Title	Thinking strategies: Its effectiveness in the teaching of multiplication facts to low achievers of mathematics
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Source	*Singapore Journal of Education, 8*(2), 32-44
Published by	Institute of Education (Singapore)

Thinking Strategies: Its Effectiveness in the Teaching of Multiplication Facts to Low Achievers of Mathematics

Fong Ho Kheong

Many students have found the learning of multiplication a difficult task. Without mastering multiplication facts, a student will always resort to fundamental strategies which may be undesirable in problem-solving. Many teachers and educators agree that learning basic facts in the four operations are the fundamental steps that precede learning harder facts. Referring to the aspect of computational errors, Knifong and Holton (1975) indicated that computational errors accounted for 49% of the total errors made by 35 students who sat for the Metropolitan Achievement Test. Suydam (1975) and NCTM (1977) stressed the importance of basic facts even though calculators and lately computers have been introduced into the school curriculum. The purpose of this study was to investigate the methodology of teaching basic facts which would alleviate the problems of teaching these facts to low achievers of Mathematics. A review of studies on thinking strategies shows that very few were carried out in learning the thinking strategies, particularly in multiplication using low achievers as the target. The success of using thinking strategies to teach multiplication number facts to most students does not imply its success in teaching all students. One of the intentions of this study is to investigate what strategies are useful in helping the low achievers in learning mathematics.

"Relationship Model"

Ashlock (1983) indicated that there is no single theory comprehensive enough to explain learning, organise content and choose a single strategy. To be effective in teaching, it is necessary to acquire relevant and suitable ideas from theories which provide basic information that can help to organise a comprehensive strategy in teaching. The following paragraphs present some theorists' views which contribute to the learning of skills in multiplication facts.

Piaget's (1972) theory of equilibrium concept indicated that ideas and thoughts in equilibrium cannot be changed easily. New experiences will reorientate learners' outlook and reorganise their way of thinking. These experiences will make them assimilate and accommodate new information to reach equilibrium state.

Skemp extended Piaget's work on cognitive development in mathematical learning. Skemp (1971) considered experiences as fundamental to concept formation. Understanding and hence learning occurs when children can relate a new concept or skill to other concepts they already know. The new concept is said to be assimilated and accommodated so that it becomes part of the whole cognitive structure.

In Bruner's (1966) theory, the basic element is the importance of exposing the learner to the structure of a subject which stresses relationships between parts rather than presenting the subject as a string of unrelated concepts or skills. Seeing relationships between parts helps the child to comprehend, remember and apply them to a new situation.

According to Gagne' (1968), mathematical tasks can be organised into hierarchies of component skills. A child would not be able to acquire certain capability until all the subordinate skills have been learned. To learn a higher order skill, a child is required to relate it to its subordinate skills.

Ausubel's (1968) meaningful learning emphasized the teaching of the structure of a dis-

cipline. For him, the precondition for learning is that the learning task must be meaningful through its relation to the learner's existing cognitive structure. By relating new mathematical concepts and principles to previous mathematical structures, students are able to assimilate and accommodate the new concepts into the old structures. According to Ausubel, earlier meaningful learning provides the means to relate the new concepts or principles to be learned.

Analysis of the theories contributed by Ausubel, Bruner, Gagne', Piaget and Skemp seems to indicate the emphasis of the 'relationship' factor. To them, earlier skills, structures, concepts or principles when properly taught or have been internalised in the learner's cognitive structure, are essential contributory factors to the understanding and acquiring of new concepts or skills. Thinking strategies are in fact, when carefully analysed, capitalising on this model ("relationship") in their structure of teaching multiplication facts.

Thinking Strategies

Thinking strategies were first introduced by Brownell and Chazel (1935) in solving basic facts. In 1943, Brownell and Carper carried out a study on the thinking strategies used by students in solving multiplication combinations. Eight strategies identified were meaningful habituation, rote memory, guessing, using known solution, counting, reversal, reciting tables and visualisation.

Thornton (1976) suggested that using relationships among number combinations and encouraging patterns of thinking can help children learn mathematics. The strategies used in her study (1978) included patterns, relationships, twice as much, adding on, substracting from, finger multiplication for 9's and commutativity.

Rathmell (1978) indicated that thinking strategies may be the solution to help children recall facts immediately. His thinking strategies used in teaching multiplication are skip counting, repeated addition, solution (add on), twice as much as a known fact and patterns.

A comparison of thinking strategies used by Brownell, Thornton and Rathmell shows that there are differences in the use of thinking strategies in teaching multiplication facts.

