
Title	Problem solving in mathematics: Influence of presentation
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Source	<i>Singapore Journal of Education</i> , 8(2), 67-74
Published by	Institute of Education (Singapore)

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Problem Solving in Mathematics: Influence of Presentation

Eric Plant

Introduction

Lesh (1979) suggested that the interplay between the manipulative, pictorial and symbolic modes of representation in mathematics' problems enhances meaningful learning. However, Behr (1976) and Khoury and Behr (1982) contend that it is in fact extremely difficult for many students to achieve this interplay. Students seem to proceed through a problem solution in mathematics in one mode or another and then if requested, solve problems in another mode (Behr, 1976).

The way in which a problem is presented varies in respect of the language which is used and the illustrations, such as pictures, diagrams and graphs which are incorporated in the questions. The problems invariably are specific to a topic and the solutions which pupils attempt are very specific to the task set by the problem. It is the contention of the researcher, therefore, that the way in which the problem

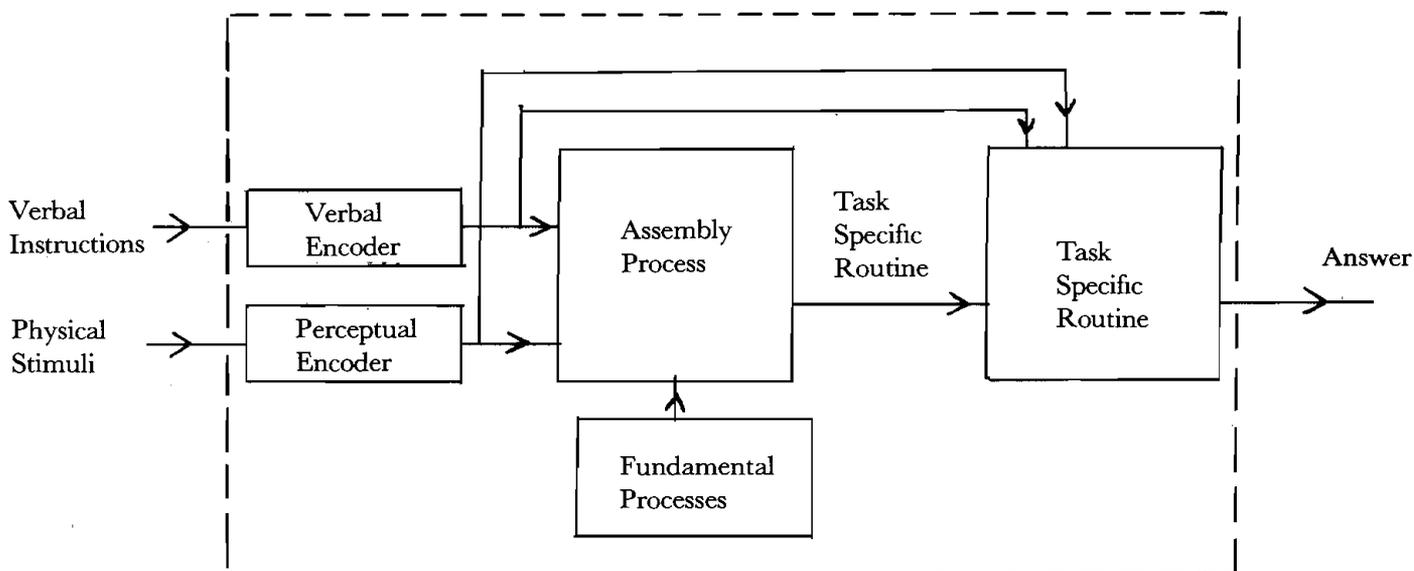
is presented to the pupil is likely to be influential in the pupil's attempt to solve the problem correctly.

The following model of Klahr and Wallace (1970) was used as a framework for this study.

In this model, Klahr and Wallace draw attention to the fact that the pupil has to process problem information in mathematics which is usually presented through the written word with illustrations. The pupil then assembles the data from the problem and calls upon processes which will enable him to solve the problem effectively. To examine the importance of the way in which pupils understand and use the input data, twelve task specific problems were constructed.

