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Elements of Ancient Chinese Mathematics for Pre-university Students

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Abstract: The purpose of this study is to investigate the effects of an Ancient Chinese Mathematics Enrichment Programme (ACMEP) on pre-university students' attitudes towards mathematics. A total of 37 students took part in this study and a mathematics attitudes scale was administered prior to and after the ACMEP which consisted of 4 sessions of one hour duration each, held once a week, conducted during curriculum time. Results of statistical analysis revealed that the post-ACMEP mathematics attitudes survey overall mean score was higher (t = 1.99, df = 31, p = 0.06) than that of the pre-ACMEP mathematics attitudes survey, with an effect size of 0.40. Overall, the conduct of the ACMEP appeared to improve pre-university students' attitudes towards mathematics.

Key words: Ancient Chinese mathematics; Attitudes towards mathematics; Pre-university

Introduction

The primary objective of the mathematics curriculum in Singapore is mathematical problem solving, and the ability of mathematical problem solving is dependent on five inter-related components of concepts, skills, processes, attitudes and metacognition (Ministry of Education, 2006a & 2006b). Although one of the aims of the Singapore mathematics education is to develop positive attitudes towards mathematics, Ng (2006) commented that there seemed to be little emphasis in the areas of mathematical nature, history, and real-world relevance, as well as a lack of concerted effort to help students develop positive attitudes towards mathematics.

On the other hand, while in a study by Civil (2006), it was found that embedding real life examples in the learning of mathematics is important for students' development of mathematical concepts, Rahmat (2000) found that secondary school students did not find the study of mathematics relevant to real life. The situation is particularly acute at the junior college (pre-university) level, as Loong (2004) found that only 51.1% of junior college students see the relevance of mathematics taught in schools. This observation was also made by the second author in his own experience as a teacher in a junior college. Students attending the two-year junior college have to acquire a tremendous amount of mathematical content knowledge and develop sufficient mathematical reasoning ability, so that they are adequately prepared for the General Certificate of Education Advanced Level (G.C.E. 'A' level) mathematics examination, an examination prior to the entry into university, at the end of the second year of the junior college education. In the new G.C.E. 'A' level mathematics syllabus, for example, students have to learn to analyse, formulate and solve different types of problems, and work with data and perform statistical analyses, from no less than 11 major topics ranging from functions and graphs to correlation and regression (Singapore Examination and Assessment Board, 2007).

Anecdotal evidence suggests that junior college students have very little time and exposure in mathematics beyond the examination syllabus in the areas of culture, history and real world relevance. This is exacerbated by the need to channel their energies between other subjects of study and co-curricular activities. Though attempts are made to incorporate these components within the mathematics curriculum, they are frequently met with remarks by students to 'not waste time and get on with the syllabuses'. In the light of this, the authors hope to examine the possible roles of ancient Chinese mathematics in the Singapore junior college mathematics curriculum to address the afore-mentioned concerns. More specifically, the purpose of this study is to investigate the effects of an Ancient Chinese Mathematics Enrichment Programme (ACMEP) on students' attitudes towards mathematics at the junior college level. The objective is to determine if there is a significant difference in the attitudes of first-year junior college students towards mathematics before and after the administration of the ACMEP.

Background

Many researchers have produced works and studies on ancient Chinese mathematics from a historical perspective and its impact on mathematics (Ang, 1969; Ng, 2006; Siu, 1993; Shen, Crossley & Lun, 1999; Straffin, 1998; Swetz, 1979). Yet, very little has been done to explore the role of ancient Chinese mathematics in the teaching and learning of mathematics. Yang (2009) presented two cases of utilizing materials from the history of ancient Chinese mathematics in modern mathematical education. Indeed, there are plenty of content from the ancient Chinese mathematical texts that are relevant to the Singapore secondary school mathematics curriculum. Ng (2006) first conceived the idea of using an ancient Chinese mathematics enrichment programme to address the apparent lack of focus in areas of mathematical nature, history, and real-world relevance in the Singapore secondary school mathematics curriculum. As a strand of his study, Ng (2006) examined the effects of Ancient Chinese Mathematics on the academic achievement of secondary two students in Singapore. He also argued that using ancient Chinese mathematics in school curriculum in Singapore, where the student majority are ethnic Chinese, would effect culturally relevant teaching which would likely increase students' interest in mathematics, and might also enhance student achievement in mathematics.

