phases: pilot study and major study. The former aimed to verify the suitability of the items and to check their hierarchical nature using a small sample. The latter study used a more rigorous analysis to verify if the items of the instruments were scalable. The following paragraphs describe the two studies.

### *Procedures and results of pilot study*

**Instruments**

The Solving Word Problems (SWP) instrument comprises 18 questions; three concerning the Order of Four Operations, five concerning Fractions, five concerning Composite Figures and five concerning Ratio and Proportion. (Note: the Order of Four Operations is viewed as the most basic topic, so the Type B information could not be applied here to construct the level 3 and level 5 questions). All of them were constructed using the hypothetical IPT features which characterise pupils' hierarchical responses into five levels. This instrument is meant to test the validity of the different hierarchical levels of the Model.

The SWP instrument was piloted to both British pupils (a class of 30 14-year-olds) as well as Singapore pupils (a class of 42 12-year-olds). The trial was carried out to ensure that the questions could be managed by high ability pupils from two different cultural groups. They had to have done some work on ratio and proportion to be considered for selection. The British pupils at 12 years old had not covered ratio and proportion problems comparable to those in Singapore.

From the results of the tests, Questions 6, 13 and 16 were revised. They were found to be either ambiguous, non-conforming to the specific level of the model, to cause difficulty in understanding the question or to cause difficulty in interpreting some geometrical figures.

**Subjects**

Three intact classes of 103 pupils were selected from two schools in Singapore for pilot testing the revised items. Pupils from these three classes were classified as low, average and high achievers on the basis of their Primary School Leaving Examination (PSLE) aggregate scores. The revised SWP test was administered to these three classes of pupils. Each pupil only answered any two topics of the revised SWP test.

**Results of pilot study**

The pupils' working sheets were marked and each item of a pupil was coded according to the coding scheme from 0 to 4. As each item corresponded to an IPT level of a topic, pupils who scored 3 or 4 in the item were said to have attained the level of the topic. For example, for the topic on fractions, if a pupil had achieved a score of 3 or 4 for the second question and failed to answer questions beyond the second question, then he had achieved level 2 of the topic. Taking each topic and counting the number of pupils who achieved each level, the percentage of pupils who attained each level was computed. Table 2 shows the percentage of the 12-year-olds who attained each level of the four topics.

The results show that for the topics the Order of Four Operations, Fractions, and Composite Figures, the number of pupils who achieve the level decreases as the difficulty level increases from level 1 to level 5. For these three topics, the hierarchical pattern seems to conform to the hypothesised IPT Model. However, for Ratio and Proportion, the question at level 3 (achieved by 23.3%) appears to be very much more difficult relative to the question at level 2 (achieved by 93%). The question at level 5 (achieved by 27.9%) appears to be easier than the previous question at level 4. However, consultations with

teachers revealed that some pupils were familiar with questions similar to those of level 5 on Ratio and Proportion at Secondary One. Some pupils who were interviewed also said that they had worked on similar types of problems.

*Procedures and results of major study*

**Instrument, groupings, coding and grading of pupils**

In the study, all pupils sat for the Chelsea Ratio and Proportion Diagnostic (CRPD) test. For the SWP test, children from each school were divided into approximately three equal groups and pupils from each group sat for one of the topics: Fractions, Composite Figures, and Ratio and Proportion. In the process of dividing pupils into groups, care was taken to ensure that an approximately equal number of boys and girls were included in each of the three groups. Using the two tests' results, each pupil was assigned two levels, one for the CRPD and the other for the SWP. The levels ranged from 0 to 4 for the CRPD test and from 0 to 5 for the SWP test. The system of assigning levels for the CRPD test was based on the Chelsea Project coding system, whereas the SWP coding system followed that described in the pilot study.

Another instrument which was also used in the study was the School Mathematics

Table 2: Percentage of 12-Year-Olds Who Achieved Each Level of the Four SWP Topics

| Order of Four Operations | | | Fractions | | |
|---|---|---|---|---|---|
| Level | Pupils Who Achieved the Stipulated Level | | Level | Pupils Who Achieved the Stipulated Level | |
| | Proportion | Percentage | | Proportion | Percentage |
| 1 | 39/39 | 100 | 1 | 41/42 | 95.2 |
| 2 | 39/39 | 100 | 2 | 30/42 | 71.4 |
| 3 | 35/39 | 89.7 | 3 | 24/42 | 57.1 |
| | | | 4 | 18/42 | 42.8 |
| | | | 5 | 10/42 | 23.8 |

| Composite Figures | | | Ratio & Proportion | | |
|---|---|---|---|---|---|
| Level | Pupils Who Achieved the Stipulated Level | | Level | Pupils Who Achieved the Stipulated Level | |
| | Proportion | Percentage | | Proportion | Percentage |
| 1 | 47/47 | 100 | 1 | 43/43 | 100 |
| 2 | 39/47 | 82.9 | 2 | 40/43 | 93 |
| 3 | 32/47 | 68.1 | 3 | 10/43 | 23.3 |
| 4 | 17/47 | 36.2 | 4 | 7/43 | 16.3 |
| 5 | 5/47 | 10.6 | 5 | 12/43 | 27.9 |