The aim was to try to determine the effect which different styles of encoding a question may have on pupil's performance. Twelve mathematics questions were devised for the

KLAHR AND WALLACE MODEL (1970)



Secondary 2 level. Four different ways were considered in which to present each of the questions. These different presentations are referred to as 4 MODES OF PRESENTATION.

Modes of Presentation

MODE 1

This mode is typical of the traditional English 'O' level Examination form of presentation when the problem is given in words only; some of the phrasing is also the style used in Singapore school text books.

MODE 2

The questions in mode 2 are again presented in English but with the expressions made as concise and as "simple" as possible, in order to convey the same problem as in MODE 1.

MODE 3

This mode is essentially a diagrammatic-pictorial one.

MODE 4

Mode 4 is a presentation with diagram and a "simple" explanation in English.

The followings are 4 out of 12 items which were constructed and each one is expressed in 4 different modes, M1, M2, M3, M4. (The remaining items are available from the author)

MODE 1	A number is chosen with some value y . The number is then doubled. 4 is then subtracted. The result of this is divided by 3 and the final value is 12. What was the original value of y ?
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MODE 2	Think of a number, double it. Subtract 4 divide by 3, the answer is 12. What is the number?
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MODE 3	Put in a value A.
	$\text{IN} \rightarrow A \rightarrow \square \rightarrow \square \rightarrow \square \rightarrow \text{OUT}$ $A =$

MODE 4	Put in a value.
	$? \xrightarrow{\times 2} \square \xrightarrow{- 4} \square \xrightarrow{\div 3} \boxed{12} \xrightarrow{\text{result is}} \text{OUT}$
	This diagram shows a sequence of operations. Put in a number with value ? double the number. Subtract 4. Divide by 3 and the result is 12. What is the value of ?

ITEM 6: PRESENTATION OF ITEM 6 IN ITS 4 MODES

MODE 1

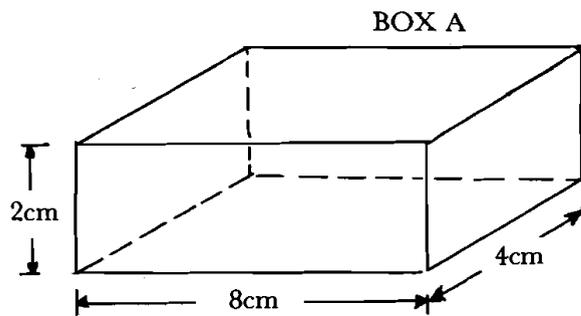
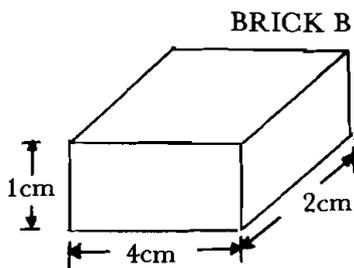
How many bricks of size 4cm in length 2cm in width and 1cm high may be fitted into a box which measures in its inside 8cm in length, 4cm in width and 2cm in height?

MODE 2

I have a box. The inside measures 8cm long, 4cm wide and 2cm high. How many plastic bricks 4cm long, 2cm wide and 1cm high can I fit in the box?

MODE 3

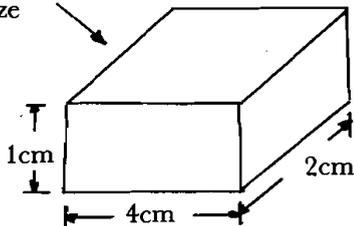
How many bricks B will fit in box A?



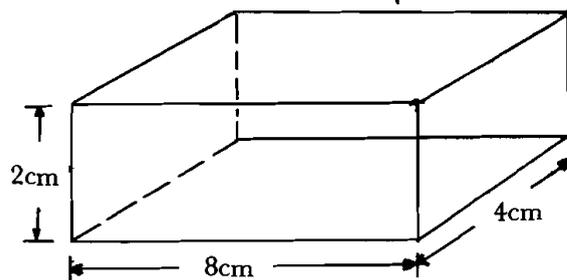
MODE 4

How many bricks will it take to fill the box?

