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Assessment of Primary 5 Students' Mathematical Modelling Competencies

Chan Chun Ming Eric
National Institute of Education
Nanyang Technological University, Singapore

Ng Kit Ee Dawn
National Institute of Education
Nanyang Technological University, Singapore

Widjaja Wanty
Faculty of Arts and Education, School of Education
Deakin University

Cynthia Seto
Academy of Singapore Teachers
Ministry of Education, Singapore

Mathematical modelling is increasingly becoming part of an instructional approach deemed to develop students with competencies to function as 21st century learners and problem solvers. As mathematical modelling is a relatively new domain in the Singapore primary school mathematics curriculum, many teachers may not be aware of the learning outcomes and competencies needed to develop in their students during mathematical modelling. This paper reports on the assessment of two groups of Primary 5 students' (aged 11) mathematical modelling competencies in their first attempt in completing a modelling task. The students' competencies are assessed to be at levels 1 and 2 of a researcher-designed rubric. Findings appear to suggest that students faced particular challenges in formulating a mathematical problem from the real-world problem through making assumptions. Implications on teacher education on the facilitation of problem formulation and mathematisation during mathematical modelling at the primary level are drawn.

***Key words**:* Assessment; Mathematical modelling; Competencies; Model-eliciting activities

Introduction

Mathematical modelling was introduced in the Singapore Mathematics Curriculum in 2007 (MOE, 2007) and is defined as "the process of formulating and improving a mathematical model to represent and solve real-world problems" (p. 8). It is seen as playing a vital role in enhancing understanding of key mathematical concepts and methods, as well as developing mathematical competencies (MOE, 2012). Students are to be provided with opportunities to "apply mathematical problem-solving and reasoning skills to tackle a variety of problems, including open-ended and real-world problems" (MOE, 2012, p.17). A distinct feature that sets mathematical modelling apart from traditional problem solving in the Singapore curriculum is that modelling provides a platform for students to "deal with ambiguity, make connections, select and apply appropriate mathematics concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data" (MOE, 2013, p.18). The benefits of mathematical modelling is seen through the rich experiences that students undertake towards deepening their mathematical understanding as well as motivation to learn mathematics as a whole as they see "relevance of what they are learning to the world outside the classroom and other subjects" (MOE, 2012). These experiences augur well for developing 21st century skills such as critical thinking, creativity and communication (MOE, 2012). In other words, modelling activities serve to develop modelling competencies such as "the ability to use a wide range of tools to solve complex real world problems and to work collaboratively with people" (MOE, 2012, p.1). However, during this formative stage since the inception of mathematical modelling into the curriculum in 2007, there have been very few such studies carried out in schools (Chan, 2008a; Ng, Widjaja, Chan, & Seto, 2012). Positional papers written about mathematical modelling by local researchers and educators (Ang, 2001, 2006; Balakrishnan, Yen, & Goh, 2011) serve to promote the use of algebraic functions, trigonometry or/and calculus in modelling real-world situations in secondary and pre-university classrooms. Researchers working with younger children in primary classrooms (Chan, 2008b, 2009, 2010; Ng, Widjaja, Chan, & Seto, 2012; Seto, Thomas, Ng, Chan, & Widjaja, 2012) have also used model-eliciting activities to reveal the ways students think about real-world situations and how they model them through making conceptual representations and mathematics. Children were found to identify goals, variables, clarify task details, conceptualize designs and interpret their model representations (Chan, 2008b) as well as test and revise their models as they go through different modelling stages (Chan, 2010). There are no local studies on the assessment of students' mathematical competencies. This is not surprising as even internationally, there are "few detailed studies on modelling competencies compared to the long and intensive discussion on connected tasks to real world problems" (Maaβ, 2006, p.119).

This paper reports on the modelling competencies of two groups of Primary 5 students in their engagement of a model-eliciting task. We analysed the mathematisation processes of Primary 5 students (aged 11) by examining their assumption making, mathematical reasoning and task interpretations as part of their modelling competencies. Students' competencies as well as difficulties they faced in managing the modelling task for the first time are identified for the purpose of teacher education.

Theoretical Perspectives

Mathematical Modelling from a Model-Eliciting Approach

This study takes on a modelling perspective based on the use of a model-eliciting activity to determine students' levels of mathematical modelling competencies. A model-eliciting activity is defined as "a problem solving activity constructed using specific principles of instructional design in which students make sense of meaningful situations, and invent, extend, and refine their own mathematical constructs" (Kaiser & Sriraman, 2006, p.306). Such an activity is designed to reveal the mathematical thinking of students in a given real-world situation. Students would generate mathematical models to describe their thinking processes, explain, manipulate, or predict the behaviour of the real-world system so as to select the most appropriate solution pathway for problem solving as interpreted within the real world context. It is also desirable for students to evaluate the possibility of applying their models to new problems.

The local context presented in a model-eliciting activity is crucial in guiding students' mathematical representations of the problem and hence also serves as a reference point in decision making during the modelling process. Such a process engages students in a cognitive situation where they express, test and refine their mathematical ideas iteratively until they develop a mathematical construct or model that is useful and meaningful for them and at the same time achieve the goal of the problem. Kaiser and Grünewald (in press) assert that model-eliciting activities draw upon the pragmatic-utilitarian perspective of problem solving while attaining psychological goals in motivating students in developing mathematics to make sense of the context in a natural way. In this respect, a model-eliciting activity is seen as being purposeful or what is termed as having an end-in-view (English & Lesh, 2003).

