Mathematical Modelling for Children: A New Curriculum and a New Perspective

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Abstract: Research in mathematical modelling involving children sees the learning of mathematics taking place *through* modelling experiences as children develop conceptual representations for making sense of problem situations towards adapting a mathematical way of thinking. The newly revised mathematics curriculum (Singapore) has stated that *applications and modelling* be part of the mathematical problem solving process for all levels, children included. This paper shares excerpts from a classroom research involving Primary 6 students via a modeling perspective to show the students' conceptual interpretations during problem solving.

Key words: mathematical problem solving, modelling, conceptual interpretations

Introduction

In the history of Singapore's primary school mathematics education, the term *mathematical modelling* is somewhat unheard of. Mathematical modelling to many teachers and students in our local schools is associated with higher level mathematics in upper secondary or pre-university classes which involve hard mathematics (pure and applied mathematics). In the primary school mathematics curriculum, model-drawing rather than mathematical modelling is more recognizable to our local teacher practitioners and students.

The Singapore Mathematics Curriculum Framework (SMCF) has mathematical problem solving as its central goal. It asserts that developing good problem solvers is dependent on the development of five inter-related components, namely, *Process*, *Skills*, *Concepts*, *Metacognition*, and *Attitude*. In the recent revision of the SMCF, *Applications and Modelling* alongside *mathematical reasonin*g, *communication*, and *making connections* have been included under the *Process* component (MOE, 2007). The curriculum document states that application and mathematical modelling "*should be part of the learning at all levels*" (p.14), and is seen as "*playing vital roles in the development of mathematical understanding and competencies*" (p. 14). These new inclusions suggest pedagogies that promote active inquiry and making students' mathematical thinking visible. It also implies that children should be actively gaining experiences through modelling, reasoning, communicating, and making connections.

What does mathematical modelling look like and what does it entail especially in the teaching and learning of mathematics in the primary school? This article attempts to provide insights from a modelling perspective that increasingly is making headway into mathematics

classroom practices in the US, Australia, and Europe (Mousalides, Sriraman & Christou, 2007).

The Modelling Perspective

Mathematical modelling has been defined variously in literature. The modelling perspective asserts that thinking mathematically is about interpreting situations mathematically (Lesh and Doerr, 2003). This paper argues from the perspective that modelling is a problem-solving activity that involves the process of using mathematical language or mathematical terms to develop representational descriptions of specific situations as students construct mathematical ideas and processes. When children are given a modelling task to solve, they would be engaged in purposeful descriptions or explanations to make mathematical sense of that situation. Unlike traditional modelling which has been described as a direct modelling method from givens to goals through the use of definitive procedures as well as having only one way of interpreting the problem (English, 2003), the modelling endeavour of today encompasses multiple cycles of constructing, testing and revising of ideas (Lesh & Zawajewski, 2007). Thus when students go through such cycles, they are said to be constructing models. A *model* in this paper is referred to as conceptual interpretations or representations of elements of ideas, concepts, or constructs made explicit through the establishing of relationships between them. The relationships can be seen in the form of text, diagrams, metaphors, abstractions, or verbal explanations through the connections and operationalization of these elements.

For model development to take place, the task needs to be specially designed for such a purpose since modelling as problem solving is a goal-oriented process. The modelling tasks are thought-revealing tasks where upon engagement will usually involve mathematizing, that is, it requires students to quantify, dimensionalize, coordinate, categorize, algebratize, and systematize relavant objects, relationships, actions, patterns, and regularities in realistic situations (Lesh & Doerr, 2003).

The mathematical modelling activities are designed for students to work in small groups. When students first engage a modelling task, their initial conceptualizations can be naïve (Lesh & Doerr, 2000). They sometimes make unwarranted assumptions and impose inappropriate constraints on the products they are to develop (English, 2006). These are to be expected as the tasks are not meant to be straightforward. They may contain incomplete information or have too much data or even require students to make plausible assumptions. Thus when students get stuck, these occasions are seen as opportunities for the group to offer feedback and critique ideas towards revising and refining those ideas. Students will have to express, test and revise their ideas and thereby develop models to describe, explain or predict the behaviour of the system. Research on modelling involving children although scant (Lehrer and Schauble, 2003) have found that children can model successfully and display different levels of sophistication of conceptualizations and show development in representational fluency (Chan, 2008; English and Watters, 2005; Lehrer & Schauble, 2000).

The modelling perspective therefore shifts attention beyond mathematics as computation towards mathematics as conceptualization, description and explanation (English, 2006), and is asserted to be the new direction in problem-solving research (Lesh & Zawojewski, 2007).