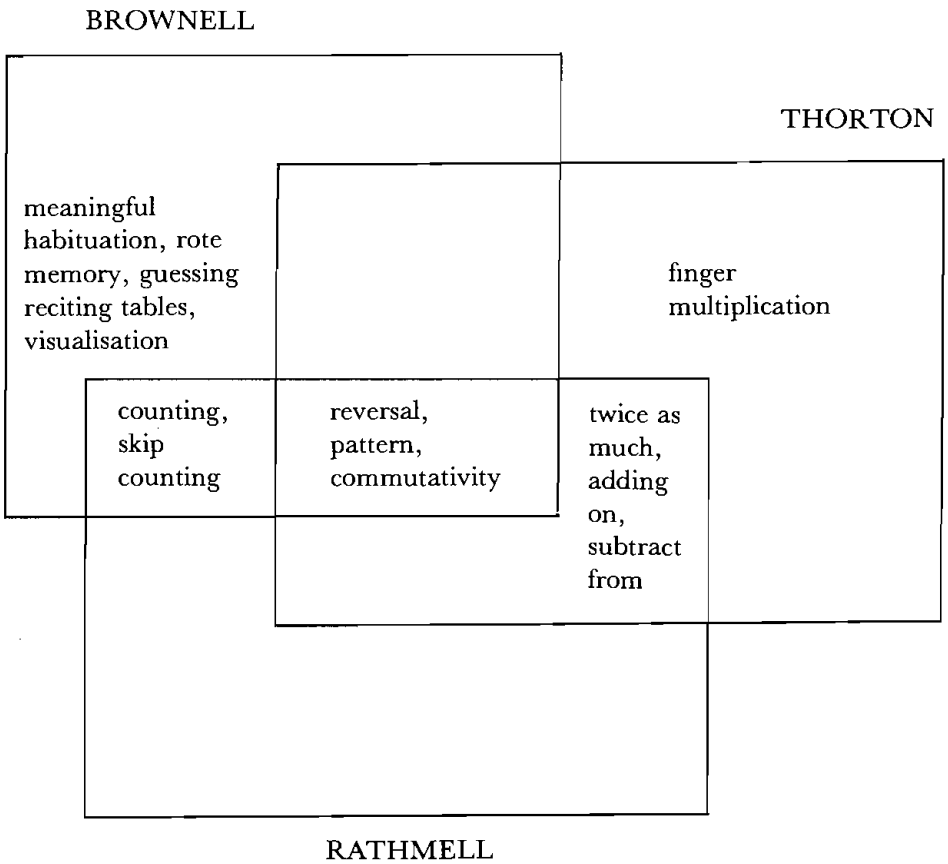


Figure 1

Thornton included the use of strategies that required some form of manipulation of numbers or aids (such as fingers). These strategies were not used in Brownell's and Rathmell's studies. Rathmell's approaches seem to lean toward the 'relationship model' whereas Brownell and Thornton included other strategies such as rote memory and finger multiplication. Figure 1 shows the different strategies used by them.

The difference in the use of thinking strategies shows that there were no consensus in the exact definition of thinking strategy. In this study, pattern, commutativity, finger multiplication, skip counting for five and deriving multiplication facts from known facts are classified as thinking strategies. The following examples illustrate the use of thinking strategies to solve multiplication.

According to Thornton (1978), $7 \times 8 = 56$ is a difficult multiplication fact to be mastered by low achievers in mathematics. However, if they apply the thinking strategy, and through practice, they should be able to recall it. To learn this multiplication fact, a learner is required to recall $7 \times 9 = 63$. This learner is expected to be able to recall $7 \times 9 = 63$ which is classified as an easier strategy (using 'finger' strategy). Hence, 7×8, is obtained by substracting 7 from 63 (i.e. $7 \times 8 = 63 - 7 = 56$). Similarly 7×6 can be obtained by recalling $7 \times 5 = 35$ and then 7 is added to 35 i.e. $7 \times 6 = 5 + 7 = 42$.

Previous Research

Research which has been carried out on thinking strategies seems to cluster around a few areas which include: (a) the effectiveness of the strategies on the four operations of basic facts; (b) the correlation of the maturity levels of the strategies (simple and complex levels of strategies) and the achievement of the subjects; and (c) the variety of strategies.

Brownell and Carper (1943) indicated in their study that thinking strategies were effective methods for solving basic combinations. Their claim was supported by Rathmell (1978) and Thornton (1978). Thornton's study (1978) also showed that there was significant effect in favour of the experimental group (thinking strategy) on the retention test of the four operations. One of her prominent findings was that many students seemed to adopt strategies that were explicitly taught or encouraged during instruction. Positive findings were also reported by Thiele (1938), Swenson (1949), Cook and Dossey (1982) and Carmine and Stein (1981) regarding the effectiveness of the use of thinking strategies.

With respect to the maturity levels of the thinking strategies, Brownell showed that there was no correlation between students' achievement scores and the maturity level of the thinking strategies students used to solve the combination. However, Anderson (1980) and Kalin (1978) refuted Brownell's claim. They showed that achievement and maturity levels in multiplication strategies were closely tied.

Analysis of students' responses revealed the different types of strategies used by them. Thornton (1978) reported that the variety and number of strategies described by the experimental group (using thinking strategies) was greater than that of the control group (non-thinking strategies). Anderson's (1980) results also indicated that students tended to use a variety of strategies and not just one simple single type.

Hypotheses

Thornton (1978) and Cook and Dossey (1982) reported that their results were in favour of using thinking strategies to teach multiplication facts. However, no research study has been carried out specifically to investigate their effectiveness in teaching multiplication facts to help low achievers in mathematics. Issues were raised by Cifarelli and Wheatley (1979) regarding the use of the thinking strategies in teaching basic facts. One of them was the drill method which Cifarelli and Wheatley thought would be effective and that thinking strategies might not be necessary. Reports by Thornton and Cook and Dossey did not indicate whether such a drill element was used in their approach. Pupils' are expected to retain their facts if they are taught using the thinking strategies approach. In view of the issues above, the study seeks to answer these questions:

(1) Is there any difference in the performances between students who use thinking strategies and the traditional approach in teaching multiplication facts to low achievers in mathematics?
(2) Is there any difference in the performances between students who use thinking strategies followed by systematic drill and the traditional approach followed by systematic drill in teaching multiplication facts to low achievers in mathematics?
(3) Is there any difference in the performances between students who have acquired multiplication facts through the thinking strategies' approach and those who do not make use of the strategy in retaining the facts?
(4) Is there any difference in the performances between students who used thinking strategies to solve harder facts (the 10 monsters) and those who use the traditional approach in solving the same facts?