Models

In model-eliciting activities, models are "conceptual systems that generally tend to be expressed using a variety of interacting representational media, which may involve written symbols, spoken language, computer-based graphics, paper-based diagrams or graphs, or experienced-based metaphors. Their purposes are to "construct, describe or explain other system(s)" (Lesh & Harel, 2003, p. 158). Models are both internal and external, meaning that they reside in the mind and are expressed using external notation systems to construct, describe or explain the behaviours of other systems. Thus the conceptual system (in the mind) and the conceptual system expressed through a representational media (written, verbal, pictorial, etc.) are to be treated as a single cognitive unit and constitutes the structure of a model for knowledge development (Carmona, 2003). The models that learners develop are constantly subject to tests and revisions as greater sense-making experiences occur through interpretation and re-interpretation of them. These revisions result in models that are better and more stable than those initially conceived which are usually unsophisticated and naïve (Chan, 2010; Lesh & Doerr, 2000). When learners develop mathematical models, it implies how they are able to develop mathematical ideas and make mathematical translations through mathematising to give meanings to their conceptual representations and therefore interpret and solve the real-world problem.

Mathematisation

The term *mathematisation* is closely tied to mathematical modelling. Mathematisation is not modelling but part of the modelling process where students work towards having models that fit the real-world contexts. The mathematical modelling process, according to the Ministry of Education (MOE, 2012) comprises four elements, namely, formulating, solving, interpreting, and reflecting as modellers move between the real-world and the mathematical world. Thus, *mathematisation* is defined as students working within the modelling process to translate a real-world problem into a mathematical one by formulating a mathematical model which includes understanding the problem, making assumptions and representing the problem in mathematical form (Balakrishnan et al., 2011). From a model-eliciting point of view, mathematisation is seen as students making multiple cycles of interpretation, descriptions, conjectures, explanations and justifications that are iteratively redefined and reconstructed as learners interact with others (Doerr & English, 2003) and it involves quantifying, dimensioning, coordinating, categorising, algebratising, and systematising relevant objects, relationships, actions, patterns, and regularities (Mousoulides, Sriraman, & Christou, 2007). Van den Heuvel-Panhuizhen (2003) asserts that mathematisation plays a central role in assisting students to move along a continuum of developing models in form and function where students will be developing a "model of" and a "model for" the problem-solving context presented. Simply put, a student uses a model to investigate the problem situation but later transforms the model to relate to other situations and/or towards providing a way to better understand the situation at hand. This view of mathematisation is also echoed by Murata and Kattubadi (2012) where they see "model of" as the conceptualising of the situation model that is intuitive, detailed, and focusing on the problem context and "model for" the situation as focusing on the critical mathematical information of the problem and the general solution method.

Modelling Competencies

A central goal of mathematical modelling is the promotion of modelling competencies (Kaiser & Grunewald, in press). Depending on the perspective that mathematical modelling takes and the goals to be fulfilled, the development and the assessment of modelling competencies may appear different but at times overlapping. Most of the descriptions of modelling competencies are related to the ability to mathematise and work with mathematical models during the modelling process. The Program for International Students Assessment (PISA) (OECD, 2009) perceives modelling as having a symbiotic relationship with mathematical competence. PISA identified a modelling competency cluster characterised by the ability to structure the situation, mathematise and de-mathematise, work with mathematics to tackle the model, validate the model, reflect, analyse, critique the solutions, monitor and control the modelling process and communicate the model and results. Similarly, Lingefjard (2006) likened modelling competencies to students doing mathematics during the modelling process, using everyday knowledge and validating mathematical models through reflecting, critiquing, explaining, describing and communicating those models. This is further echoed by Kaiser (2007) who identified modelling competencies to showing understanding of real-world problems, creating mathematical models, solving problems within the model, interpreting mathematical results in the real-world model or situation and challenging solutions. At a micro level, Niss, Blum, and Galbraith (2007) viewed mathematical modelling competency as the ability to identify relevant questions, variables, relations or assumptions about a real world situation, to translate these into mathematics and to interpret and validate the solution.

We found Maaβ's (2006) discussion of modelling competencies a useful framework for this paper. She defined modelling competencies as "skills and abilities to perform modelling processes appropriately and are goal oriented as well as the willingness to put these into action" (p. 117). Specifically, modelling competencies are based on the actions involved within movements between the real-world and the mathematical world. She identified sub-competencies in the modelling process such as to understand the real problem and set up a model based on reality, to set up a mathematical model from the real model, to solve mathematical questions within the mathematical model, to interpret results in a real situation and to validate the solution. In addition, Maaβ (2006) also raised the importance of metacognitive competencies during the modelling process. These are competencies to (a) 'structure real-world problems and work with a sense of direction for a solution', (b) 'argue in relation to the modelling process and to write down this argumentation', and (c) 'see the possibilities that mathematics offers for the solution of real world problems and to regard these possibilities as positive' (p 117).

Niss, Blum, and Galbraith (2007) highlighted the mutual impact of mathematical and modelling competencies. In other words, growth in modelling competence is dependent on mathematical competence and at the same time helps develop mathematical competence. This puts forth the argument that modelling is a potential vehicle for student learning of mathematics and that learning mathematics develops competency in applying mathematics and developing models. When students are engaged in modelling activities, it is not just a case of students expressing their mathematical competencies but is one that concurrently develops students' competence further (Lingefjard, 2006; Swan, Turner, Yoon, & Muller, 2007). In this regard, it also suggests that competence development as a continuous process (Blomhøj & Jensen, 2007). The criteria for assessing the students' modelling competencies in this study are discussed in the next section.

Research Design and Methodology

This study took on, as the over-arching research framework, a three-tiered corroborative teaching experiment framework in implementing a modelling activity with Primary 5 students and embraced design research methodology to support the analysis of data.

Multi-tiered Teaching Experiment

The multi-tiered teaching experiment considers the development of all participants involved in the research as well as hinges on the creation of conditions that optimise the chances that development will occur without dictating the directions for (a) developing new conceptions of participants' (students, teachers, researchers) experiences, (b) structuring interactions to test and refine constructs, (c) providing tools that facilitate the construction of relevant models, and (d) using formative feedback and consensus building to ensure the constructs develop in directions that are continually better (English, 2003). The main principle underlying this framework is to seek corroboration through triangulation. In this regard, all participants or learners worked interdependently with "each of them engaged in a common goal of trying to make sense of, and learn from, their respective experiences" (English, 2003, p. 227). In the three tiers, the researchers, teachers, and students were engaged differently in their own form of learning, but all of them were involved in making sense of their experiences by developing models that were used to generate descriptions, explanations, constructions, and justifications using a variety of representational systems.