Conceptual Framework

As an instructional strategy, I situated mathematical modelling in a problem-based learning (PBL) platform. PBL has three main tenets, namely, an unstructured and complex task (in this case the modelling task), student collaboration, and teacher-scaffolding. The interaction between these three tenets would make visible the students' mathematical thinking during their modelling endeavour. Their mathematical thinking manifests as conceptual interpretations evidenced through the way they mathematize and operationalize data to create models.

Method

Participants

The participants were Primary 6 students from two classes of two neighbourhood schools. Each class comprised small groups of four or five students. One group from each class was selected by their respective mathematics teacher to be the target group for video-recording.

Modelling Task

The task was situated in the context of a contest to determine which group could make the biggest box and justify why their box was the biggest to the teacher who would play the role of the judge. Rulers, calculators, scissors, and tapes were provided. Two 50cm by 50cm vanguard sheets were also provided but the dimensions were not made known or indicated to the students. The task details provided to the students are framed in Figure 1 below.

> Your team is participating in a math project competition where you will need to present your findings in two days' time. In the competition, each team has been given only two square sheets made of vanguard. The team can decide if they want to use one vanguard sheet for trial. The team is supposed to make the biggest box (volume) using only ONE vanguard sheet. How would your team plan to solve the problem of making the biggest box? Show in detail how you reach a solution to convince the judge.

Figure 1. Task details

Data Collection and Analysis

The target groups were video-recorded during their engagement of the model-eliciting task. Other sources of data included my field notes, the students' written work and journals. The video data were transcribed and reviewed several times for evidence of conceptual interpretations that suggested the progression of the students' thinking. This paper addresses

the various conceptual interpretations displayed by one group of students during the modelling process.

Findings

Students' Mathematical Conceptions

In this section, I describe using examples from one of the target groups' involvement in the said modelling activity. From a modeling perspective, the students' mathematical thinking and reasoning are reported as models of conceptual interpretations. These conceptual interpretations are conceptual tools that they have developed along the way towards attaining the final solution.

Model 1: Assessment of properties

To solve the problem, students firstly need to break down the task details for consumption. The group was quick to assess what was needed when provided with the vanguard sheet. They took their rulers and measured the dimensions (length and breadth) of the vanguard sheet. What followed was a deliberation about the shape of the box and whether the box should be shallow or huge:

"But what is the shape of the box? Because 50, 50, 50. Would you like it to be shallow or huge or…?"

"Depends on what shape you want."

"Find breadth times length time height."

The group was throwing out ideas about what they knew about making a box. The initial brainstorming saw some ideas that were flawed as they tried to conceptualize a box:

"Divide by 5. 2500 divide by 5, how much? 2500 divide by 5."

"Now we're finding area, right? So 5 parts. 5 pieces because the cover we don't need. So the area is 500. One side area is 500."

The above saw a member having a correct notion of a net of a box in that it ought to have five equal faces without the cover. He argued that if the area of the vanguard sheet was 2500 cm^2, then to get five equal square faces, it should be divided by five. However, although the reasoning sounded plausible to the members at first, it was actually flawed because the group later realized that a square vanguard sheet of 2500 cm^2 could not be cut into five equal squares of 500 cm^2 each.

Assessing properties of the variables and mathematics components was an important starting point towards developing their models. The students had identified important variables but had yet to tie them together to form a mathematical relationship and thus the assessment of properties at this juncture constituted an underdeveloped model.

Model 2: Analysis of Design

The group had to contend with shape the box should take. They raised various options like whether the base of the box could take the form of a square, rectangle, triangle and even a circle. Although those were ideas worth thinking about, there did not delve deeper into them as the students found them too difficult and that there would be "endless possibilities" to consider.

"Triangle base is very hard."

"But rectangle, the possibilities is like you have to continue counting."

They chose to design a box with a square base as they deemed that to be within their abilities to construct by applying what they had learned about nets. Using the geometrical concept of nets, they formed relationships between the respective dimensions of length, width, height, and volume.

Model 3: A list of dimensions to test volume

The group became aware that different combinations of dimensions of length, width and height would give a different volume. They thus resorted to list and test the range of dimensions that would give them the biggest volume.

"Must I list out the all the whole number possibilities?"

"List all out. Height, length and breadth can all be the same right?"

Model 4: Establishment of key relationships

It was during the testing that a member discovered a pattern:

"So every time the length decrease by 2, the height will increase by what?"

"Increase by 1."