METHODS

Sampling

From the whole level of Primary 3 pupils of the Serangoon Garden South Primary School, those who scored below 75% in the pre-test were selected for the study (In this study, pupils who scored below 75% in the multiplication facts tests were considered as low achievers). Using the blocking technique, they were randomly assigned to 2 different groups (experimental and control groups). Table 1 below shows the distribution of number of students in each range of scores (10 — 25, 26 — 50, 51 — 74 and 75 — 100) of the pre-test and table 2 shows the distribution of the number of students in each lower and higher group with respect to the range of scores.

TABLE 1 DISTRIBUTION OF NUMBER OF STUDENTS IN EACH RANGE OF SCORES OF THE PRE-TEST

Range of Scores	No. of Students		Total
	Experimental Group	Control Group	
0 — 25	1	1	2
26 — 50	3	3	6
51 — 74	9	9	18
75 — 100	29	30	59

TABLE 2 DISTRIBUTION OF NUMBER OF STUDENTS IN EACH GROUP (LOWER AND HIGHER) WITH RESPECT TO THE RANGE OF SCORES

Range of Scores	No. of Students			
	Experimental Group		Control Group	
	Lower	Higher	Lower	Higher
0 — 58	7		7	
60 — 74		7		7

Instruments

Six instruments were used in this study for the collection of data required in the analysis.

(1) *The Digit Span Test*
This test contained 14 questions and was used to determine the retention power of the subjects. Each question required the students to recall the digits read to them. The number of digits in each question ranged from 3 to 9 digits.

(2) *The Multiplication Facts Test*
This test was used in pre-test, mid-test, post-test and retention test. Each test consisted of 50 questions which were randomly selected from the one hundred multiplication basic number facts. It was to test the student's achievement in the multiplication facts. During each test, the subjects were only given six seconds to respond to each question.

(3) *The Diagnostic Profile of Multiplication Facts*
This clinical instrument was developed to probe into the students' concept, understanding and strategies used in solving multiplication facts. This instrument consisted of four sections. The first two parts concerned the concept of rows, columns and multiplication using the array model. The third and fourth sections of the instrument dealt with the structure and pattern of the multiplication facts respectively.

(4) *Teaching Materials 1*
Teaching Materials 1 was constructed to teach the experimental group. The rationale of designing these materials was based on the thinking strategies approach (pattern, commutativity, finger multiplication, skip counting for 5 and deriving multiplication from known facts). The sequence of presenting the multiplication number facts was in accordance with Thornton's and Bolduc's approaches. The facts were presented in the following sequence: 0 and 1 facts, 2 facts, 5 facts, 9 facts, square facts, commutative property and ten monsters.

(5) *Teaching Materials 2*
This was construced and developed to teach the control group on the basis of the responses of the subjects who had undergone the pre-instructional clinical interview. The strategies emphasised in these materials were mainly rote memory and using the immature strategies such as skip counting and repeated addition in solving multiplication facts. Students were found mainly to apply the skip counting and reciting of table strategies in solving multiplication facts.

(6) *CAI Drill Programmes*
Two graded CAI programmes (Mul Drill 1 and Mul Drill 2) were developed by the writer for use during the drill sessions. Mul Drill 1 was designed and developed in accordance with Thornton's strategy of sequencing number facts. In the Mul Drill 2, multiplication number facts would appear at random. Both multiplication programmes were used in both groups.

Procedures

The pre-test post-test control group design was adopted to investigate the effectiveness of the thinking strategy method in assisting pupils with difficulty in multiplication combinations. Thinking strategies treatment was administered to the experimental group. The method used in remediating those students in the control group was the traditional approach (i.e. the method used by students before the study). Drill and practice using CAI was administered to both groups of children after formal instruction.

A series of tests was implemented at each particular time. A pre-test and a digit span test were administered before remedial work was conducted. Just before the CAI drill was introduced to the students, a mid-test was administered. All students completed the post-test at the end of the drill session. Two weeks after the post-test, a retention test was again administered to both the experimental and control groups.

Clinical interviews were conducted with a sample of the selected subjects. The first inter-

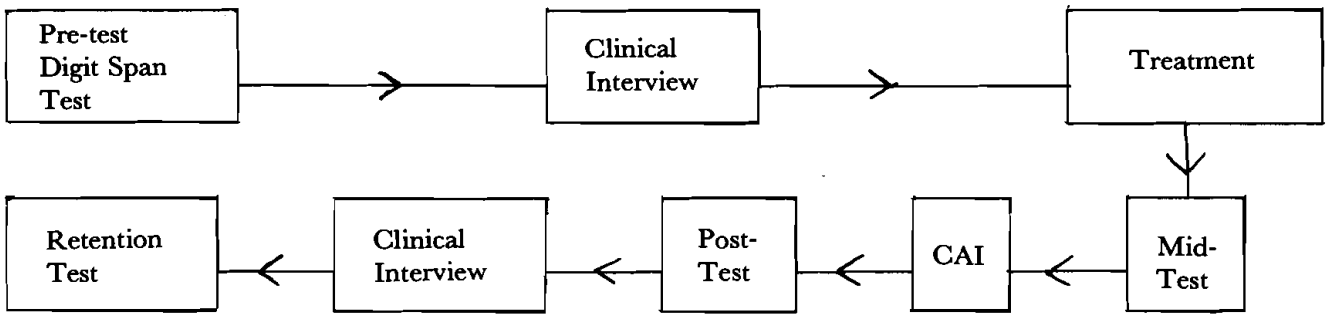


Figure 2

view, conducted before the formal instruction, was to find out the strategies used by the students and their weaknesses in solving multiplication facts. The second interview, conducted after the post-test, was to determine whether the students in the experimental group used the thinking strategies in solving multiplication number facts. Figure 2 summarises the sequential stages in conducting the study.