Design Research Methodology

In this study, the design research methodology (Dolk, Widjaja, Zonneveld, & Fauzan, 2010) was embraced within the Multi-tiered Teaching Experiment framework to guide the analysis and interpretation of data. One key aspect of design research is its focus on the retrospective analysis that sees researchers and teachers working together to produce meaningful change in the context of classroom practice and instruction (Design-Based Research Collective, 2003). In this respect, the process of the interaction involves cycles of phases comprising Knowledge, Design, Experiment, and Retrospective Analysis. Engagement for retrospective analysis include eliciting the teacher's knowledge of mathematics and students, the features of the problem tasks (in this case, the modelling tasks), the potentials and challenges by way of their experiences with the task, and the scaffolding towards advancing students' learning. In this paper, the main focus of the analysis will be on the teaching experiment phase.

The Modelling Task

The modelling task, (see Appendix) was designed by the research team, Chan, Widjaja, and Ng (2011), by adapting the modelling design principles of Lesh, Cramer, Doerr, Post, and Zawojewski (2003). Considerations include whether (1) the task warrants sense-making and extension of prior knowledge (reality principle), (2) the situation creates the need to develop (or refine, modify or extend) a mathematically significant construct (model construction principle), (3) the situation requires self-assessment (self-evaluation principle), (4) the situation requires students to reveal their thinking about the situation (construct documentation principle), (5) the elicited model is generalisable to other similar situations (construct generalisable principle) and, (6) the problem-solving situation is simple to carry out (the simplicity principle). The task was designed based on these principles and posited to reveal significant information about the students' solution processes.

As the research design embraced a multi-tiered teaching experiment approach, the teacher personally went through the task with her colleagues and provided feedback towards refining the design of the task. The design of the task also took into consideration the assessment of students' modelling competencies at three levels.

Subjects, Data Collection and Assessment

The subjects were Primary 5 (Grade 5) students in a mixed-ability class who worked at the task in groups of four and five. The students did not have any prior experience with modelling tasks as the problem-solving lessons in their class mainly conformed to solving structured problems. Data from video recordings and written work of the students were collected from two target groups of students. Three modelling sessions each lasting an hour were conducted. The teacher's retrospective interviews were audio-recorded after each session. Assessment of the students' work was made by interpreting the transcribed videos and written work against the criteria set in the task-specific rubrics.

Assessment of Students' Modelling Competencies

The researchers employed different means such as rubrics (Lesh & Clarke, 2000) as well as a qualitative approach analysis of the data from transcribed audio and video tapes (English, 2007) to assess modeling competencies. In this study, the modelling competencies of the students were assessed as a group where students' verbalisations and representations of their thinking were made observable through their verbal discourse and written work. Criteria descriptors are developed and framed as an assessment rubric since it can "help teachers analyze and describe students' responses to complex tasks and determine students' levels of proficiency" (NCTM, 2000, p.22). Classroom discussion on the use of such an assessment rubric helps differentiate between an excellent response and one that is mediocre in students' approaches to solving complex problems (NCTM, 2000).

For the purpose of this study, we developed the modelling competencies criteria for assessing group work with reference to the mathematisation abilities necessary for successful completion of the modelling task. We took into consideration that there is a broad spectrum of modelling competencies and that the students involved in this study were novice modellers and had not been exposed to model-eliciting tasks before. We were also keen to assess competencies that were more atypical of normal teacher-led mathematics lessons, for example, making assumptions, mathematical reasoning and interpreting solutions. The competencies identified (shown in Table 1) are aligned with the processes as spelt out in the Ministry of Education's framework as well as those corroborated from literature.

These modelling competencies, *making assumptions*, *interpreting of the task and solution using real world knowledge* and employing *mathematical reasoning and computations* differentiate them from the typical assessment of students who were engaged in structured tasks in investigative group work. The descriptions of the three competency components are as follows:

Table 1
Mathematising Behaviours During the Modelling Process

Elements in the Modelling Process (MOE, 2012)	Distinctive features during mathematical modelling (MOE, 2012)	Competencies identified for this study
Understanding and simplifying the problem (Formulating)	Making assumptions, recognising variables, constructing relations between variables, distinguishing relevant and irrelevant information. Understanding the problem, making assumptions to simplify the problem, representing the problem.	Making assumptions towards understanding and, simplifying the real-world problem
Manipulating the problem and developing mathematical model (Solving)	Making appropriate representations, reasoning, using mathematical knowledge to solve the problem, making accurate computations. Selecting and using appropriate mathematical methods and tools, solving and presenting the solution.	Mathematical reasoning and computation
Interpreting problem solution (Interpreting)	Communicating the solution through mathematics or descriptively. Interpreting the mathematical solution in the context of the real world, presenting the solution of the real-world problem.	Interpreting task and solution using real world knowledge
Verifying and Validating (Reflecting)	Questioning the model or solution, checking and reflecting on the model. Reflecting on the real world solution, improving the model.	(We see this as occurring throughout and is subsumed in the earlier phases)

Competence in making assumptions. This refers to students developing an awareness of assumptions as they seek to understand and simplify the problem. The development of an awareness of assumptions is seen to have a dual role: (a) as the bridge to connect the *real world* to the *mathematical world* and, (b) the promotion of activities that reflect on the formulation stage in the mathematical modelling process (Seino, 2005).

Competence in interpreting task and solution using real-world knowledge. This refers to the students making appropriate representations via their real-world and mathematics knowledge as they formulate variable relationships. In this task, specifically, students can establish distance-time relationships, distance-cost relationships, and use real-world knowledge such as the number of bus-stops, duration at bus-stops, or even population density to manipulate their solutions.

Competence in mathematical reasoning and computation. This refers to the ability to confront multiple variables mathematically towards solving the problem with the plausible recommendations made. The accuracy of the computations is also taken into account.