Examination (SME). This was applied as an instrument to grade pupils into different levels of mathematical abilities. The CRPD has been widely tested in Britain and Taiwan, so it is a reasonably good standardised test of pupils' mathematical abilities, especially of ratio and proportion. The pupils' scores in the test were correlated with the CRPD test using the Correlation Procedures of the SAS Package. The results showed that the scores of the CRPD correlated highly with the SME scores which were significant at the level of 1%. When the data were analysed by schools, correlations between the tests were also significant at this level. According to this result, it is reasonably safe to use the scores from the SME result as a criterion for grading pupils' level of ability.

**Subjects and sampling**

In the Singapore primary school system, after the Primary 4 end-of-year examination, pupils are streamed into EM1 (for the brighter pupils who can handle two languages very well – English and the mother tongue at first language level), EM2 (pupils taking English at first language level and mother tongue at second language level) and EM3 (pupils taking English at first language level and mother tongue at oral proficiency level). Some schools are specially selected to concentrate on certain categories of pupils. For example, an "above average" school normally takes in pupils from the EM1 and EM2 only. On the other hand, the "below average" schools will have more classes to cater to pupils from EM2 and EM3 streams. In the major study, seven schools (two "above average", two "average" and three "below average") were selected on the basis of the types of classes which the school had. The classes were identified in each school such that they reflect the average standard of the school. This was achieved by selecting pupils from the whole range of different abilities in each school. Thus, a vast distribution of pupils was obtained so that a representative sample with a wide range of pupils' abilities was possible. From the seven schools, 499 pupils (Age: 11+) were sampled at random from a total of 876 pupils. Out of the total number selected, 249 were boys and 250 were girls. Using the School Mathematics Examination (SME) and the Chelsea Ratio and Proportion Diagnostic (CRPD) test results as criteria for sampling, 149, 219 and 131 pupils were categorised according to high, average and low ability respectively. (Note: The CRPD test results conducted in the Singapore sample showed a similar profile to the CSMS results except for a slightly better result at the upper end. The detailed results are not reported here).

This sampling procedure resulted in classifying a list of pupils into ability and topical groups. From them, stratified random sampling was carried out to pick an approximately equal number of boys and girls for the study. Table 3 shows the distribution of pupils classified into the various groups.

Table 3 shows that the distribution of pupils taking the three topics reflects a wide dispersion of pupils of different ability groups and that an approximately equal number of boys and girls were selected from each ability group.

**Methods of analysis**

The Guttman procedure was used to test the five levels of the hypothetical level. This is a procedure used to determine the degree of deviation of the ideal model (the Taxonomy) from the representation of empirical data. Three statistics were applied as guidelines for an acceptable Guttman scale (Guttman, 1947; Mciver and Carmines, 1981): Coefficient of Reproducibility (CR) > 0.9, Minimal Marginal Reproducibility (MMR) < 0.9, and Coefficient of Scalability (CS) > 0.6.

To verify the hypothetical IPT Model, the bivariate relationships among items using the correlation coefficients were also examined. According to Mciver and Carmines (1981), when the correlation coefficients are small the relationship between any two items must be a weak relationship. Thus when this is the case, they are said to be far apart in level of difficulty.

**Results of major study**

Correlation procedures were processed using SAS and the coefficients were checked. Generally, most bivariate coefficients were found to be less than 0.4. According to Burns and

Dobson (1981), any correlation coefficients below 0.4 are considered to be low and the relationship between any two items is said to be small. The overall results (with small relationship between two items) seem to indicate that a higher level item could discriminate better pupils from weaker ones.

The Guttman procedure helps to determine if the items in each of the three topics are scalable or arranged in hierarchical levels. The three recommended statistics (CR, MMR and CS), which are used as guidelines for an acceptable Guttman Scale, were first processed using the SAS Guttman Procedure on each of the samples (Fractions, Composite Figures and Ratio and Proportion) using Items 1 to 5 as class variables (SAS, 1984). The results are shown in Table 4.

The overall results were found to be satisfactory. Although the coefficients of reproducibility for the fractions and ratio and proportion samples are 0.878 and 0.861 respectively, they only deviate marginally from the recommended standard (CR > 0.9). On the whole, the findings imply that the items for each of the three samples are scalable.