Singapore's Ministry of Education (2006a & 2006b) stated that students' attitudes towards mathematics are shaped by their learning experiences. Hence, making the learning of mathematics fun, meaningful and relevant will go a long way in inculcating positive attitudes towards mathematics. Currently, there is little research on students' attitudes towards mathematics at the junior college level in Singapore. There is also no research as yet on the effects of ancient Chinese mathematics on students' attitudes towards mathematics. The present study is intended as a small scale study to examine the effects of the use of ancient Chinese mathematics in a different context. It adopts the theoretical framework of Ng's (2006) study to: (a) generate interest in mathematics through cultural and historical real-world relevance, thereby improving students' attitudes towards mathematics; (b) encourage lateral thinking in problem solving by demonstrating early mathematicians' creativity in solving real-world problems; and (c) assist students in problem-solving metacognition. By highlighting the use of history of ancient Chinese mathematics, the present study hopes to enthuse students at the post-secondary level to understand and appreciate ancient problem-solving methods, thereby improving students' attitudes towards mathematics.

Researchers have offered many arguments for the use of history of mathematics in the teaching and learning of mathematics. Yet it is not easy to incorporate the use of history of mathematics in the mathematics classroom. Studies on actual implementation and evaluation of the use of history of mathematics in mathematics teaching and learning were limited compared to the many theoretical studies. A short review of using history in mathematics and ancient Chinese mathematics have highlighted the potential of using the history of ancient Chinese mathematics in mathematics learning in the current Singapore mathematics curriculum. By launching the ACMEP with a brief account on the history of ancient Chinese mathematics, students would hopefully be enthused to learn mathematics. By solving puzzles and questions from ancient Chinese mathematical texts, and understanding and appreciating the ancient Chinese problem solving methods and approaches, interest would hopefully be generated among students through cultural and historical real-world relevance, thereby improving their attitudes towards mathematics. At the same time, lateral thinking in problem solving are encouraged through the creative ancient Chinese problem solving methods and approaches, and problem-solving metacognition enhanced, through the discourse on ancient Chinese mathematics.

Studies on attitudes have stressed the importance of attitudes as a construct in student learning, and research has been fertile in exploring teachers' and students' attitudes towards mathematics in teaching and learning. Some researchers seemed to suggest a relationship between using history in mathematics teaching and learning, and students' attitudes towards mathematics. However, such studies are limited and inconclusive. In particular, there is no research to date on the effect of ancient Chinese mathematics on students' attitudes towards mathematics. A study at this point would provide an understanding of the possible issues, and would be useful in making mathematics learning more relevant and interesting to students.

Ancient Chinese Mathematics

The earliest Chinese traditional mathematical text, *A book on numbers and computation* (算数书 Suan Shu Shu), was written during the Han (汉) Dynasty more than two thousand years ago (Ng, 2006). Other subsequent prominent early traditional mathematical texts include *The arithmetical classic of the Zhou gnomon* (周髀算经 Zhou Bi Suan Jing) and *The nine chapters on the art of mathematics* (九章算术 Jiu Zhang Suan Shu). In particular, the influence of *The nine chapters on the art of mathematics* on oriental mathematics may be likened to that of Euclid's *The Elements* on western mathematical thought (Swetz, 1979). In the next phase of development of ancient Chinese mathematics from about 220-960 AD, important texts such as Liu Hui's (刘徽) *Commentary on the nine chapters on the art of mathematics* (九章算术注释 Jiu Zhang Suan Shu Zhu Shi) and *The sea island mathematical manual* (海岛算经 Hai Dao Suan Jing), as well as *Zhang Qiujian's mathematical manual* (张丘建算经 Zhang Qiu Jian Suan Jing) were written. At the zenith of mathematical progress around the Song (宋) Dynasty, the text *Mathematical treatise in nine sections* (数书九章 Shu Shu Jiu Zhang) by Qin Jiushao (秦九韶) was written in 1247 AD (Ng, 2006). These ancient Chinese mathematical texts offer rich and vast mathematical content in the form of problem situations and solutions that not only provide a glimpse of the society in those times, but more importantly, highlighted the advances in mathematical know-how in ancient China.

The importance of ancient Chinese mathematics in the evolution of mathematics cannot be understated. Swetz (1979) wrote:

If comparisons must be made among the societies of the pre-Christian world, the quality of China's mathematical accomplishments stands in contention with those of Greece and Babylonia, and during the period designated in the West as pre-Renaissance, the sequence and scope of mathematical concepts and techniques originating in China far exceeds that of any other contemporary society. (p.17)

Research studies in ancient Chinese mathematics in the past 40 years have focused on theoretical discussions of traditional Chinese mathematical texts and their impact on the evolution of modern mathematics. Examples of studies ranged from translating and evaluating traditional mathematical text (Ang, 1969), acknowledging the sophisticated inductive knowledge evident in arithmetic and algebraic approaches (Swetz, 1979), sampling proofs with a view to using them for the enrichment of the teaching of mathematics (Siu, 1993), to focusing on exceptional works of an individual mathematician as a tribute and honour to the mathematician (Straffin, 1998). In comparison, there are few studies on the potential of using ancient Chinese mathematics in teaching and learning in the classroom. Some elements of ancient Chinese mathematics that can be used for teaching and learning in the mathematics classroom under the current Singapore mathematics curriculum would form the basis of the ACMEP content for the present study. They include the tangram, counting rods, translation and solving of problems from ancient Chinese mathematics classics, and geometrical proof of mathematical results. More details will be provided in the next section.