Although *reflecting* is a key element in the Ministry of Education's mathematical modelling framework, we see aspects of validating and verifying as situated within the *formulating*, *solving* and *interpreting* elements of the modelling process as without validating and verifying, revisions cannot be made towards improving the models.

The assessment of modelling competence can be characterised into three levels and depicted as a multi-dimensional rubric as ascertained during the task design and handholding session with the teacher facilitator. The lowest level is Level 1 suggesting the lack of the characteristics and the highest Level 3 the explicit manifestation of the characteristics (see Table 2).

Table 2
Rubric for Assessing Modelling Competencies

Competencies	Level 1	Level 2	Level 3
Assumptions	* No assumptions made. * Incorrect notions of assumptions.	* At least 2 assumptions made and explained based on real-world interpretations of task. * Assumptions stated are relevant to model.	* Comprehensive list of assumptions made and explained based on real-world interpretations of task. * Assumptions stated are relevant to model.
Interpretation of task and solution using real-world knowledge	* There is no evidence or only one real-world constraint (e.g. number of bus-stops, time duration, etc.) in the presentation of work.	* Evidence of two real-world considerations (e.g. number of bus-stops with time duration) in examining variables that will impact interpretation and solution of modelling task.	* Evidence of three or more real-world considerations (e.g. number of bus-stops, time duration, population, etc.) in examining variables that will impact interpretation and solution of modelling task.
Mathematical reasoning and computation	* 1 variable considered. * Appropriate use of mathematics but with some minor errors in computation. * Mathematical reasoning somewhat logical. * Attempted recommendation but not well substantiated with the mathematical reasoning.	* 2 variables considered. * Appropriate use of mathematics with hardly any computational errors. * Mathematical reasoning is logical. * Recommendation substantiated with the mathematical reasoning.	* 3 or more variables considered. * Appropriate use of mathematics. * Mathematical reasoning is logical and computation is clear and very accurate. * Recommendation substantiated with strong mathematical reasoning.

Findings

This section presents the assessment of two groups (A and B) of students concerning their mathematical modelling competencies. The assessment was carried out based on the students' written work as well as from their collaborative discourse with respect to the three criteria outlined in the rubric. One group was assessed to be at Level 1 competence while the other at Level 2 competence.

Exemplification of Level 1 Modelling Competence (Group A)

Group A was assessed to be at Level 1 competence as they were only able to determine the shortest route through the use of the string to compare distances. The analyses of the data revealed several aspects that the students had difficulties with in regard to managing real-world problems.

Assumptions. The students made the assumptions that the buses were in good conditions and they started travelling at the same time (see Figure 1). These are credible assumptions made by the students as the conditions of the buses or the timing of the day (e.g. peak or non-peak periods) could have an impact on the meaning regarding *the most efficient route*. Unfortunately, the assumptions raised were not followed-up in the discussion to reveal how they could have impacted their understanding of the task and subsequently in their planning of the solutions. The third assumption that there were no junctions did not make realistic sense as the map clearly showed road junctions. Other key assumptions such as the number of bus-stops and population density that could be inferred from the map were not discussed.

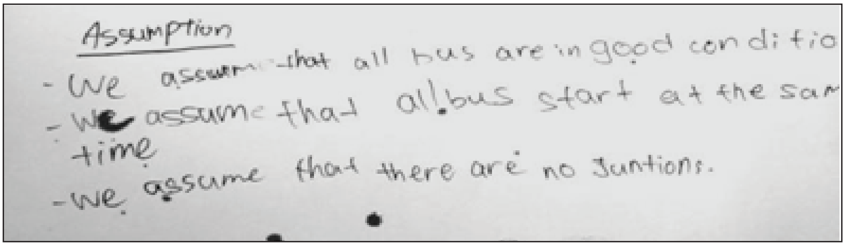


Figure 1. Assumptions made by Group A

Interpretation of task and solution. Generally, the students had difficulties understanding what the scale (shown as a black-and-white strip) given at the bottom of the map meant. The students misinterpreted the scale icon to be another road found in the map when actually the striped line in the map refers to the train track. It is probable that the scale icon being represented as a black-and-white strip bore semblance with the black-and-white train track that led to their misinterpretation. It was observed that they had difficulty trying to reconcile how to use the striped road (train track) for Ms. Chang to travel from her home to the school (see excerpt below).

1	*S3*	*But from here (pointing at Ms. Chang's flat and the track), never put the thingy (inferred that the striped road did not start near Ms Chang's house).*
2	*S1*	*But are you sure it is this or not? (checking with S3 if the striped road has anything to do with the scale).*
3	*S3*	*But initial is this you know? (aligning Ms. Chang's flat to be the starting point where the striped road should start).*

This misinterpretation surfaced again later in their consideration to measure the routes: "*If we measure this (train track), then we cannot measure all these (the three bus routes). Because the black lines, here don't' have, here also don't have (referring to the coloured bus routes), so only here have (where the train track and the road coincide). So, very difficult. So only here we can find (where the train track and the road coincide) and somewhere like here.*"

The students had been exposed to scale representations before in social study lessons but they were not able to link it to this problem situation in this case. There is a possibility that the scale icon that the students were familiar with was just a line drawing instead of a black-and-white strip. However, students in other groups did not face this issue. Even though the scale icon is presented as a black-and-white strip, the scale figures should provoke them to think about scale representations instead of just the visual black-and-white icon. In moving on, the students dropped the consideration of the striped road as they could not get it to feature in the network of roads that led to the school.

Another aspect that concerns their interpretation of the route was the notion that roads with smaller turns (they referred to the yellow route) had shorter distances compared to roads with larger turns (the pink route).

