Developing Creativity In The Singapore Primary Mathematics Classroom -Factors That Support And Inhibit-

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Abstract:
The worldwide trend for reforms in mathematics education is also happening in Singapore. A growing concern within mathematics education is that teaching methods that focus on standard textbook questions and solving problems through drilling only encourage the development of procedural knowledge. This paper describes the use of short open-ended tasks that teachers can implement in the classroom to develop students' mathematical creativity. Three cases of primary teachers implementing such activity are shared to elicit from these experiences the classroom-based factors that could support or inhibit creativity in the pupils' work.

Introduction

The Third International Mathematics and Science Study (TIMSS) put Singapore on the world map when the achievement of her pupils in mathematics exceeded that of competitors from more than 40 countries. The mathematics curriculum in Singapore schools can be said to be traditional with strong emphasis on content that has well-defined inner structures. To challenge pupils' analytical skills numerous complex multiple-step problems are normally given. Pupils are trained to classify problems into types and to tackle them according to some specific methods. These aspects of mathematical learning in developing basic concepts, skills and problem solving have been very successful in our Singapore curriculum for the TIMSS mathematics achievement data released in 1996 and 1997 ranked the performance of Singapore pupils at ages 13 and 9 respectively in the top place. The fact that the TIMSS items had high test-curriculum match with what we were teaching in Singapore schools, was of course an advantage to our pupils participating in the study.

As to the teaching methods in our mathematics classrooms, the prevalent practice is using whole class teaching, textbooks and regular testing (Chang, Kaur, Koay and Lee, 2001). Pupils do a lot of practice sums, mostly one-method and one-answer kind, to consolidate and reinforce the mathematics concepts or procedures taught by the teacher through an expository method. A growing concern within mathematics education is that teaching methods that focus on standard textbook questions and solving problems through drilling only encourage the development of procedural knowledge. Desirable learning outcomes such as meaningful conceptual understanding, critical and creative thinking in problem solving, as well as giving students' ownership in their learning cannot be brought about through such product-oriented methods. Supporters of process-based curriculum are now arguing for creativity in the mathematics classrooms through more open-ended, practical and investigative tasks in mathematics to supplement textbook questions that teachers have been so dependent upon. Students can benefit in many ways while working on the open-ended tasks that would require them to make their own decisions, plan their own strategies as well as apply their mathematical knowledge. In this paper, I will share the experiences of three primary teachers who had tried open-ended questions on their pupils and to elicit from these experiences the classroom-based factors that could support or inhibit creativity in the pupils' work.

Creativity and Open-ended Questions

What does it mean to have creativity in the mathematics classroom? To answer this question will depend on one's view of the nature of mathematics. To many people creativity is not usually associated with the traditional image of school mathematics which is often seen as static structured systems of facts, procedures and concepts. This portrayal of school mathematics has led to lessons where students tediously learn a collection of techniques by following predetermined rules. However there is a view emerging with increasing acceptance that mathematics is an exciting and dynamic science (de Lange, 1993) which focuses on the active generative processes engaged by the learners as they do mathematics. One cannot count on the standard mathematical textbook questions used by teachers to support this new view of mathematics education. Pupils must encounter rich mathematical problems where they can reason and offer evidence for their thinking, communicate and present their ideas in mathematics, and find connections across mathematics as well as in real life. Foong (2002) has been advocating the use of short open-ended questions that teachers can convert from their textbook sums to engage pupils in higher level thinking and creativity. Take for example a standard closed textbook question:

There are 12 oranges to put into 3 bowls. Each bowl must have the same number of oranges. How many oranges are there in each bowl?

By leaving out an element we can transform it to an open-ended version that has higher level cognitive demands:

Open-ended Version

Bowls of Oranges

There are 12 oranges to put into bowls. Each bowl must have the same number of oranges. Show how you could put the oranges into bowls.