Design of the Study

Purpose of Study

The intent of the study was to examine the effects of an Ancient Chinese Mathematics Enrichment Programme (ACMEP) on first-year junior college students' attitudes towards mathematics. The objective is to determine if there is a significant difference in the attitudes towards mathematics of first-year junior college students, before and after the administration of the ACMEP. The independent variable in the present study will be the ACMEP intervention, while the dependent variable is the overall attitudes towards mathematics score as measured by five out of the nine domain-specific attitudinal subscales from the

Fennema-Sherman Mathematics Attitudes Scales (FSMAS), as developed by Fennema and Sherman in 1976.

Research Design

In order to answer the research question of whether the ACMEP improves first-year junior college students' attitudes towards mathematics, a single-group pretest-posttest design was adopted. This design was the best the authors could adopt due to implementation constraints as experienced in the junior college in which the study was carried out. The junior college administration had planned for the ACMEP to be conducted by the second author within curriculum time as part of their Pastoral Care and Career Guidance (PCCG) enrichment programme. A one-hour single period each week was set aside for the PCCG programme, which took place on Fridays from 12.45pm to 1.45pm. As there were other enrichment programmes being carried out concurrently, it was not possible to obtain a control group from the same first year junior college (JC1) cohort. As a result, the author had little choice but to adopt the single-group pretest-posttest design for the study.

The authors were fully aware of the limitations of this design as control of extraneous variables was difficult. In order to mitigate the threats to the internal validity of this design, a full-scale pilot study mirroring the main study was carried out. With the benefit of this pilot study, major confounding factors for this design due to history, maturation, regression to the mean, testing, instrumentation and mortality were identified. Efforts were made to effect changes to the main study so that those threats to internal validity in the main study were minimised.

Research Sample

This study was conducted in a junior college in Singapore. Students from this college were of above-average academic ability, based on their General Certificate of Education Ordinary level (G.C.E. 'O' level) examination aggregate scores taken at the secondary four level.

The participants chosen for this study were 17 year-old JC1 (grade 11) students. As the ACMEP was one of the electives under the college PCCG programme, students were given the option to participate in the ACMEP. The authors had requested the college administration to select the participants randomly from the JC1 cohort. A total of 37 students, 19 female and 18 male, were eventually selected for the ACMEP. Fourteen of the 37 students opted to participate in the ACMEP on a voluntary basis, while the rest were selected at random from the JC1 cohort. Of the 37 participants, 6 were ethnic Indians, 1 was ethnic Malay, with the remaining students all ethnic Chinese.

Instrumentation

The participants of the ACMEP were required to complete a mathematics attitudes scale consisted of questions adapted from the Fennema-Sherman Mathematics Attitudes Scales (FSMAS), as developed by Fennema and Sherman (Fennema & Sherman 1976).

The FSMAS is a 108-item multi-dimensional self-rating scale with nine subscales. The nine subscales are:

1. Attitude Toward Success in Mathematics (AS)
2. Confidence in Learning Mathematics (C)
3. Effectance Motivation in Mathematics (E)
4. Father (F)
5. Mathematics Anxiety (A)
6. Mathematics as a Male Domain (MD)
7. Mother (M)
8. Teacher (T)
9. Usefulness of Mathematics (U)

Each of the nine subscales consists of 12 attitudinal statements, of which six are worded positively and the other six worded negatively. The nine subscales can be used individually, in groups, or as a complete set, as determined by the different variables within FSMAS. In this study, five subscales from FSMAS, namely Attitude Toward Success in Mathematics (AS); Mathematics Anxiety (A); Confidence in Learning Mathematics (C); Effectance Motivation in Mathematics (E); and Usefulness of Mathematics (U), were used.