RESULTS

Pre-instructional Clinical Interview

Results show that not all of the subjects interviewed had mastered the multiplication concepts using the array models. They seemed to mix up columns and rows. They tended to show the number of pieces instead of the number of columns and rows. For example, 8 pieces of chips were arranged into 4 × 2 array. When the subjects were asked to identify the number of rows and the number of columns, they responded with 2 rows and 8 columns respectively.

Students were found to have difficulty in verbalising and writing the array model symbolically. Most of the students were found to be poor in commutative property. None of them had knowledge of associativity and distributivity.

The strategies generally used by some of the students were skip counting and repeated addition. None of them were familiar with the five pattern and the strategies used in solving 9 facts.

Pre-, Mid-, Post- and Retention Tests

Figures 3 to 8 show the comparison of the mean scores of the pre-test, mid-test, post-test and retention test of the lower/higher/combined group of the experimental/control group.

Figure 7 indicates that subjects from the combined experimental group improved in the remedial programme particularly 9 facts, 5 facts, square facts and the 10 monsters (3 × 4, 3 × 6, 3 × 7, 3 × 8, 4 × 6, 4 × 7, 4 × 8, 6 × 7, 6 × 8, 7 × 8). In this group 75% criterion score was only achieved after the CAI drill was administered. In the Mid-test, only the higher experimental group (Figure 5) achieved the 75% criterion score with respect to all facts except 10 monsters. For the post- and retention tests, subjects from higher experimental group achieved 75% score for all the facts. However for the lower experimental group (Figure 3), subjects did achieve the criterion score in the simple facts such as 0 & 1, 2 facts and 5 facts.

For the combined control group (Figure 8), the mean scores of the 9 facts, square facts, the 10 monsters and the 100 facts were still below the 75% criterion score in the mid-, post- and retention tests. This traditional programme had some effect for the easier facts (2 and 5 facts). On the whole, the result indicates that both the lower and higher control groups still had not mastered the harder facts.

Table 3 shows the result of the t-tests on the mean differences between pre-/mid-post-tests and mid-/post-/retention tests. Little or no improvement was observed in the control group during the pre-retention and post-retention stages. The results show improvement for