Cognitive demands of the open-ended task

- Pupils to make own assumptions about the missing data: the number of bowls/number of oranges in each bowl
- Pupils to access relevant knowledge multiplication, division, fraction, factors etc..
- Pupils to display number sense & equal grouping patterns
- Pupils to use the strategy of systematic listing
- Pupils to communicate their reasoning using multiple modes of representation
- Pupils to display creativity in as many possible strategies and solutions

The characteristic features of such open-ended questions are many possible answers and it can be solved in different ways. They should offer pupils room for own decision making and natural mathematical way of thinking. There are three categories of open-ended questions that teachers can create from textbook sums:

- **Problem with missing data or hidden assumptions**
- **Problem to explain a concept, procedure or error**
- **Problem Posing**

For the purpose of this paper, we will not discuss the construction of such problems which can be found in Foong (2004). Many teachers who had attended the author's workshop on using open-ended questions had tried such question on their own pupils. Initially many of them were apprehensive about giving their students such tasks. It had never been part of their teaching where they required pupils to give explanations and reasons for their solutions. Neither were they used to giving pupils more room and time for group discussions in mathematical problem solving. The practice had always been giving students problems that had only one answer and one taught method of finding it, and mostly as individual work. After the teachers had trialed the open-ended questions with their students, many of them were surprised by the rich responses that most of their students could give. Of course, there were also reports by some teachers that their pupils did not engage in the tasks as intended and were not able to show their reasoning and communication skills. In the following section we will discuss classroom-based factors that support or inhibit thinking and creativity when teachers attempt to use open-ended tasks with their pupils

Classroom-based factors that support and inhibit thinking and creativity

Research has shown that the mere presence of high level tasks in the classroom will not automatically results in pupils' engagement in high-order thinking. The same can be said of open-ended questions that hope to develop critical and creative thinking even though the tasks have been specifically designed for such outcomes. Henningsen and Stein (1997) suggest that although attention to the nature of mathematical instructional tasks is important, attention to the classroom processes surrounding the tasks is equally important. One must create the ambient classroom environment to balance classroom management needs and the academic demands. In their study they found that high level tasks were more susceptible to various factors that could cause a decline in pupils' engagement to less demanding thought processes. High level tasks such as open-ended problems involve more ambiguity and higher level of personal risks for pupils than they are normally exposed to in routine problems. A very important phase in carrying out open-ended high level tasks in the classroom is the **set-up** phase where the teacher announces the task in such a way as to encourage pupils to use more than one strategy, to use multiple representations, to supply explanations and justifications. And during the **implementation phase** to sustain pupils' high level engagement in the task, teacher must be willing to let pupils struggle and that pupils are willing to persevere in their struggle. From their analysis of successful and non-successful classroom processes that aimed to encourage high-level mathematical thinking and reasoning, Henningsen and Stein, observed the following inhibiting and supporting factors:

Inhibiting factors that lead to decline in high-level cognitive demand

- Tendency of pupils not used to ambiguity in the open-endedness of the task would pressure the teacher to provide explicit procedures for completing the task.
- Tendency of teacher to "taking over" difficult parts of the task and performed them for pupils in order to manage their anxiety.
- Tendency of pupils and teacher to drift away from a focus on meaning and understanding toward an emphasis on accuracy and speed; sometimes ignoring the challenging part of the task to quickly get to the answer.

- Lack of alignment between tasks and pupils' prior knowledge, interest, motivation and learning disposition thus leading decline into unsystematic exploration.
- Lack of time for pupils to grapple with the important mathematical ideas embedded in the task.
- Lack of accountability for high level products when pupils were not expected to justify their methods or when their unclear or incorrect explanations were accepted; or when pupils thought such work would not be "counted".

Supporting factors that maintain high-level demands

- Scaffolding actions by teacher or more capable peer whenever a pupil was stuck; providing assistance without reducing the cognitive demands of the task.
- Modeling of high-level performance by teacher or selected pupils.
- Sustaining pressure on pupils to provide meaning, explanation and justification to demonstrate their understanding of the mathematics underlying the task.
- Encouraging pupils' self-monitoring as they progressed through the tasks, giving them a sense of competence and control which in turn motivated them to remain engaged in the task.
- Providing and planning the appropriate amount of time for the whole process.

Polar Bear Problem

For the purpose of this paper, we discuss the classroom experiences of three primary school teachers who had tried ***the Polar Bear*** problem (figure 1) on their pupils.

A Polar bear weighs 500 kg. How many children from our class weigh together as much as the polar bear?