The 60 items from the five FSMAS subscales were arranged randomly to form a mathematics attitudes scale. The distribution of the items is shown in Table 1. A 5-point Likert-type response format was used in the mathematics attitudes scale. Responses from the subjects were converted into numerical values with weights of 5 (strongly agree), 4 (agree), 3 (neutral), 2 (disagree) or 1 (strongly disagree). Negatively worded items were inversely weighted and scored accordingly. The score for each subscale ranged from 1 to 60, and the maximum possible overall score for the five subscales was 300. Changes were made to the wording of some of the items, without deviating from the original meaning, so that they could be understood with ease.

Table 1
Distribution of FSMAS Items in the Mathematics Attitudes Scale

Weight	Item no. in mathematics attitudes scale				
	Attitude Toward Success (AS)	Anxiety (A)	Confidence (C)	Effectance Motivation (E)	Usefulness (U)
+	59	34	40	33	3
+	30	51	53	44	60
+	24	12	14	37	23
+	46	32	48	18	56
+	17	2	7	42	41
+	9	57	27	6	16
–	28	19	22	25	52
–	39	43	45	4	10
–	15	26	1	55	29
–	21	5	38	13	20
–	50	36	31	11	35
–	54	49	8	58	47

Procedures

Participants were given about 15 minutes to complete the mathematics attitudes scale, which was collected immediately upon completion. Participants were assured by the second author, who administered the scale, that the mathematics attitudes scale was not a test in itself, but more of a reflection of their general attitudes towards mathematics.

The ACMEP intervention, the content of which was designed by the authors, consisted of four sessions of one-hour duration each, held once a week, conducted during curriculum time. The intervention was conducted by the second author to ensure consistency in content delivery. At the end of each session, the participants were asked to reflect on the content of the session. These questions served to sustain interest in the ACMEP by keeping the participants involved over the entire duration of the ACMEP. More importantly, the qualitative data obtained acted as a means to triangulate the quantitative data obtained from the mathematics attitudes scale.

Immediately after the last session of the ACMEP intervention, the same set of mathematics attitudes scale was again administered. Similarly, participants were given about 15 minutes to complete the post-ACMEP mathematics attitudes scale, which was collected immediately upon completion.

Statistical Analysis

For both pre-ACMEP and post-ACMEP mathematics attitudes scales, the scores of negatively weighted items were converted using the formula ($y = 6 - x$), where x is the raw score of each negatively weighted item. The mean scores for each of the five subscales, as well as the mean score for the entire 60-item scale, were calculated. The means scores obtained, with values that ranged between 1 and 5 inclusive, were used for the purpose of analyses. Higher scores in the Attitude Toward Success (AS), Confidence (C), Effectance Motivation (E) and Usefulness (U) subscales would indicate more positive attitudes in each of the particular dimension of attitudes towards mathematics, while a higher score in the Anxiety (A) subscale would mean a lower anxiety level towards mathematics. In order to check the internal reliabilities of each of the five subscales and the scale as a whole, Cronbach's alpha reliability analyses were performed. Pearson correlation coefficients were calculated among the five subscales and the scale as a whole, for both pre-ACMEP and post-ACMEP mathematics attitudes scales, for the purpose of inter-correlation analyses. The parametric paired samples t-test was used to determine if there were any significant differences between the pre-ACMEP and post-ACMEP mean scores of each subscale, and also the pre-ACMEP and post-ACMEP overall mean scores of the mathematics attitudes scales. In the present study, the effect size (d), where d is the mean divided by the standard deviation, is calculated to measure the magnitude of the treatment effect.

ACMEP Intervention

The content of the ACMEP consisted of collages of mathematical advancements and achievements from ancient China, including puzzles, counting methods and mathematical solutions to problems as encountered by the ancient Chinese civilisation. These contents were adapted from materials used by the first author in his study on the effects of ancient Chinese mathematics on secondary school students' achievement in mathematics (Ng, 2006).

The ACMEP was divided into four main components: tangram, counting rods, translation and solving of problems from ancient Chinese mathematics classics, and geometrical proofs of mathematical results. In each of the four components, the second author gave a brief introduction to the history of the component, and went through some examples, before engaging the participants in hands-on activities to solve puzzles, questions and problems.

Tangram (七巧板)

The tangram puzzle was included in the ACMEP content as an introductory activity in the first session of the ACMEP to inject fun and excitement into the programme. It was intended to create positive experiences for the participants, as well as to entice the participants to the ACMEP. Participants were encouraged to use a tangram to form shapes that relate to the Chinese culture, from animals representing the Chinese zodiac like a rabbit or goat, to people and objects like a farmer and a Chinese junk.