1	*S2*	*But yellow has many turns, maybe that's the short cut.*
2	*S3*	*This one (pink) turns half a round, this one (yellow) got many turns… directly.*
3	*S4*	*This one (yellow) got many turns.*
4	*S3*	*Many turns, so?*
5	*S4*	*Slower.*
6	*S1*	*Maybe you here (referring to one of the pink turnings) , turning also very slow.*
7	*S4*	*Aiya, don't know.*
8	*S1*	*Maybe the outer one (pink) is slower, the inner ones (yellow turns) are faster.*

Although the excerpt shows some disagreement concerning the relationship between the number of turns and speed, they later concluded that the yellow route was the shortest *"…because it is in the middle because the pink is like half a round*". Their interpretation is interesting as seemingly the yellow route was positioned in the middle of the pink and blue routes and appeared to be a more direct route to the school compared to the buses on the pink and blue routes which apparently needed a "larger turn" (inferred to be like the circumference of a semi-circle) to reach the school. It was not until the moment when they did the actual measurement with the strings did they realise that the yellow route was in fact longer.

Mathematical reasoning and computation. The students were initially lost with respect to finding the distance of the routes. One student estimated (without basis) that the distance between two pink bus stops was 1 km but her reasoning was challenged by her friends.

1	*S3*	*All this long one is all 1 km (uses the distance between thumb and index finger to suggest distance between two pink bus stops).*
2	*S2*	*How you know?*
3	*S3*	*I think so.*
4	*S2*	*But then not possible. Too short already.*

They later realised that they could use the strings to meet their objective. Figures 2 and 3 show the students determining the distances by tracing the routes before measuring them with the metre-rulers. The students had already known from comparing the marked strings which route was the longest but they measured them with the rulers for conversion purposes.

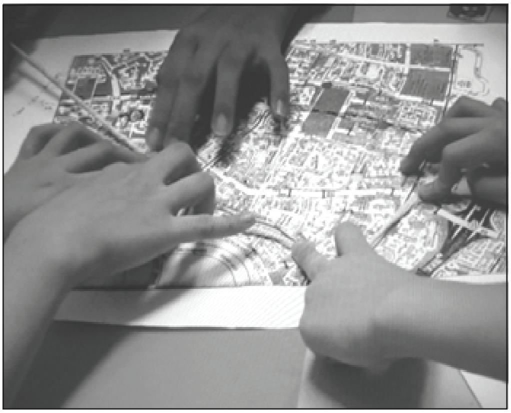


Figure 2. Measuring distances of the routes.

Figure 3. Measuring distances with metre-ruler.

It was observed that they had some problems with the conversions of units. For example, they measured the pink route to be 116.5 cm (which could not be the case if measured in cm as the route was designed to be less than 50 cm) but they converted the distance into kilometres by dividing by 1000 to get 0.1165 km. It was evident that they lacked reflection and validation here as 0.1165 km is essentially about 116 m and a traveller would rather walk than take a bus.

Their final piece of written work is shown in Figure 4 where they concluded that the pink route was the most efficient route when originally they had thought that it was the yellow route.

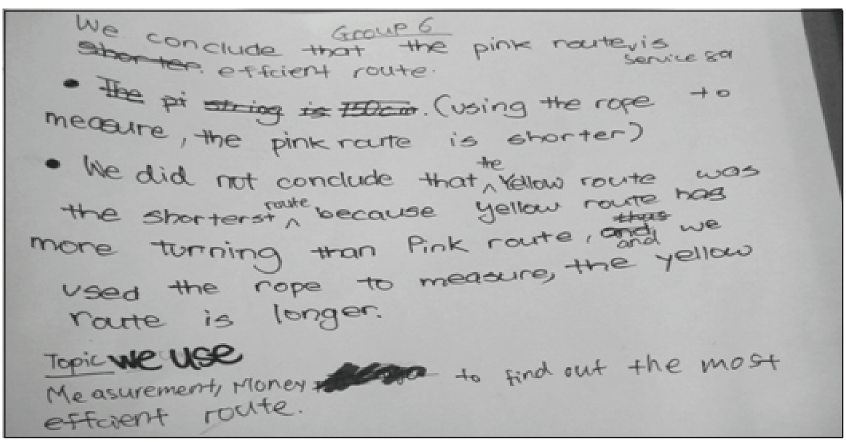


Figure 4. The written solution of Group A.

Overall assessment. The students in Group A showed some knowledge of modelling as they related distance and time although distance was the more dominant variable they investigated on. To meet their goal of finding the most efficient route, they measured and compared distances. Some plausible assumptions were made but they were not expanded towards understanding how they impacted their deliberations and solutions. Their mathematising efforts revealed some misconceptions with respect to scale- and map-reading as well as making unjustified estimations for distances. They were also unsure how to make unit conversions. Nevertheless, their actual measurement effort helped them to realise that their initial conceptions were incorrect (e.g. having more turns means that the route was shorter) which in a sense enabled them to revise their thinking. Their effort was generally free of making mathematical translations as seen in Figure 4. In this regard, Group A is assessed as having Level 1 competence.

Exemplification of Level 2 Modelling Competence (Group B)

Group B was assessed to be at Level 2 competence. Their mathematising effort was richer with respect to the deliberations made concerning making assumptions as well as formulating mathematical relationships with two variables.

Assumptions. The excerpt below shows that the students were trying to grapple with the meaning of making assumptions in the context of the problem situation.

1	*S2*	*Two assumptions. You write. We assume that the distance is…we assume that the route is in km. (S1 writes as S2 dictates). Now your turn. Assumptions.*
		(S2 takes over the pen to write).
2	*S2*	*We assume that all the buses' speed is the same.*
3	*S3*	*But this is correct what? (as a fact).*
4	*S2*	*No. We assume that pink is the fastest. No, we can assume again.*
5	*S3*	*If we assume again… like…*
6	*S2*	*No. It may be...Maybe the pink is wrong. Maybe it's blue. We're just assuming.*
7	*S2*	*We assume that all the buses' speeds are the same.*
		(S1 records the assumption).
8	*S2*	*We assume all the buses reach the bus stop at the same time. And we assume pink is the shortest distance.*
9	*S2*	*We assume all the buses' fares are the same.*
10	*S3*	*You measure all already, right?*
11	*S4*	*It may not be because you see, some distances are shorter, some distances are longer.*

12	*S2*	*No. You must look at the total.*
13	*S2*	*Yeah. The total fares are the same.*
14	*S3*	*How can they be all the same?*
15	*S2*	*Assumption.*
16	*S3*	*Here already write they are different.*

It was observed that S2 presented most of the assumptions. It was inferred that the students were not too certain about making assumptions with respect to the constant in the problem situation that is needed for comparisons to be made. While it was logical for the students to assume that the buses travelled at the same speed, to assume that a particular route was the shortest or that the buses ended at the same time, however, was invalid as those were precisely the aspects to be investigated upon (see Figure 5).