Figure 1: Polar Bear Problem

This open-ended activity aims to elicit pupils' high level cognitive processes and mathematical creativity. It represents an important goal of mathematics education. In an open-ended situation, pupils in addition to applying calculation procedures are also required to solve realistic problems where there is no known solution beforehand and not all data is given. It would require pupils' own contributions, such as making assumptions on the missing data. Without giving the children's weight, it becomes a real problem and the pupils have to think about and estimate the weight of an average child. There is no cue word for pupils to figure out which operation to use as in a closed question. The Polar bear context is also an interesting situation that could arouse curiosity in pupils to want to investigate. From a curriculum perspective the task has been designed specifically for the following high-level cognitive pupils' outcomes:

- Pupils to identify the missing information critical to the question
- Pupils to make own assumptions about the missing data: weight of a child
- Pupils to access relevant knowledge and estimate the average weight of a child in relation to themselves
- Pupils to display number and measurement sense
- Pupils to communicate their reasoning
- Pupils to display creativity in using possible strategies and solutions

Three teachers from different schools used this problem with their P4 classes, of sizes ranging from 39 to 40 pupils. In all the three classes, the pupils worked in small groups. Three different levels of pupil outcomes resulted due to the different orientations of teachers in setting up the task. We compared features of the teachers' task implementation, pupils' engagement on task and outcomes for factors that supported or inhibited high level thinking and creativity in the pupils' work.

Three Cases

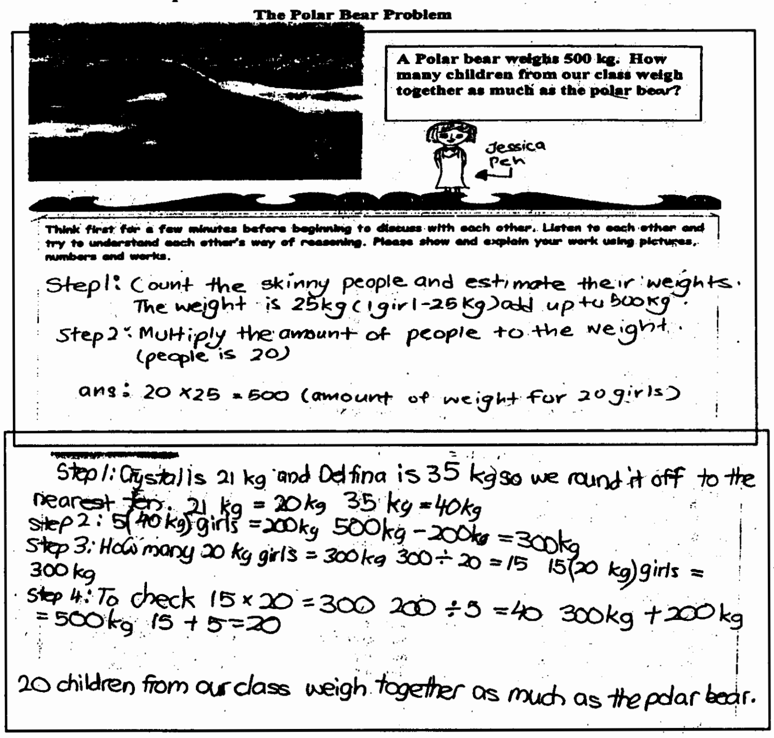
Data for this study were drawn from the case reports written by the teachers themselves. This was part of a project undertaken by the author involving in-service teachers enrolled in her course. The participants had to report on how they set up and implemented the task; made observation of pupils at work, took notes of pupils' behaviors and cognitive processes; interviewed some for their attitudes towards such tasks and their participations in group work. They had also to analyse pupils' work; evaluate and reflect on their lesson outcomes.

Case of Teacher A: Successful

Teacher A set up the task using an interesting introduction to stimulate interest and arouse curiosity in her pupils. She used a catch-theme for them to identify themselves as "*We are math investigators*". She told them that as math investigators they were required to solve problems and explore as many approaches as possible. They were also required to explain, compare and justify their solutions. Before introducing the actual problem she showed pictures of various animals and discuss with pupils their habits and sizes. She was able to use cooperative learning strategy and provided the appropriate scaffolding whenever the pupils needed help to move on. Initially there were looks of frustration in the pupils who found the question ambiguous. She appeased them with clarifications without giving away too much or reducing the cognitive level of the task. She asked the pupils why they thought the question in the problem was not clear. She also asked them what was needed to make the problem solvable. She observed that after clarification of initial doubts, pupils were more proactive and clear in their activity, there was an obvious change in attitudes. While pupils worked in cooperative groups, she discouraged them from dismissing any of their member's ideas without first assessing it critically. They had to ask the "solver" to explain in detail and checked the solution before moving on. At the end of the group session, she selected samples of pupils' work to share with whole class and allowed other groups to comment on their friends' work.