Counting Rods (算筹)

Counting rods were used in ancient China for the purpose of calculation long before the invention of the abacus. Calculations were arranged from left to right, exactly like the present day Hindu-Arabic system. Under this system, two sets of symbols, all formed using rods, were used to represent digits 1 to 9, and multiples of 10 from 1 to 9. These symbols were alternated for successive powers of 10. Pictorial representations of the symbols, together with some examples of numbers using rods representation, are shown in Figure 1. There was no symbol for zero initially, and a blank space was used for zero.

	1	2	3	4	5	6	7	8	9
Symbols for positions of units, 100, 10,000 etc.	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱
Symbols for positions of 10, 1,000, 100,000 etc.	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
182	𝍩𝍧𝍪								
6075	𝍥 𝍦𝍭								
493,846	𝍣𝍱𝍢𝍰𝍣𝍮								

Figure 1. Symbols in Rod Arithmetic and Examples of Numbers using Rod Representation

In the ACMEP, the participants were taught to use counting rods only to perform the four basic operations of addition, subtraction, multiplication and division, without the assistance of modern hand-held calculators. The objective was for participants to appreciate the advances of counting in ancient China, thereby providing inspiration in mathematics learning and fostering positive attitudes towards mathematics.

Translation and Solving of Problems from Ancient Chinese Mathematics Classics

Problems were selected from the renowned and influential early traditional text, *The Nine Chapters on the Art of Mathematics* (九章算术 Jiu Zhang Suan Shu); *Zhang Qiujian's Mathematical Manual* (張丘建算经 Zhang Qiu Jian Suan Jing), a text written during the development phase of mathematics; and the *Mathematical treatise in nine sections* (数书九章 Shu Shu Jiu Zhang) by Qin Jiushao (秦九韶), a text written during the peak of mathematical development in China. A total of eight problems from these texts were selected as examples for the ACMEP in the present study and are summarised in Table 2.

Table 2
Problems Selected as Example for ACMEP

Example	Ancient Chinese Mathematical Text	Problem
1	*The Nine Chapters on the Art of Mathematics*	Chapter 6 Problem 12
2	*The Nine Chapters on the Art of Mathematics*	Chapter 6 Problem 26
3	*The Nine Chapters on the Art of Mathematics*	Chapter 7 Problem 1
4	*The Nine Chapters on the Art of Mathematics*	Chapter 7 Problem 18
5	*The Nine Chapters on the Art of Mathematics*	Chapter 9 Problem 20
6	*Zhang Qiujian's Mathematical Manual*	Volume 3, Problem 38
7	*Mathematical Treatise in Nine Sections*	Section 3 Problem 20
8	*Mathematical Treatise in Nine Sections*	Section 4 Problem 34

Ng (2006) mentioned that the ethnic Chinese participants for the ACMEP in his study found it challenging to translate classical Chinese language, as classical Chinese language is very different from the modern daily conversational Chinese language used by the participants.

As a result, participants in the present study were first encouraged to come up with a version of their translation, before the second author shared a version for the benefit of participants who were not able to complete the translation.

Non-ethnic Chinese participants were given the translation of the problems immediately to work on, while their ethnic Chinese peers attempted to translate the same problems.

Geometrical Proofs of Mathematical Results

In this component, participants of the ACMEP were asked to provide a proof, using the modern western approach, for a theorem as shown in Figure 2. The theorem can easily be proved by using the concept of similar triangles (see Proof 1 in Figure 2). The authors subsequently shared with the participants an alternative geometrical proof provided by Liu Hui (刘徽), a Chinese mathematician from the third century AD.

In his proof, Liu Hui first added a second triangle to obtain a rectangle of area bc square units. The rectangle was then manipulated to form another rectangle of area $s(b+c)$ square units. Hence $s = \dfrac{bc}{b+c}$ (see Proof 2 in Figure 2). The purpose of this component is to contrast the algebraic approach used today as opposed to the simple and elegant geometric approach by Liu Hui more than 1700 years ago.

Theorem:

The length of the side of a square inscribed in a right-angled triangle is the quotient of the product and the sum of the legs.

Proof 1 (Modern western approach)
Given a right-angled triangle *ABC* with inscribed square ADEF where $AB = c$, $AC = b$, $AD = s$ as shown in the diagram

Using similar triangles,

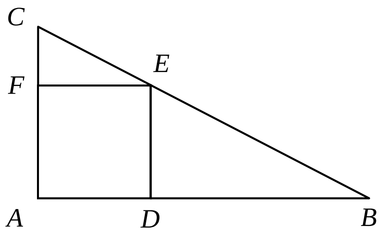
$$\frac{s}{c} = \frac{b-s}{b}$$

$$sb = bc - sc$$

$$sb + sc = bc$$

$$s(b+c) = bc$$

$$\therefore \quad s = \frac{bc}{b+c}$$



Proof 2 (刘徽Liu Hui's proof, 3rd century AD):