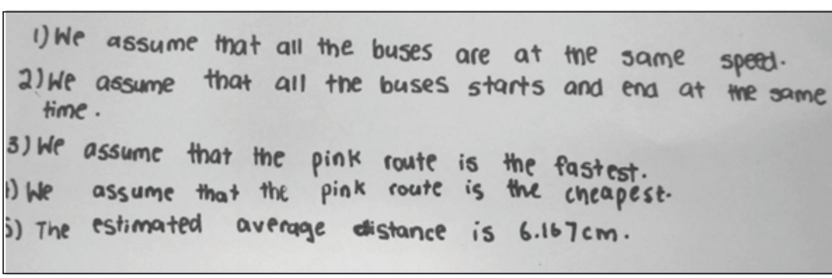
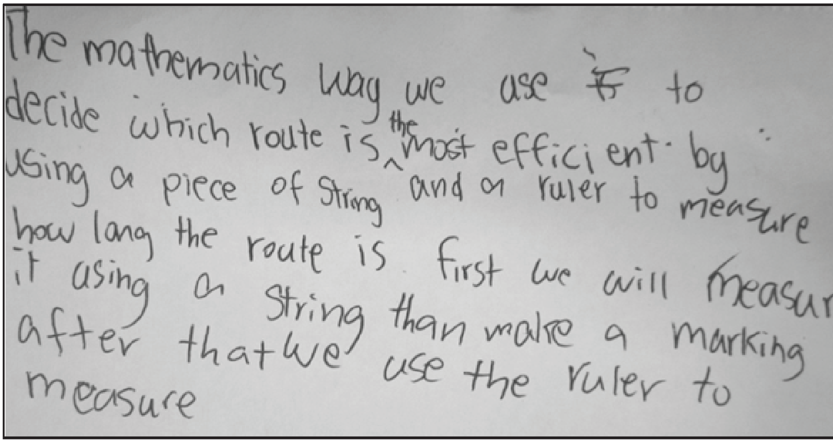


Figure 5. Assumptions made by Group B

It is noted that the students vigorously argued about the plausibility of their assumptions as seen in the excerpt above. The students' confusion about making assumptions became clear when the teacher explained that they could make assumptions only when they did not have the data given in the task sheet: "*When you have proper data and when you can get proper answers, you don't have to assume anymore*".

Interpretation of task and solution. Group B made use of the strings to measure and compare the distances of the respective routes. As seen in Figure 6, it was the method they described to decide which route is the most efficient.



The mathematics way we use ~~is~~ to
decide which route is the most efficient by
using a piece of String and a ruler to measure
how lang the route is. first we will measure
it using a String than make a marking
after that we use the ruler to
measure

Figure 6. Group B's explanation of their method

The group did not just stop at measuring distances. They also related the bus fares as a variable that would affect efficiency. The excerpt below exemplifies what the students were interpreting with respect to the step-rate table provided in the task sheet, that is, for the first kilometre, the bus fare remained the same ($1.10 as given in the task skeet).

1	S4	*All the fare will be the same, correct? Just like average what.*
2	S1	*Seems like you saying…*
3	S4	*( ) it into a average. Then after that we…*
4	S1	*Seems like you saying the… like you know one kilometre all the fare is the same for all the bus.*

They were able to formulate distance-cost relationship based on using the table. Their solutions are seen in the next part exemplified below.

Mathematical reasoning and computation. The students' communication of their mathematical ideas and computations was more explicit in written form. This is shown in Figure 7.

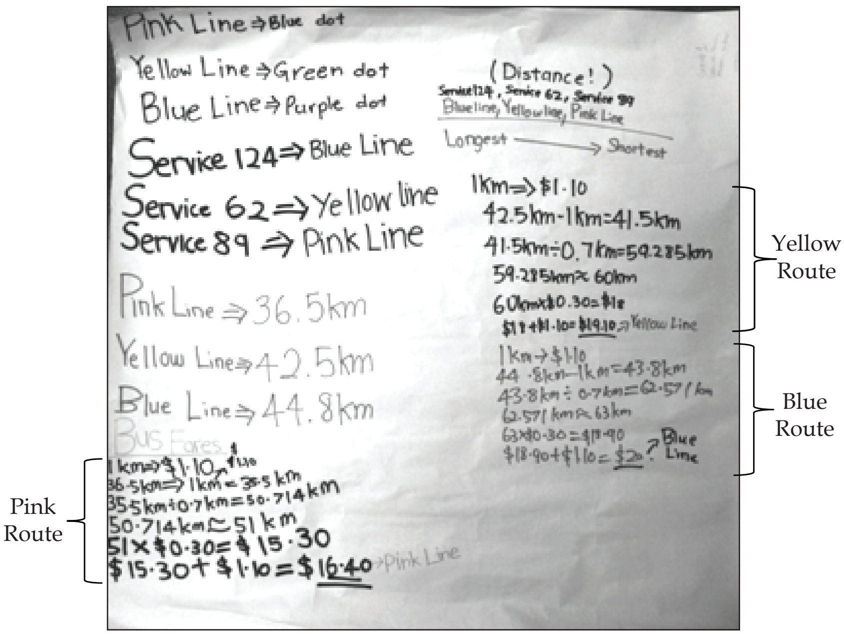


Figure 7. Group B's final solution

In Figure 7, it was evident that the routes were measured and the distance variable was tied to the costs needed to travel the three routes. It can be seen clearly that the distance of a particular route (Pink) had 1 km subtracted first (for example, 36.5 km → 1 km = 35.5 km) before dividing by 0.7 km as they attempted to find the variable rate. The arrow (36.5 km → 1 km = 35.5 km) was the students' way of depicting the exclusion of the first kilometre. This method of working out the travel cost is consistently used for the other two routes. For the computation of the cost for the pink route, they obtained 50.714 km. This statement was mathematically incorrect as the division should give the number of sets of distances 0.7 km long and should not end with the km. unit of measurement but it did not affect the follow-up computation. Following on, the group rounded this figure to the nearest whole before multiplying it by $0.30 to get the total cost for the route. The mathematical workings for the other two routes were consistently carried out and the group was able to compare the costs as well.