FIGURE 3 COMPARISON OF MEAN SCORES OF THE PRE-, MID-, POST- AND RETENTION TESTS OF THE LOWER EXPERIMENTAL GROUP

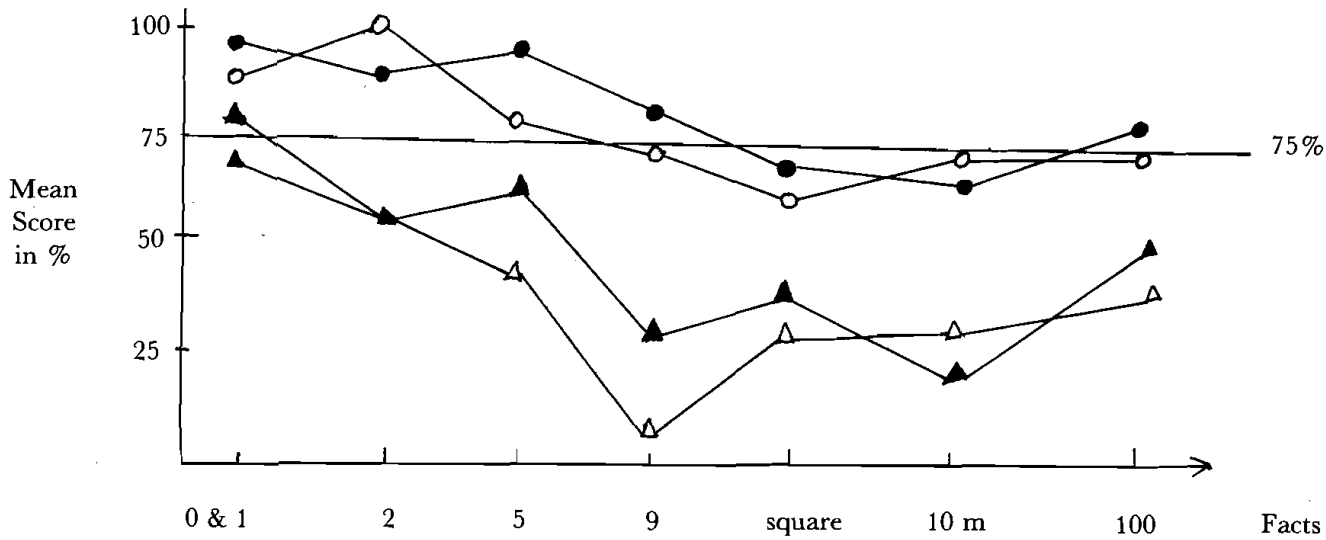


FIGURE 4 COMPARISON OF MEAN SCORES OF THE PRE-, MID-, POST- AND RETENTION TESTS OF THE LOWER CONTROL GROUP

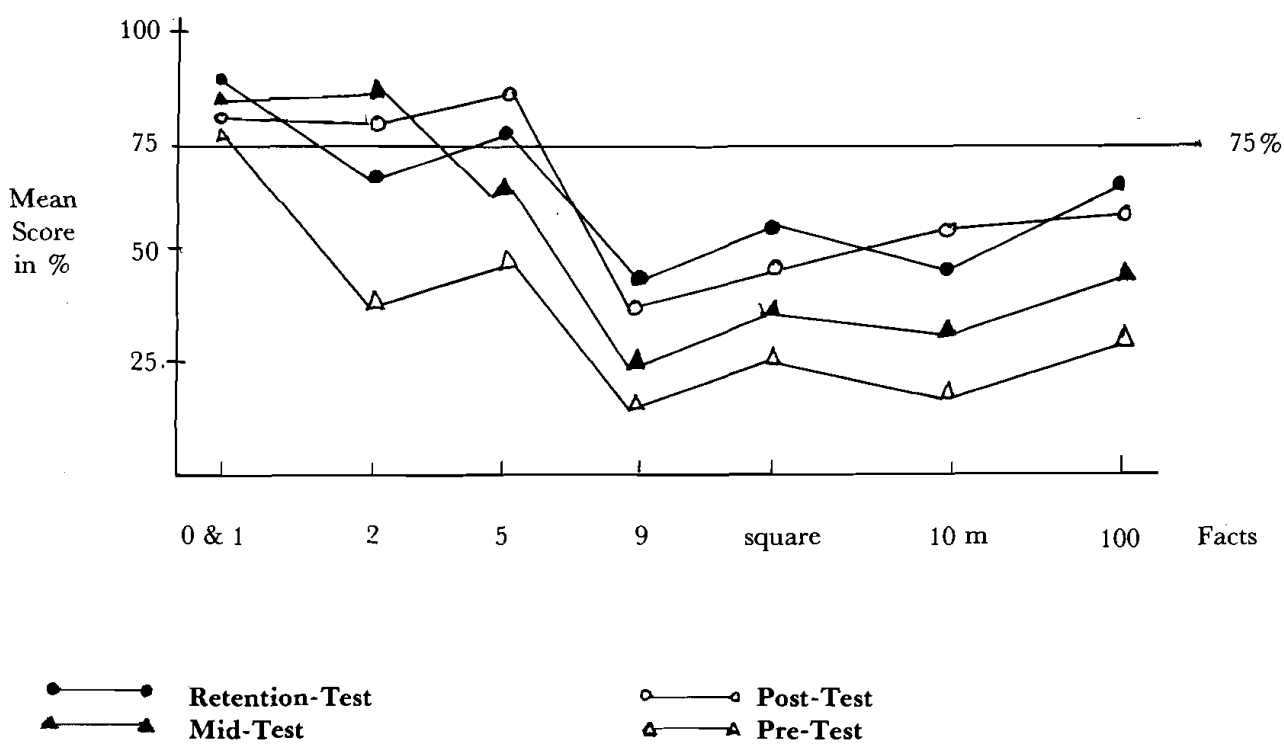


FIGURE 5 COMPARISON OF MEAN SCORES OF THE PRE-, MID-, POST- AND RETENTION TESTS OF THE HIGHER EXPERIMENTAL GROUP