It is filled with some bricks, this size



This is a diagram of a box



ITEM 8: PRESENTATION OF ITEM 8 IN ITS 4 MODES

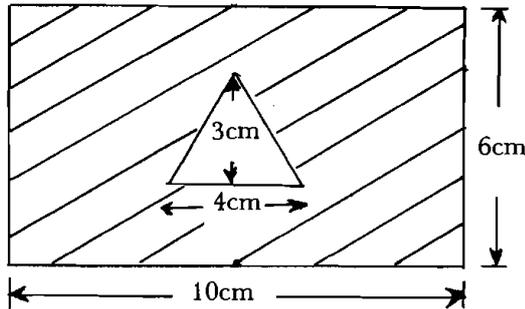
MODE 1

What is the surface area which is left after a triangle with a base of 4cm and a height of 3cm is cut out from a rectangular piece of card which measures 10cm in length and 6cm in its width?

MODE 2

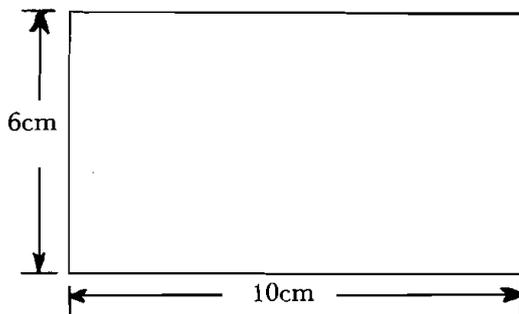
I have a piece of cardboard which is a rectangle in shape. It measures 10cm long and 6cm wide. I cut out a triangle from it. The triangle has a base 4cm long a height 3cm. What is the surface area of the cardboard which I have left?

MODE 3

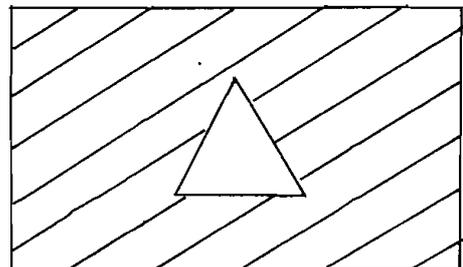
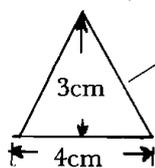


Shaded area = cm²

MODE 4



I cut this size \triangle from the rectangle.
What area is left?



ITEM 10: PRESENTATION OF ITEM 10 IN ITS 4 MODES

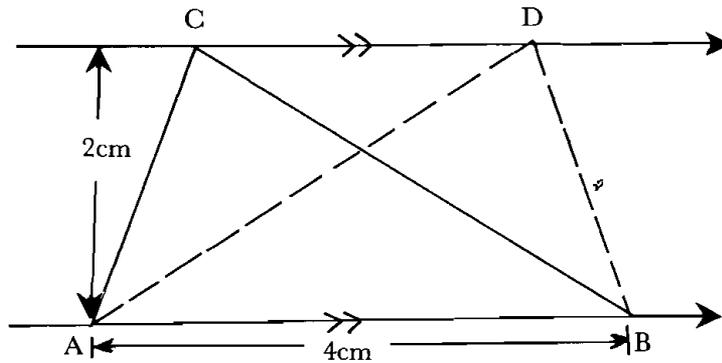
MODE 1

According to a theorem, all triangles are equal in size if they have the same base and the third point in anywhere on a line parallel to the base. What are the areas of two triangles ABC and ABD, if they have AB as their common base and the two points C and D are on a line XY which is parallel to AB?

MODE 2

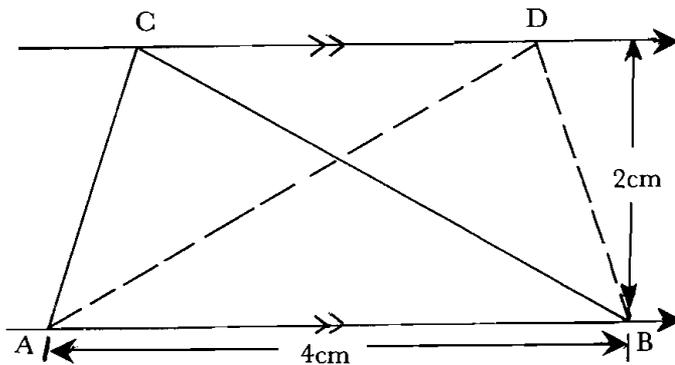
XY and AB are two parallel lines. All triangles with AB as a base and any point C on the line XY as its top are equal in area. If AB is 4cm and the distance between AB and XY is 2cm. What are the areas of two triangles ABC and ABD, if points C and D are two different points on XY?