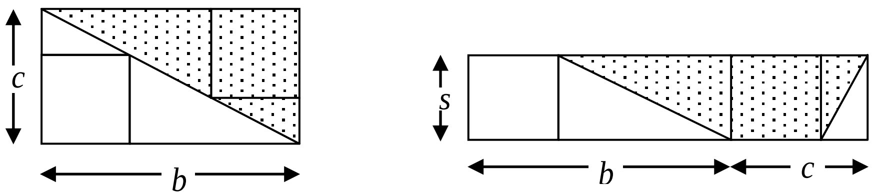


Figure 2. Proving Mathematical Results

Results and Discussions

The means and standard deviations of subscale scores of the pre-ACMEP mathematics attitudes survey, together with Cronbach's alpha reliability indices for each of the subscales measuring the five attitudinal domains that influence students' attitudes towards mathematics, are presented in Table 3. The reliabilities of the five subscales ranged from 0.88 to 0.93. These values indicated that the five subscales used in the present study are sufficiently reliable. The pre-ACMEP overall reliability of the mathematics attitudes scale was 0.95.

Table 3
Statistics on Main Study Pre-ACMEP Subscale and Overall Scores

	Attitude Toward Success	Anxiety	Confidence	Effectance Motivation	Usefulness	Overall
Mean Score	3. 80	3. 35	3. 44	3. 47	3. 59	3. 53
Standard Deviation	0. 60	0. 59	0. 67	0. 64	0. 71	0. 48
Reliability	0. 90	0. 88	0. 91	0. 88	0. 93	0. 95

No. of items: 60 No. of cases: 37

The pre-ACMEP overall mean score on the mathematics attitudes scale was 3.53 (3 being neutral) out of a maximum of 5, with a standard deviation of 0.48. The order of the five attitudinal domains based on the mean score of each domain, from the highest to the lowest score, was: Attitude Toward Success, Usefulness, Effectance Motivation, Confidence, Anxiety. The mean scores ranged from 3.35 to 3.80, while standard deviations ranged from 0.59 to 0.71. Overall, the participants found all the five attitudinal domains crucial in influencing their attitudes towards mathematics.

Table 4 presents the descriptive statistics and reliabilities (Cronbach's alpha) of the post-ACMEP mathematics attitudes scale. The reliabilities of the five subscales ranged from 0.78 to 0.93. These values suggested that the five subscales used in the present study are fairly reliable. The post-ACMEP overall mathematics attitudes scale reliability was 0.96 which suggested that the mathematics attitudes scale was reliable for the purpose of the present study.

Table 4
Statistics on Main Study Post-ACMEP Subscale and Overall Scores

	Attitude Toward Success	Anxiety	Confidence	Effectance Motivation	Usefulness	Overall
Mean Score	3. 88	3. 64	3. 79	3. 66	3. 72	3. 74
Standard Deviation	0. 57	0. 48	0. 55	0. 45	0. 62	0. 42
Reliability	0. 93	0. 87	0. 92	0. 78	0. 90	0. 96

No. of items: 60 No. of cases: 32

In the post-ACMEP mathematics attitudes survey, the rank order of the five attitudinal domains from highest to lowest mean scores was: Attitude Toward Success, Confidence, Usefulness, Effectance Motivation, Anxiety. Compared to the pre-ACMEP mathematics attitudes survey, there were no changes in the first and last ranked domains namely Attitude Toward Success and Anxiety domains. The Confidence domain changed from fourth to second rank. The Usefulness domain dropped from second to third rank, and Effectance Motivation domain dropped from third to fourth rank. The mean scores ranged from 3.64 to 3.88. An increase in mean score from pre-ACMEP to post-ACMEP was observed for each of the five attitudinal domains, with the Confidence domain registering the highest increase of 0.35, from 3.44 to 3.79. The standard deviation of the domain scores in the post-ACMEP mathematics attitudes scale ranged from 0.45 to 0.62, in comparison with 0.59 to 0.71 in the pre-ACMEP mathematics attitudes scale.

Table 5 reports the Pearson correlation coefficients between the five attitudinal subscales and the overall attitudes towards mathematics scale for the pre-ACMEP mathematics attitudes scale.

Table 5

Pre-ACMEP Inter-correlation among Subscales

Subscale	Attitude Toward Success	Anxiety	Confidence	Effectance Motivation	Usefulness	Overall
Attitude Toward Success	--	. 07	. 33*	. 16	. 36*	. 51**
Anxiety		--	. 82**	. 81**	. 27	. 80**
Confidence			--	. 68**	. 40*	. 87**
Effectance Motivation				--	. 43**	. 83**
Usefulness					--	. 68**
Overall						--

* $p < .05$, two-tailed test. ** $p < .01$, two-tailed test.