Overall assessment. Generally, the group managed to determine their most efficient route by formulating relations between distance and costs and suggesting why their route was shortest and cheapest. The mathematical reasoning was aptly applied (see Figure 7) to give meaning to the problem situation as well as to solving it. They also negotiated ideas about assumptions made although they still had difficulties with the notion of making plausible assumptions in relation to the problem situation. Their modelling competence was assessed to be at level 2.

Discussion and Closing Remarks

The modelling competencies identified for assessment in this study were those that cannot be developed using traditional instruction and are essential to the development of competencies in the modelling domain. In this regard, the outcomes from this study have significance in improving curricula and promoting meaningful modelling activities with students. It also serves to enhance our understanding of developing and assessing students' modelling competencies.

From the study, it was found that although groups did display various ways of thinking and working, groups A and B had some difficulties making plausible assumptions. Making plausible assumptions helps towards simplifying the problem as well as situating the problem solving in a more realistic setting. According to Seino (2005), setting assumptions is important otherwise the nature of the situation would be distorted and the problem could not be solved appropriately. An awareness of assumption therefore acts as a bridge that connects the real world and the mathematical world. Some students in this study either did not know how to employ real-world knowledge or they superficially considered them. Even when they did come up with some assumptions, some of these were found to contradict one another. This observation was also affirmed by the teacher during her retrospective interview session as she spoke about how one student went about making assumptions: "*You know he was talking about assumptions when it's already calculated and proven that it's correct and the others could get it. They could see it. But he in particular couldn't and … he wasn't really listening to why he was in the wrong track*". Although making assumptions might create tensions between students with regards to whether they are plausible, it is an essential part of working towards clarifying what might be reasonable for the problem to be solved. It is possible that too much traditional instruction could have led to the students' lack of ability in applying real-world considerations or students having the notion that mathematics is a discipline strictly comprising formulas and rules to be used in mathematics lessons. Nonetheless, Seino (2005) pointed out that it is possible to develop an awareness of assumptions by making students recognise conditions in the problem as assumptions, by helping students realise the question of whether an assumption affects certain functions, and whether there are other assumptions to consider. The purpose, as he puts it, is to make students appreciate the necessity and importance of setting up assumptions, and to appreciate the usefulness of mathematics. Another way to develop this aspect is for teachers to develop students' ways of thinking by asking them "what-if" questions or getting them to make suppositions as these aspects are tied to ideas of making assumptions.

Students' interpretations of the task and solution in terms of the real-world situation and the mathematical world are interwoven. It was encouraging to see the mathematising aspects as they worked out the most efficient route based on the assumptions they had established. One group was successful in making unit conversions with the scale while the other group could not figure out the scale. The group (Group B) that could make out the scale progressed to using the step-rate system to calculate the bus fares while the other group made their own distance estimations that did not lead to plausible solutions. Group B's efforts saw them consider two variables, distance and cost, with the mathematics aptly applied towards finding a solution.

In making generalisations, students also learned to justify why their model worked. They recommended the most efficient route for residents travelling by bus from the vicinity where Ms. Chang was staying to the vicinity where the school was located. Such a generalisation was made possible after the students considered the mathematical aspects of the model by investigating various relationships established between appropriate variables alongside other factors in the problem situation. In this sense, they learned to validate their work by comparing various solution options for decision making and to fulfil the goal they had set out to achieve.

As first time modellers, it was not surprising that the students did not exhibit Level 3 competence. The findings were consistent with what other researchers have noted concerning novice modellers in that they have the tendency to not complete the modelling cycle once they perceived they had answered the question (Hodgson & Harpster, 1997) or that their thinking about the problem can be rather inconsistent and disorganized (Lesh et al., 2000). Notwithstanding, the modelling platform provides opportunities for students to describe, explain and make-decisions about the problem situation enabling them to apply their learning in authentic contexts.

Competence has to develop over time and over a series of tasks, and is seen as a continuous process (Blomhoj & Jensen, 2007). Providing more of such opportunities for students would bring their mathematical conceptual system into play and work towards a representation of the system as local competence gradually increases (Lesh & Doerr, 2003). In a sense, this implies a longer period of being involved in various modelling activities and a longer period of study for researchers to validate this. It will be a process of unlearning and re-learning in such modelling sessions for students to be better modellers and even for teachers as they learn to appreciate how students learn and apply mathematics in a more holistic way. According to Maaβ, (2006), apart from providing teachers with such tasks, teachers need to gain independent experience with modelling first. In other words, they need to get to know and test teaching methods with modelling examples towards supporting the relevant modelling competencies.

In conclusion, this study has shown that novice modellers are capable of completing a modelling task albeit at different levels of competence shown. It must be acknowledged that modelling is a sophisticated endeavour for children. It involves students working in groups towards understanding the problem, framing appropriate questions, making assumptions to simplify the problem, formulating models through establishing variable relationships, comparing mathematical outcomes, and revising the conceptualisations until an adequate solution is obtained. Students require a host of competencies to solve modelling problems successfully. Generally, the novice modellers have shown that their competence of making assumptions is rather weak. As mathematisation cannot be said to be dissociated with mathematical competence, students need to exercise a mix of modelling and mathematical competencies to manage and address the many different aspects of the task. In this regard, this assessment has presented what students are capable of in terms of making assumptions, mathematical reasoning and interpretation compared to a traditional paper-and-pen assessment or other group activities. Since such modelling activity platforms are meant to help students develop their modelling competencies, it can only be viewed with optimism that the students' competencies will be further developed through more of such engagement. As studying students' modelling competence is complex, further work is still needed to add to the repertoire of this knowledge domain. It will be interesting to consider Blomhoj and Jensen's (2007) three dimensions to describing and supporting progress in students' mathematical modelling competency in the future analysis of this work as well, namely, by examining (1) the degree of coverage, according to which part of the modelling process the students work with and the level of their reflections, (2) the technical level, according to which kind of mathematics the students use and how flexible they are in their use of mathematics, and (3) the radius of action, according to the domain of situations in which the students are able to perform modelling activities.