MODE 3



The area of ABC is cm².
 The area of ABD is cm².

MODE 4



The diagram shows two lines which are parallel to each other and 2cm apart. Between these lines are 2 triangles with common base.

The area of ABC = cm².
 The area of ABD = cm².

TABLE 1 — SCORING FOR EACH ITEM

ITEM NO.	Problem Solving Requirements			TOTAL POSSIBLE SCORE
	Sequence of Steps	No. of Skills	No. of Concepts	
	Q. 1	2	2 Arithmetic Operations	
Q. 2	1	3 Arithmetic Operations		4
Q. 3	1	2 Arithmetic Operations	2	5
Q. 4	1	2 Arithmetic Operations		3
Q. 5	2	Reversibility	2 Area Perimeter	6
Q. 6	3	3 Arithmetic Operations		6
Q. 7	4	2 Arithmetic Operations	2 Fraction Volume	8
Q. 8	3	2 Arithmetic Operations	1 Area	6
Q. 9	3		3 Height Area Volume	6
Q. 10	2		2 Parallel Lines Area	4
Q. 11	3	3 Arithmetic Operations	2 Area	8
Q. 12	4	3 Arithmetic Operations	Area	8

The different item score does not affect the analysis since each item is compared only across its 4 modes.

Scoring Protocol

Each item was appraised to see how many steps, concepts or skills could be identified on a pragmatic basis. Each of these was assigned a score of 1, so that the total number of marks which a test item may score depends on this appraisal (see table 1). Each item was marked by a different marker who went through each paper in the four different modes for that item. This procedure reduced variability and increased consistency in marking.

The Experimental Procedure

Description of the Sample of Students

347 Secondary 2 pupils were tested in an examination type situation. These pupils were from 11 classes in the school. Pupils in 4 of the classes studied Chinese Language 1 (CL1) and pupils in 7 classes studied English Language 1 (EL1) but all received mathematics lessons in English, and all were Express Stream pupils.

177 were boys with ages ranging from 12 years 5 months to 16 years 11 months, while 170 were girls with ages ranging from 13 years 0 months to 15 years 10 months.

Procedures

12 questions were devised and these were written and illustrated pictorially and diagrammatically. They were then expressed in four modes for presentation as already described. Test papers were then compiled by selecting three different questions in each mode and grouping together into four separate test papers.

Findings

The results of the tests based on the 4 different modes of presentation are given in the following tables. The scores for the twelve items are presented for each of the modes, MODE 1, MODE 2, MODE 3 and MODE 4. Table 2 gives the number of pupils(N), mean(M) and standard deviation (S.D.) in each cell of the matrix for the combined results of all pupils.

Table 3 shows a subsequent re-arrangement of the results according to the highest mean scores. These are in descending order from left to right and are represented by the mode for which the appropriate mean score was achieved.

The trend very much suggests that the pictorial plus word (Mode 4) facilitated the

**TABLE 2 — PUPILS' SCORES ON 12 ITEMS
COMBINED SCORES FOR ALL CLASSES**

ITEM	MODE 1			MODE 2			MODE 3			MODE 4		
	N	M	SD									
1	86	3.21	1.49	86	2.95	1.73	88	2.11	1.60	84	2.98	1.17
2	85	1.62	2.01	87	1.72	1.72	84	2.29	1.94	90	2.40	1.84
3	88	1.92	2.36	83	4.12	1.71	87	2.54	2.44	84	2.80	2.37
4	84	2.02	1.39	84	2.14	1.25	98	2.68	2.28	91	2.48	1.04
5	84	2.60	3.24	86	2.54	2.30	90	2.31	2.61	83	2.60	2.51
6	84	3.61	2.57	89	4.37	2.35	83	4.53	2.46	87	4.38	2.41
7	89	4.18	3.48	83	4.14	3.30	88	4.05	2.98	84	3.82	3.14
8	82	3.56	2.56	86	3.85	2.69	83	4.54	1.99	90	4.62	1.68
9	84	0.50	1.14	85	0.69	1.20	90	0.90	1.31	84	1.14	1.53
10	83	0.34	0.96	89	1.10	1.72	84	3.00	1.62	88	3.02	1.68
11	89	0.88	2.65	85	0.88	1.82	89	2.29	2.38	84	2.01	2.59
12	90	3.78	3.69	86	4.13	2.85	84	4.46	3.45	90	4.50	3.40