The correlations between the subscales ranged from 0.07 to 0.82. Most of the correlations were statistically significant at the 1% level and a few significant at the 5% level. Three pairs of subscales, Attitude Toward Success and Anxiety (r = 0.07); Attitude Toward Success and Effectance Motivation (r = 0.16); and Anxiety and Usefulness (r = 0.27), did not exhibit significant correlation. The results indicated that there was no linear relationship between the Attitude Toward Success and Anxiety subscales, the Attitude Toward Success and Effectance Motivation subscales, and the Anxiety and Usefulness subscales.

The non-correlation between the Attitude Toward Success and Anxiety subscales appeared that the Attitude Toward Success and Confidence subscales, the Attitude Toward Success and Effectance Motivation subscales, and the Anxiety and Usefulness subscales, could be taken as separate subscales independent of each other, measuring different attitudinal domains. Hence, future studies in attitudes towards mathematics using the FSMAS could include these considerations in their instrumentation. The correlation between the Confidence and Anxiety subscales (r = 0.82) remained the highest among all subscale pairs. This result again suggests that the Confidence subscale and Anxiety subscale are very similar in nature, as noted by Fennema and Sherman (1977) and O'Neal, Ernest, McLean and Templeton (1988). This provided a strong indication that only one of the Confidence and Anxiety subscales need to be used in subsequent studies on attitudes towards mathematics.

Table 6 shows the Pearson correlation coefficients for the post-ACMEP mathematics attitudes scale.

The correlations between the subscales ranged from 0.31 to 0.85. Most of the correlations were statistically significant at the 1% level or at the 5% level. The only subscale pair with no significant correlation was between Confidence and Effectance Motivation (*r* = 0.31). This provided an indication that the Confidence and Effectance Motivation subscales are independent of each other, with each measuring a separate attitudinal domain. Both the Confidence and Effectance Motivation subscales could thus be considered in future research on attitudes towards mathematics using the FSMAS. More importantly, the confidence and anxiety subscales remained highly correlated in all four correlation analyses conducted in the pilot and main study. This is consistent with previous findings on the FSMAS that in relation to mathematics, the confidence and anxiety subscales are very similar (Fennema & Sherman, 1977; O'Neal, Ernest, McLean & Templeton, 1988). It might thus not be useful to consider the Confidence and Anxiety subscales independently in future studies of this nature.

Table 6
Post-ACMEP Inter-correlation among Subscales

Subscale	Attitude Toward Success	Anxiety	Confidence	Effectance Motivation	Usefulness	Overall
Attitude Toward Success	--	. 46*	. 52**	. 44*	. 49*	. 75**
Anxiety		--	. 79**	. 45*	. 60**	. 83**
Confidence			--	. 31	. 70**	. 85**
Effectance Motivation				--	. 44*	. 64**
Usefulness					--	. 84**
Overall						--

* $p < .05$, two-tailed test. ** $p < .01$, two-tailed test.

Paired sample *t*-tests (N = 32) were conducted to evaluate whether mean scores of each subscale differed after the ACMEP. The results are shown in Table 7.

Table 7
Results of Paired Sample t-tests on the Pre-ACMEP and Post-ACMEP Mathematics Attitudes Scales

Subscale	Paired Differences (post-ACMEP – pre-ACMEP)		t	Significance (2-tailed)
	Mean	Standard deviation		
Attitude Toward Success	0. 05	0. 74	0. 36	0. 72
Anxiety	0. 29	0. 58	2. 52	0. 02
Confidence	0. 32	0. 67	2. 41	0. 02
Effectance Motivation	0. 12	0. 58	1. 07	0. 30
Usefulness	0. 03	0. 79	0. 21	0. 84
Overall	0. 17	0. 41	1. 99	0. 06

The differences in the pre-ACMEP and post-ACMEP mean scores of the Attitude Toward Success subscale, the Effectance Motivation subscale and the Usefulness subscale are all positive, although not statistically significant. The mean scores for the Anxiety subscale differed significantly (t = 2.52, df = 31, p = 0.02) after ACMEP, with the post-ACMEP mean score being significantly higher than the pre-ACMEP mean score. The mean scores for the Confidence subscale also differed significantly (t = 2.41, df = 31, p = 0.02) after ACMEP, with the post-ACMEP mean score being significantly higher than the pre-ACMEP mean score. The overall mean scores between pre-ACMEP and post-ACMEP mathematics attitudes scales differed (t = 1.99, df = 31, p = 0.06) with the post-ACMEP overall mean score being higher than the pre-ACMEP overall mean score. The calculated effect size for the main study was 0.40, a small value (Cohen, 1992).