Appendix

Determining the Most Efficient Bus Route

Ms Chang has recently moved to Block 297C Punggol Road. She is going to start teaching at Punggol Primary school next week and needs to know how to travel to the school. However, the MRT is always too crowded for her to take and it also requires her to take a feeder bus which results in inconvenience. Ms Chang realises that there are three bus services that ply different routes to her school. Help her to find the most efficient route to travel by bus from her home to the school. The location of her home is marked in the map. Currently the three bus services that are available for Ms Chang to choose are Service 124, Service 62 and Service 89. The routes for Service 124, Service 62 and Service 89 are marked as blue, yellow, and pink lines respectively on the map. The bus stops along each bus route are marked with stickers with corresponding colours.

Your task is to give Ms Chang a proposal consisting of the following:

1. How your group determines what is meant by the "*most efficient*" bus route
2. *Assumptions* about the problem your group made in order to help Ms Chang
3. The *mathematics* used to decide which route is the most efficient
4. How your group *justifies* that the selected route is the most efficient
5. The final *recommended* route for Ms Chang

For us to better understand your work, you can attach the following to your proposal:

(a) A map containing the chosen bus route.

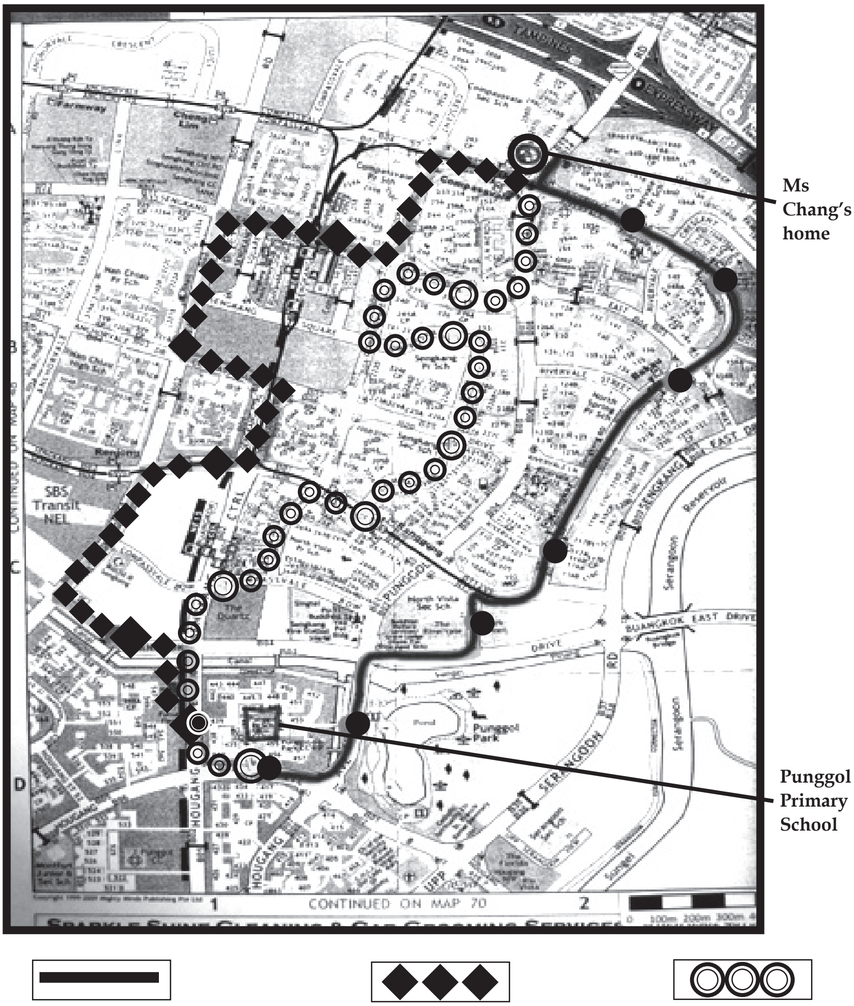
(b) The information you found useful for this task

Bus Fares

Distance Range	*Bus Fare (Cash)*
First 1 km	$1.10
Up to every 0.7 km increase	30 cents increase

Note: The student-groups were given a map in which the three different routes for selection were marked using pink, blue, and yellow colours. Bus stops were indicated along the routes using coloured stickers. In Singapore, bus fares are calculated according to the distance travelled.

The map showing Ms Chang's residence, the school, and the bus routes



Note: The larger shapes along each line are the bus stops.

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Authors:

Chan Chun Ming Eric; PhD, National Institute of Education, Nanyang Technological University, 1 Nanyang Walk, Singapore 637616
e-mail: eric.chan@nie.edu.sg

Ng Kit Ee Dawn; PhD, National Institute of Education, Nanyang Technological University, 1 Nanyang Walk, Singapore 637616
e-mail: dawn.ng@nie.edu.sg

Widjaja Wanty; PhD, Faculty of Arts and Education, School of Education Deakin University (Melbourne Campus), 221 Burwood Highway, Burwood, Victoria, 3125, Australia
e-mail: w.widjaja@deakin.edu.au

Cynthia Seto; MSc, Master Teacher, Academy of Singapore Teachers, Ministry of Education, 2, Malan Road, Singapore 109433
e-mail: cynthia_seto@moe.gov.sg