## Discussion and Implications

Assessing the mathematical abilities of pupils has long been of concern to educators. One approach to studying children's intelligence is the psychometric method (Barnett, 1975; Krutetskii, 1976; Thurstone, 1938). This method of identifying pupils' intelligence was not so popular in the last decade as it was considered to lack rigour in studying children's problem-solving abilities. It measures the product of children's outcomes rather than the processes, and merely determines and classifies different types of predictors of behaviour rather than revealing the thinking mechanisms of problem-

Table 3: Distribution of Pupils Classified into Topical, Sex and Ability Group

| Topic | Category of Pupils | No. of Boys | No. of Girls | Sub Total | Total |
|---|---|---|---|---|---|
| Fractions | Low | 19 | 23 | 42 | 158 |
| | Average | 28 | 40 | 68 | |
| | High | 31 | 17 | 48 | |
| | Total | 78 | 80 | 158 | |
| Composite Figures | Low | 20 | 24 | 44 | 175 |
| | Average | 36 | 39 | 75 | |
| | High | 34 | 22 | 56 | |
| | Total | 90 | 85 | 175 | |
| Ratio & Proportion | Low | 24 | 21 | 45 | 166 |
| | Average | 33 | 43 | 76 | |
| | High | 24 | 21 | 45 | |
| | Total | 81 | 85 | 166 | |

solvers.

The Bloom and SOLO taxonomies are two techniques which have been used by teachers and researchers to evaluate the mental abilities of children (for SOLO taxonomy see Collis, Romberg and Jurdak, 1986; Chicks, Watson and Collis, 1988). The taxonomies are used for designing questions to assess the levels of pupils' achievements in terms of Bloom's classifications, or to assess the quality of students' responses to items in the case of SOLO. In both cases, the designing of items does not appear to involve the mental operations of knowledge and their inter-relationships. Thus, the distribution of items to the assigned levels is subjective and is not arranged according to the pre-defined features of items that can elicit specific pupils' responses. As a result, it is difficult to detect marked differences among the items. However, to date they are the best available taxonomies which can be used as models for the construction of test items.

The IPT Model is intended to provide a more comprehensive psychological tool to supplement some deficiencies in the production system, information processing and schematic theories. With regard to the assessing of children's ability to solve mathematical problems, the Model could determine their levels of thinking in terms of the processes of mathematical knowledge underlying some specific features, such as the primary and secondary production of information and types A and B information retrieved from the LTM. This form of evaluation is more objective than the foregoing psychometric theory and the Bloom and SOLO taxonomies. Besides, the Model's features are identifiable from pupils' responses. For example, the secondary production of information might be identified from a child's response, so it can be inferred at the level at which he/she operates. However, in the previous models subjects are classified at certain levels according to pre-defined categories, but there are no specific and identifiable features which characterise the corresponding levels.

In connection with children's ability to solve problems, the IPT model helps to construct more precise items to assess pupils' mathematical abilities. The ability to design more precise items

Table 4: The Guttman Scale Analysis to Determine the Acceptance of CR, MMR and CS Statistics of Each Topic

| Topic | Statistics | Value | Acceptance (Y/N) |
|---|---|---|---|
| Fractions | CR | 0.878 | Marginally Accepted |
| | MMR | 0.706 | Y |
| | CS | 0.586 | Marginally Accepted |
| Composite Figures | CR | 0.933 | Y |
| | MMR | 0.775 | Y |
| | CS | 0.702 | Y |
| Ratio & Proportion | CR | 0.861 | Marginally Accepted |
| | MMR | 0.701 | Y |
| | CS | 0.534 | Marginally Accepted |

is contributed by the well-defined features at each level of the model. Although Bloom's taxonomy has often been applied by teachers and evaluators, it has not provided any specific features to guide them to construct the items. As for SOLO items, they are not defined in terms of mental operations like the IPT Model. For example, the unistructural stage described the application of one piece of information in response to the question requirement. It did not feature the actual mechanisms of mental operations. The IPT Model, however, is able to fulfill all these short-comings and provide a more accurate tool for assessing pupils' abilities.

Besides providing more definite features in designing assessment items, the IPT Model has implications for the teaching and learning of mathematics. Using the Model, mathematics problems which are classified into specific levels of thinking according to types of features demand corresponding operations in solving the problems. For example, some strategies operated at the fourth level of the Model require the secondary production of information. This implies that only pupils who can operate using the secondary production of information can solve the problems. However, if they are aware of this problem-solvers can alter the procedures of the strategies which require operation at the fourth level to the second level of the Model (by writing generated information externally). This can be carried out by transferring the condition statements from the STM to the ES. This helps to keep children from using secondary productions of information. Instead, the primary production of information is applied by retrieving information from the ES. Thus, future research in teaching and learning may apply these features of the Model in designing strategies for solving mathematics problems.

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