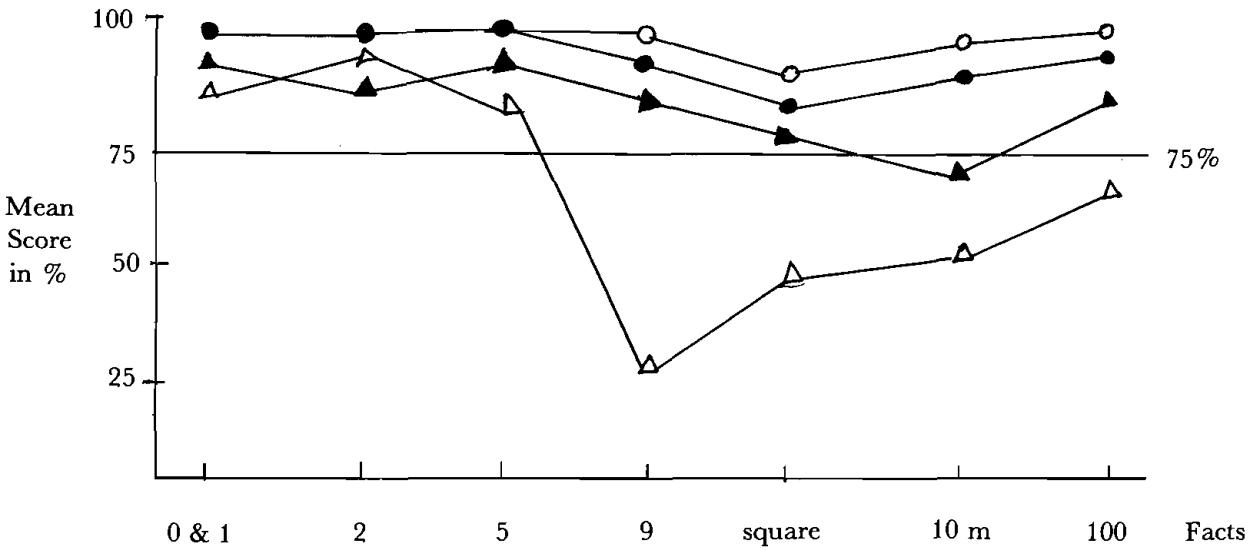


FIGURE 6 COMPARISON OF MEAN SCORES OF THE PRE-, MID-, POST- AND RETENTION TESTS OF THE HIGHER CONTROL GROUP

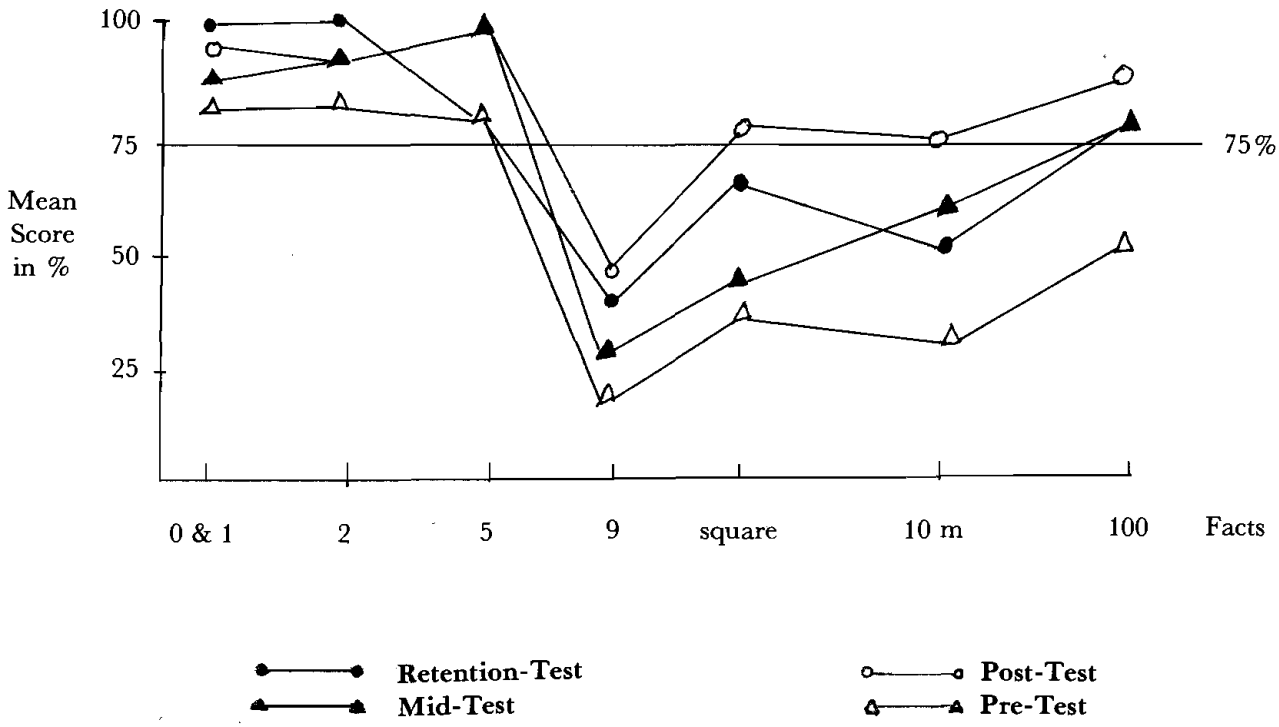


FIGURE 7 COMPARISON OF MEAN SCORES OF THE PRE-, MID-, POST- AND RETENTION TESTS OF THE COMBINED GROUP (LOWER & HIGHER) OF THE EXPERIMENTAL GROUPS