Pupils' Work

As the Polar Bear problem asked for "how many children from *our* class…" most of the pupils tended to personalize the situation by using actual weights of themselves or friends in the class. They could see that there was a wide range of sizes in the class. They did not see a global situation involving children in general. However there were about 8 different solutions from the groups. Some groups estimated weight of a small-sized or a big-sized pupil and using a reasonable proportion of such sizes to work out the answer. The four operations were used for calculation. Some seemed to have worked mentally on the numbers first to make up the 500kg. Some estimated the weight of a "skinny" girl to 25 kg, or a "fat" girl to 50kg, then using mental sum of multiplication to work out the answer as 20 skinny or 10 fat pupils showing number sense in working with compatible numbers. There was attempt to find an"average" weight among the members in the group, but was distracted by one member who was larger than normal. There was a group who performed at the low level without thinking strategically. They attempted to add all the big-sized pupils' weights first and then added others to make up 500kg. There were lots of addition computations and tedium set in that led to incomplete solution. Below are two successful works of the pupils:



The Polar Bear Problem

A Polar bear weighs 500 kg. How many children from our class weigh together as much as the polar bear?

Jessica Peh

Think first for a few minutes before beginning to discuss with each other. Listen to each other and try to understand each other's way of reasoning. Please show and explain your work using pictures, numbers and works.

Step1: Count the skinny people and estimate their weights. The weight is 25kg (1girl-25Kg) add up tu 500kg.

Step2: Multiply the amount of people to the weight. (people is 20)

ans: 20 x25 = 500 (amount of weight for 20girls)

Step1: Crystal is 21 kg and Delfina is 35 kg so we round it off to the nearest ten. 21 kg = 20kg 35 kg = 40kg

step2: 5(40kg) girls = 200kg 500kg - 200kg = 300kg

Step 3: How many 20 kg girls = 300kg 300 ÷ 20 = 15 15(20 kg) girls = 300 kg

Step 4: To check 15 x 20 = 300 200 ÷ 5 = 40 300kg + 200kg = 500kg 15 + 5 = 20

20 children from our class weigh together as much as the polar bear.

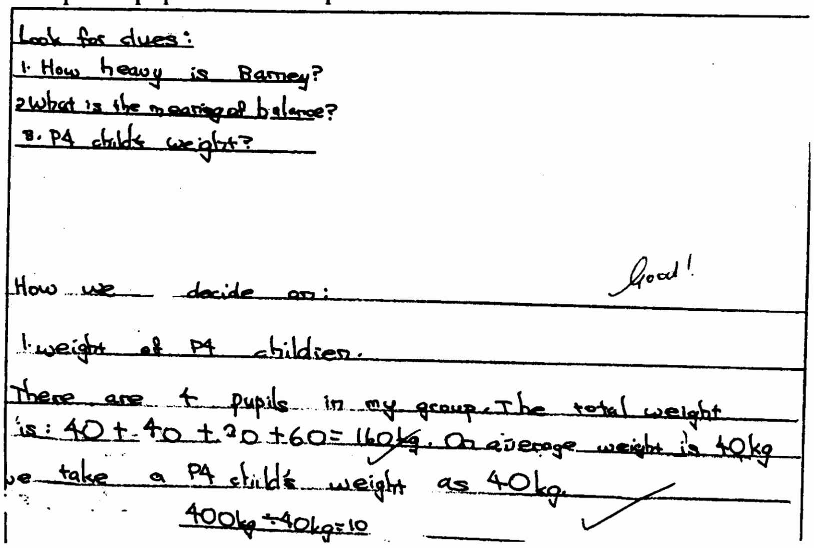
Teacher A's Reflection:

"Personally, I feel that such activity provides pupils with some kind of excitement as they are required to relate with mass of real-life objects. It actually challenges their mind to think beyond textbook answers and allow them to be creative. Honestly, I was surprised to see how active they were in getting the answer as they knew that this type of investigative question can lead to more than one solution. The moment I mentioned this, the pupils tried to 'outdo' each other in getting the best answer. During presentation, pupils were then exposed to the number of

possibilities that their classmates could think of. At the same time, their presentation skills could also improve. On the whole the lesson is a fruitful and meaningful one as it did bring out the creativity in the pupils. Pupils are able to think out of the box and provide logical reasoning."