pupils' encoding and the pupils were able to achieve more successful results. In particular, items 1 and 2 concerned with algebraic problems showed very significant differences in success solutions in favour of modes 3 and 4 presentations. This was also very evident in the data for item 10 which concerned the geometric theorem that triangles on the same base and between the same parallels are equal in area. The diagram presentation in modes 3 and 4 significantly influenced the successful solution of this problem.

The mental representations of a problem therefore, play a significant part in the number of successful solutions which pupils achieved in the problem-solving strategies. In this study there is evidence to suggest that the presentation of a mathematical problem has a significant effect on its successful solution.

Discussion of Findings

Resnick and Ford (1981) make the point that the mental representation of a problem plays an important role in directing the likely course of solution attempts. Both the task environment and individual differences in knowledge

structure influence the problem representation that is achieved. The evidence examined in this study has directed attention to the presentation aspect of the task environment. Experiments on discovery learning and functional fixedness show that verbal instructions and physical aspects of the task environment are especially influential in cuing specific initial responses (Resnick and Ford, 1981, p. 236).

Resnick and Ford (1981, p. 234) go on to ask "What are the possibilities for improving the likelihood of problem solving?" Using the three basic ingredients of problem solving namely prior knowledge, task environment and strategy, it may be possible to suggest instructional interventions that could lead to an improvement in each of these three components. Firstly, instruction could ensure the presence of "well-structured knowledge" to link together related concepts and procedures. Secondly, in a problem, one can state the mathematical details more clearly with a minimum of extraneous information, and which can be accompanied by diagrams and pictures. Pupils may then scan these features and allow their minds to make logical connections, write down thoughts, draw their own

**TABLE 3 — PUPILS' SCORES ON 12 ITEMS
RANK ORDER OF COMBINED RESULTS**

ITEM	Highest Mean Scores		Lowest Mean Scores	
	MODE	MODE	MODE	MODE
1	1	4	2	3
2	4	3	2	1
3	2	4	3	1
4	3	4	2	1
5	4	1	2	3
6	3	4	2	1
7	1	2	3	4
8	4	3	2	1
9	4	3	2	1
10	4	3	2	1
11	3	4	2	1
12	4	3	2	1

diagrams and create a richer encoding of detail with which to build a suitable problem representation. Thirdly, specific problem-solving strategies may be designed to give the pupils strong support for goal analysis, that is, teaching children to think ahead, visualise detail and deploy suitable procedures for an appropriate problem solving strategy. Wickelgren (1974) has developed problem solving routines such as working back from the solution and some of these are being examined for classroom application.

The evidence in this study points to the significance which pictures and diagrams have on the encoding used by pupils in solving mathematical problems. Further research on the relationship between skills and understanding, mental encoding and mathematical concepts will carry with it the need to examine modes of presentation and the strategies which pupils employ in their learning and performance. This implies a need to look at more evidence about the role of physical and pictorial "metaphors" used in understanding mathematical concepts.

The representation of mathematical concepts has been examined by gestalt psychologists.

Their emphasis was on a sensible structuring of mathematical ideas. Cognitive psychologists including Bruner stress the importance of simple but "mathematically correct representations" as the basis for initial learning of concepts. This led in the 1960's to the concrete representations of complex mathematical ideas so as to make sure that the concepts were more accessible to pupils. Structural apparatus such as the Dienes Multi-based Arithmetic Blocks, Dienes attribute material and Montessori materials were used. It has been assumed that working in this way may enable pupils to build up mental representations which will eventually lead pupils to successful mathematical problem solving. Many of these assertions, however, remain to be tested. The research evidence concerned with presentation which is reported in this study is a step in this direction.

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