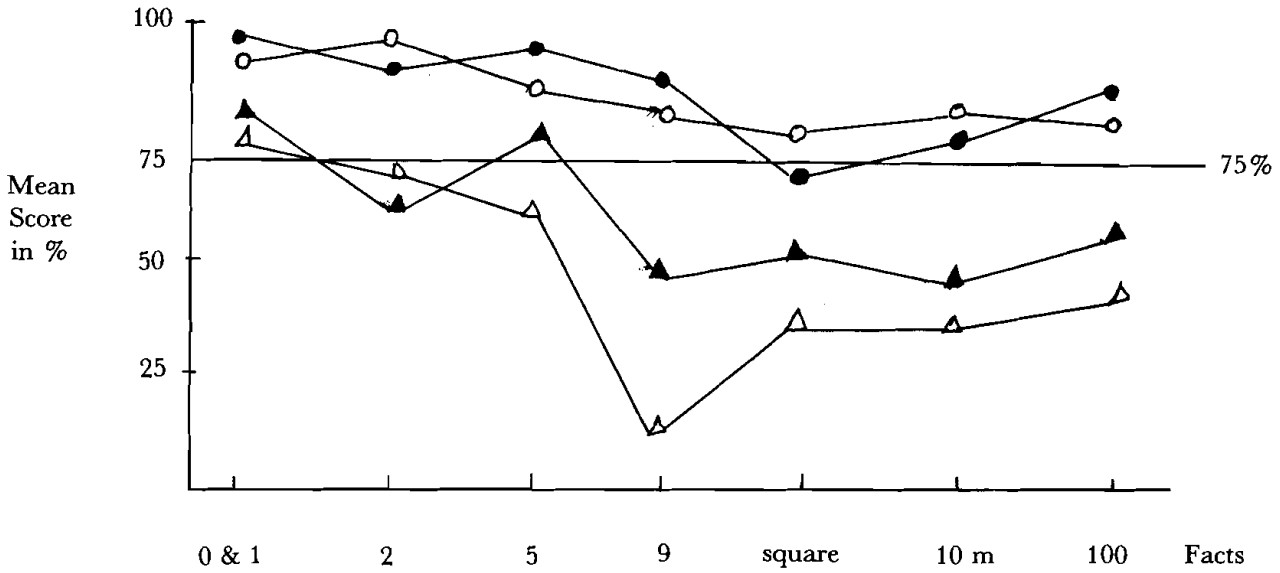
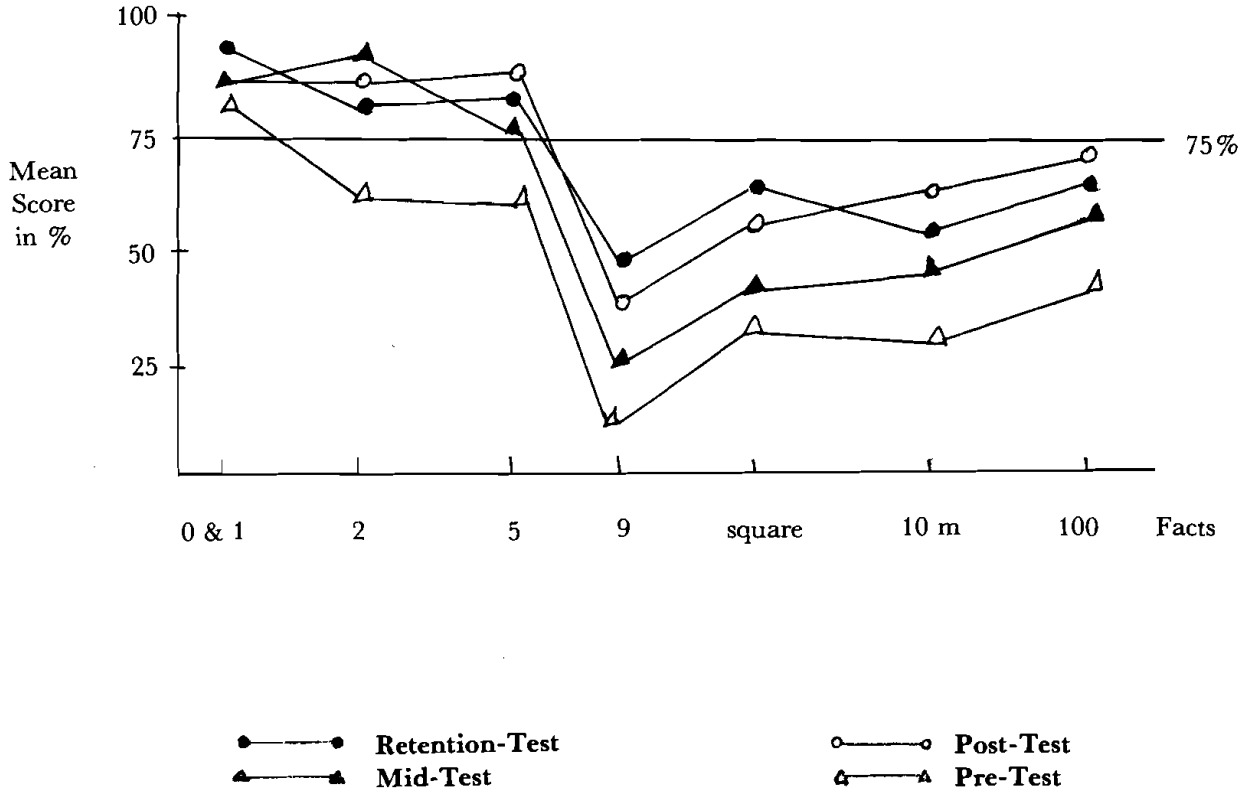


FIGURE 8 COMPARISON OF MEAN SCORES OF THE PRE-, MID-, POST- AND RETENTION TESTS OF THE COMBINED GROUP (LOWER & HIGHER) OF THE CONTROL GROUPS



the higher control group for the pre-mid test, pre-post test and the mid-post test. Detailed analysis shows that for the higher control group, 9 facts showed greatest improvement among the multiplication facts. For the lower control groups, the pupils seemed to have improved in the easier facts (2 facts) after the remedial programme. It seems that the remedial programme had some effects on the higher control group only. Statistically no effect was observed for the lower control group.

Considering the experimental lower group, significant difference was not observed at the pre-mid test (Table 3). However significance was only observed at the pre-post stage (i.e. after CAI drill had been administered). This indicates that thinking strategies did not work well for this group without going through the CAI drill. But the thinking strategies had an effect on the experimental higher group even before the CAI drill was administered. Analysis of the results shows that thinking strategies

TABLE 3 t-TESTS ON THE MEAN DIFFERENCES BETWEEN PRE-/MID-/POST-TESTS AND MID-/POST-/RETENTION TEST ON THE 100 FACTS.

Types of Tests	Group	Significant Level on Mean Differences	
		Experimental	Control
Pre & Mid	Lower	NS	NS
	Higher	0.05	0.01
	Combined	NS	NS
Pre & Post	Lower	0.01	NS
	Higher	0.01	0.01
	Combined	0.01	0.05
Pre & Retention	Lower	0.01	NS
	High	0.01	NS
	Combined	0.01	NS
Mid & Post	Lower	NS	NS
	Higher	0.01	0.01
	Combined	NS	NS
Post & Retention	Lower	NS	NS
	Higher	NS	NS
	Combined	NS	NS

For the experimental lower group (Table 3), significant difference was observed between the pre-post tests and the pre-retention tests. Referring to figure 3, it seems that the contributing factors were mainly due to the 2, 9 and 10 monsters facts.