Case of Teacher B: Too Guided

Teacher B started with a preliminary problem to discuss with pupils the nature of an open-ended question where they had to come up with their own assumptions. However when she introduced and explained the bear problem to the pupils she asked them this leading question "What information do you need to solve this problem?" She did not give pupils the opportunity to analyse and ask this critical question themselves. In a way she "closed" the problem and hence the intent of letting pupils use high level reasoning of identifying the missing information, making assumption and accessing their own knowledge in this open-ended situation was somewhat curtailed. However it was her belief that asking leading questions was useful to pupils to help them think through the steps and be systematic in their approach. She guided them in a method to find average weight using the group members' weights. She then suggested that they made appropriate assumption of their own weights by estimation. As a result most of the pupils' solutions followed a certain fixture structure modeled by her. Below is an example of pupils' common presentations:


Look for clues:
1. How heavy is Barney?
2 What is the meaning of balance?
3. P4 child's weight?

How we decide on: Good!
1. weight of P4 children.
There are 4 pupils in my group. The total weight is: 40 + 40 + 20 + 60 = 160kg. On average weight is 40kg
we take a P4 child's weight as 40kg.
400kg ÷ 40kg = 10

In a way, the performance of the pupils did decline to procedural thinking except for the estimation on the weight of a pupil. From the observation of the pupils at work, she noted that the leading questions from her guided them to look for the right clues and made the appropriate assumption about a child's average weight. There were few pupils who needed clarifications on how they could get a fair estimate of a P4 child. She observed that they were later guided by the better ones. The better ones found this sum interesting and they went on to give other estimates to find different answers. A few pupils were held up because they could not agree on the assumption. Time was wasted as they argued on the appropriate weight of a P4 pupil. The problem was resolved by voting and they consulted the teacher before making the final decision. Pupils were glad to note that all their answers were not the same as it depended on the assumptions made.

Teacher B's Reflection

> *"It was encouraging to note that pupils find the activity interesting and engaging. It was an effective way to get pupils to work independently with the proper procedures planned out. Although this is a mixed ability group, most of them could understand the problem sum. The leading questions guided them along, so they had no problem looking for the right clues and come up with the right assumptions. As such most of them were able to give the answer correctly with some variations. The task motivated pupils to want to look for clues and had engaged them in meaningful discussion to find the best assumption of the weight of a P4 pupil. It had also been very satisfying to see pupils learned while they enjoyed themselves."*

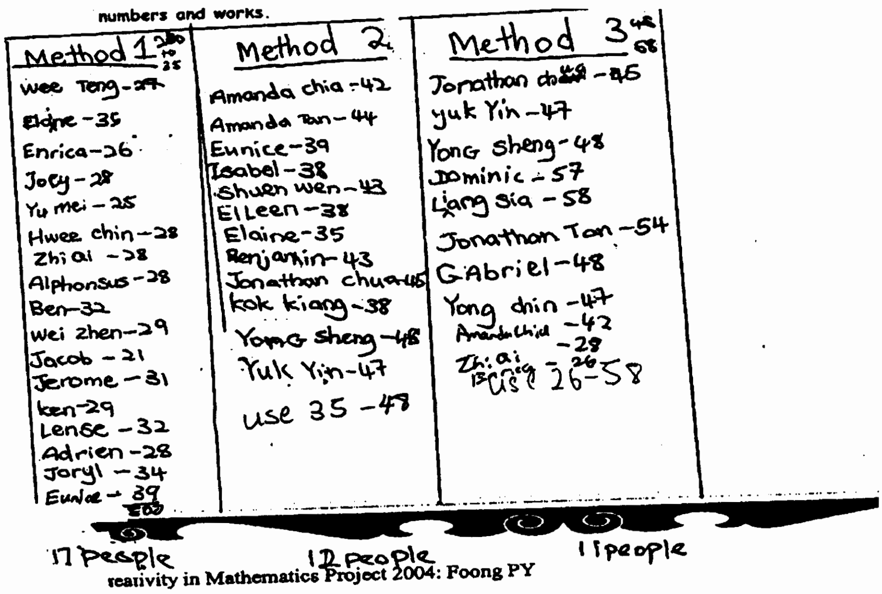
Teacher C: Unsuccessful

Teacher C might not have a full understanding of the nature of the open-ended question and the cognitive demands of this task. She did not have the correct concept underlying the problem. In setting up the task, she made a big mistake by giving out copies of the weight/height list of every pupil taken from the school's PFT (Physical Fitness Test) file. She miscued the pupils by asking them to use the list and to find as many ways as possible "*from adding the weights of the various pupils together which would bear the same mass as the bear's weight (500kg)*".