While the findings of the present study seemed to support the theoretical framework of utilising an ACMEP to generate interest in mathematics through cultural and historical real-world relevance, thereby improving students' attitudes towards mathematics, the result is not conclusive. While the four-week duration of the ACMEP intervention seemed to minimise the threats of history and maturation, it might be too short for meaningful transfer of content knowledge. If not for the constraints of curriculum time, a longer ACMEP intervention duration would do well to mitigate this limitation. Also, despite efforts to prevent participants from dropping out of the ACMEP, five participants still did not complete the post-ACMEP mathematics attitudes scale, thus affecting the completeness of the data.

The sample of the present study was not selected at random, as a portion of the participants opted to take part in the ACMEP on a voluntary basis. This situation was unavoidable due to implementation constraints at the participating junior college. While qualitative data was obtained from the participants as a means to triangulate quantitative data, a measure of biasness nonetheless remained. Also, the present study was conducted using participants from one junior college only. As a result, the findings should not be representative of students from all junior colleges in Singapore.

The content of the present ACMEP could be further refined to include a more substantial amount of ancient Chinese mathematics so that students could better appreciate ancient Chinese mathematics and its impact on the development of mathematics in general. The epistemological aspects of ancient mathematical texts can inspire one to understand and appreciate new dimensions of mathematical knowledge. Participants in the present study expressed interest in finding out the procedures used by the ancient Chinese to solve the problems, and efforts were made by the authors to devote more time to examine the ancient approaches. However, the fundamental approaches used by the ancient Chinese were not examined thoroughly, nor were comparisons made between modern and ancient approaches, due to constraints in time in the conduct of the ACMEP.

Conclusion and Recommendations

Though the present study did not justify convincingly the positive effects of the use of ancient Chinese mathematics on junior college students' attitudes towards mathematics in that it has produced inconclusive results in three of the attitudinal domains under the FSMAS, namely, Attitude Toward Success in Mathematics, Effectance Motivation in Mathematics and Mathematics Usefulness, there is reason for optimism. Results are statistically significant in a positive way in the domains of Confidence in Learning Mathematics and Mathematics Anxiety. More importantly, the effects of the ACMEP on first-year junior college students' attitudes towards mathematics are positive and statistically significant. It has provided an indication that the use of ancient Chinese mathematics could have a place in the Singapore mathematics classrooms. Though the present study was conducted at the junior college level, the use of ancient Chinese mathematics in the secondary and primary levels would not be inappropriate. With careful selection of content from the vast array of ancient Chinese mathematical texts, similar mathematics enrichment programmes could be tailored for students at the primary and secondary levels.

Ng (2006) first conceived the idea of harnessing the richness of ancient Chinese mathematics in the teaching and learning of mathematics. He investigated the effects of the use of ancient Chinese mathematics on the achievement in mathematics of secondary two students. In his study, Ng also looked into the effects of an ACMEP on students' achievement in other subjects like Science, History, Higher Chinese and English. Various possibilities as regards the use of ancient Chinese mathematics could be explored further. Firstly, future studies could extend the study by Ng (2006) to investigate the effects of ancient Chinese mathematics on secondary school students' attitudes towards mathematics. Secondly, the present study could also be a basis for future studies in examining the effects of an ACMEP on junior college students' achievement in mathematics. The potential of using ancient Chinese mathematics in the Singapore primary school mathematics curriculum has earlier been highlighted. Hence, an interesting third option could be to investigate the effect of an ACMEP on primary school pupils' attitudes towards mathematics and achievement on mathematics. With proper planning and implementation, an ACMEP would likely create an effect in pupils' learning of mathematics at the primary school level.

In Singapore, the Project Work (Ministry of Education, 1999) initiative calls for an integrated learning experience that encourages students to break away from the compartmentalisation of the different disciplines, where they learn to apply creative and critical-thinking skills, improve communication skills, foster collaborative learning skills and develop self-directed inquiry and life-long learning skills. Project Work has been implemented in primary, secondary and pre-university levels since 2000. The present study has provided yet another possible example, where an enrichment programme in mathematics took place within the Pastoral Care and Career Guidance curriculum, to improve students' attitudes towards mathematics. Curriculum planners in schools could thus consider adopting or adapting the present study, or use the present study as a stepping board for other innovative ideas, to plan for and achieve curriculum integration in their schools.

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