For the experimental higher group, significant difference was observed between pre-mid, pre-post, pre-retention and mid-post tests. Figure 5 indicates that the 9, squares and 10 monsters facts contributed to the significant improvement.

are required to help the lower group to learn multiplication facts.

Tables 4, 5 and 6 below summarise the difference in mean of the mid-/post-/retention tests between experimental and control groups using analysis of covariance. The two covariates inserted into the design to remove extraneous variables from the dependent variables were the subjects' scores in the digit span-test and the pre-test.

In Table 4, no mean difference was observed (except for the 9 facts,) between the combined

TABLE 4 SUMMARY OF DIFFERENCE OF MEANS OF MID-TEST BETWEEN EXPERIMENTAL AND CONTROL GROUPS USING ANCOVA

Facts	Significance Level Between 2 Means		
	Lower Group	Higher Group	Combined Group
100	NS	NS	NS
0 & 1	NS	NS	NS
2	NS	NS	NS
5	NS	NS	NS
9	NS	0.01	0.05
Square	NS	0.05	NS
10 m	NS	NS	NS

TABLE 5 SUMMARY OF DIFFERENCE OF MEANS OF POST-TEST BETWEEN EXPERIMENTAL AND CONTROL GROUPS USING ANCOVA

Facts	Significance Level Between 2 Means		
	Lower Group	Higher Group	Combined Group
100	NS	0.01	0.05
0 & 1	NS	NS	NS
2	NS	NS	NS
5	NS	NS	NS
9	0.05	0.01	0.01
Square	NS	0.05	NS
10 m	NS	0.01	0.01

TABLE 6 SUMMARY OF DIFFERENCE OF MEANS OF RETENTION-TEST BETWEEN EXPERIMENTAL AND CONTROL GROUPS USING ANCOVA

Facts	Significance Level Between 2 Means		
	Lower Group	Higher Group	Combined Group
100	NS	NS	0.01
0 & 1	NS	NS	NS
2	NS	NS	NS
5	NS	NS	0.05
9	0.05	0.01	0.01
Square	0.05	NS	NS
10 m	NS	0.01	0.05

experimental and control groups. With respect to higher groups, only for the 9 facts and square facts, the mean difference was significant at 1% and 5% levels respectively. Results seem to show that thinking strategy was effective for certain facts (5 and 9) in the higher group only before drill was introduced.

Post-test results (Table 5) show that there is a difference in the mean scores between the experimental group (using thinking strategy followed by CAI) and the control group (using traditional approach followed by CAI) in the teaching of 9 facts, 10 monsters and the 100 facts. The trend persists in the higher group

but for the lower group only the 9 facts strategy was found to be effective. For the retention test, difference of means was significant but the contributing factors were the 9 facts and the 10 monsters.

With respect to hard facts (9, square and 10 monsters), the strategy in teaching the 9 facts was found to be effective. The strategy used to teach 10 monsters to the higher group was also effective.

It can be inferred from the results in Table 4 and Table 5 that CAI drill plays an important role in improving the experimental group's general performance (100 facts mastery). The significant gain was mainly due to the 9 and 10 monsters strategies.

Post Clinical Interview

Results from the interview indicated that students from the higher group were able to apply the thinking strategies during the remedial instruction. Strategies used were doubling the multiplication, using known solution, finger multiplication for nine and pattern method.

Discussion

With respect to the 4 hypotheses, the study shows that thinking strategy seems to be more appropriate for the higher category of the low achievers in solving harder facts (9 and square facts). Experimental group (using thinking strategy followed by CAI drill) did better than the control group (using traditional approach followed by CAI drill) on the 100 facts. Furthermore it was found that CAI drill could effectively raise the performance of the lower group although thinking strategy may not be appropriate. Students from the higher experimental group should retain facts learned better than the control group. Furthermore investigation through clinical interview showed that this group of students did use thinking strategies in solving multiplications. Hence this supports Thiele and Swenson study that by using thinking strategies facts could be retained better. The study also showed that the experimental group did better in the harder facts. This could be due to the fact that they made use of the thinking strategies.

The study also gives a further insight into the controversial claim by Cifarelli and Wheatley that thinking strategies may not be necessary in the learning of basic facts. It shows that drill can raise the performance of the experimental lower group evidenced by the mid-test and post-test results of the 2 and 9 facts. However for the lower control group, the mean difference between the mid-test and post-test was not significant in any of the categorised facts. The results indicate that thinking strategies helped the lower group to perform well in certain facts after the CAI drill sessions. However, without thinking strategies, no gain was observed after the drill sessions. This refutes Cifarelli's and Wheatley claim when the lower group is considered. This study, therefore, supports Brownell's claim that thinking strategies are effective for solving multiplication facts.

One implication which can be drawn from the results is that thinking strategy is an effective intermediate aid in mastering multiplication facts. Drills, although considered to be traditional, do have their place in teaching.

The subjects in this study had acquired some strategies in multiplication combinations. The interview results showed that some pupils tended to fall back on their previous learned strategies. One question for further research is whether the thinking strategies will produce the same or better results if they had not been exposed to other strategies.

In this study only CAI drill was administered. The question which may arise is whether other type of drill method(s) will produce the same effect as CAI drill. It is possible that the better performance was due to the use of the microcomputer.

The time (in implementing the treatment strategies and drill and practice) and sex variables were not considered in this study. These could be incorporated into further research.

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