She observed that three groups of pupils seemed did not have the slightest idea of what to do and took some time to get started. She encountered class management problem, as some pupils did not work well with team members. Some pupils tended to dominate other weaker pupils within the same group. Some weaker pupils when working alone could not do anything. There was no evidence of any scaffolding actions from the teacher in her report to help pupils see the actual demand of the task.

Pupils' Outcome

Pupils' engagement with the task declined from the very start as Teacher C misconstrued the question and miscued the pupils. With the given list of their classmates' names and weights, pupils were only interested in getting an answer by selecting names and weights that would make up 500. Hence most of their solutions were lists of names of pupils and their weights with attempts to find the right combination that will add up to 500kg. By way of "different" solutions the pupils tried different combinations of their friends in the class but mostly unsuccessful. The whole task was reduced to tedium of adding many two digit numbers without strategic processing to get a sum of 500. Many of the pupils' workings were unsystematic and were abandoned halfway. A common example of pupils' work that declined to low level performance is shown below:



Summary

Teacher C did not provide any reflection of her lesson but when she handed in her lesson report to the author she expressed doubts about what she had implemented was correct. In her case, she might not have a good understanding of the mathematical thinking embedded in the task and the kind of cognitive demands to be made of the pupils. Her pupils were not held accountable for the required high cognitive processes as she accepted all the unsystematic trials of her pupils. The activity declined and pupils became disengaged. In the case of Teacher B, the lesson was partly successful with pupils' engagement in a systematic way, although too guided to allow room for pupils' creativity. In her reflection, Teacher B still believed that it was important to guide pupils with a planned procedure especially when the class was of mixed abilities. At a deeper level, she curtailed the openness of the problem by providing cues that would reduce the task to more procedural outcomes. Teacher A had implemented the lesson successfully with the appropriate set-up of the task and provided scaffolding that maintained pupils' high level engagement. She did not give in to pupils' anxiety over ambiguity in problem situation at the beginning.

Conclusion

Cases of Teacher A, B and C support what research has found that by merely giving pupils high level tasks in the classroom will not automatically result in the pupils' engagement in high level thinking, reasoning and sense making. A very important phase in carrying out open-ended high level tasks in the classroom is the **set-up** phase where the teacher announces the task in such a way as to encourage students to use more than one strategy, to use multiple representations, to supply explanations and justifications. This was very evident in the case of Teacher A, who had successfully implemented the lesson. In the cases of Teacher B and C, pupils' engagement with tasks declined to lower levels of processing. In this study, the task selected was appropriate to the mathematical prerequisite knowledge of the pupils although this was the first time they came across such an open-ended situation. Time was not an issue as the task was not complicated that demanded much pupil exploration. All teachers allotted the appropriate time for the pupils to complete the problem.

For the desired outcomes to be achieved, these three classroom experiences have implications for the role of the teacher in implementing open-ended tasks in which pupils are expected to actively engage in high level mathematical reasoning and creative thinking. Teachers must have a paradigm shift towards a more process-based approach where getting a correct answer to a problem is not the main criteria. Teachers often have a misguided idea that if they can come up with a set of procedures that pupils can follow to solve mathematics problems, then pupils will be well equipped to solve problems. It might not help them solve open-ended problems which often do not have a precise answer or a specific method of solution. If teachers carry this misconception into a process-oriented problem-solving classroom they will be reducing a high level problem to applying procedural skills. Importantly in implementing open-ended tasks, teachers must know the mathematical ideas embedded in the task and connections that might evolve, in order to create an encompassing classroom environment that allows pupils to take risk and ask appropriate questions. In order not to inhibit pupils' creativity in mathematical thinking, teachers must proactively and consistently support pupils' cognitive activity without reducing the high level demands of the task. They should not give in to pupils' anxiety and take over to show them how to get the answer. They should sustain pressure on pupils to provide meaning, explanation and justification to demonstrate their deep conceptual understanding and high order mathematical thinking.

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