Although the terminologies "decrease" and "increase" used by the student were not mathematically sound, it was found that the student realized that every time two lengths of the square cut-outs were removed from the corners of the vanguard sheet, one length actually corresponded to the height of the box.

The discovery of a certain pattern during the testing was an important breakthrough as they began to list *48, 48, 1; 46, 46, 2; 38, 38, 6*, and so on with ease. One member began to picture that one centimetre of the cut-out would make a shallow box which implied how they see certain relationships among the dimensions in the light of the net of a box:

"Shallower means we should just take one cm of the height."

Another student was able to generalize what happened to the volume when they continued to cut bigger square cut-outs:

"Once you reach a certain place, then it will stop, then it will start to decrease."

That observation enabled the group to know with more certainty that they had reached a point where a certain set of dimensions would give them the biggest volume.

Model 5: Refinement of the workable model

The group had obtained their biggest volume by identifying from their list of variable relationships. They concluded that 34cm by 34cm by 8cm would give them the biggest volume and began to make the artifact of the box. However, the teacher-facilitator at that interjected by asking a question to extend their thinking:

"And what makes you think that in the number system, you only have whole numbers. In the number system, other than whole numbers, what do we have?"

That was when the students brought up the idea of decimals and they quickly got down to make smaller partitioning to see if they could get bigger volumes:

"33.1 x 33.1 x 8.45. Now we count 33.2."

"33.5 x 33.5 x 8.25 (…) 9258…""

The students were able to work within a specific narrow range between 33cm and 34cm using decimal fractions and arrive at their revised dimensions for an even bigger volume although only marginally.

Model 6: Extended model

Just when the group thought they had obtained the biggest volume, the teacher-facilitator dropped another poser to extend their thinking. She challenged them to think about what would happen if they could reuse the square cut-outs and determine if that could increase the volume of the box:

"What if don't sacrifice (the square cut-outs)? Everything counts."

The students were rather stumped by what the teacher meant. They could not see how the square cut-outs could be re-used:

"But if you have to sacrifice to make the biggest box, then you have to sacrifice to make the biggest box."

Their deliberations paid dividends when one member suggested that the square cut-outs could be fixed to the box:

"Yeah, so why not we make a smaller box at first, then we can use the part to fix it up?"

The idea of fixing the cut-outs to the box became a reality when they discovered each square cut-out could be divided into four equal strips and those strips could be taped just above the each face of the box to increase the height of the box.

"Cut one square into 4."

"Hey, can. You cut one square into 4 then you paste on top!"

That resulted in an increase in height and thus resulting in an even bigger volume.

Discussion and Concluding Points

Mathematical modelling is one possible direction that we can undertake to provide children with modelling experiences while promoting mathematical modelling as problem solving. From the classroom research, it illustrates that children can develop conceptual interpretations mathematically. By situating the mathematical modelling in a PBL setting, the problem solving becomes more authentic because the students are working in groups and solving an actual problem that needed them to produce and present their findings. As they do so, the student-student interaction, the student-task interaction, as well as with the student-teacher interaction elicits the students' problem-solving behaviours. Unlike relying on a standard or memorized procedure to solve problems as in a word-problem situation, students are developing powerful mathematical modelling tools such as the assessing of properties, strategizing through listing, identifying relationships, revising, improving and extending their models which are manifested as representational conceptual systems by way of the students' mathematical descriptions and explanations. These processes show that the students are engaged in mathematizing – where evidence of their application of mathematical reasoning aspects such as quantifying, dimensionalizing, coordinating, categorizing, algebratizing, and systematizing relevant objects, relationships, actions, patterns, and regularities in realistic situations are seen.

By working on mathematical modelling tasks in a PBL setting, not only do the students' mathematical thinking becomes explicit, they develop and heighten their metacognitive thinking as well through the interaction as there is greater need to check, query, validate, and monitor the problem-solving situation. In fact, the classroom activity also saw the students working independently of the teacher most of the time. The students therefore are more self-directed and the teacher offered scaffolding only on a few short occasions to elicit, support, and extend their thinking.

Although recent reviews of local research in mathematical problem solving have indicated that more teacher-practitioners are trying out alternative pedagogies (Foong, 2007; Fan & Zhu, 2007) there have been no reports of research on mathematical modeling carried out with children. The new inclusion of the "*application and modeling*" process in the Singapore Mathematics Curriculum Framework would require students to reason, communicate and

make connections as they formulate and improve mathematical models to represent and solve real-world problems. For teacher-practitioners to enact these processes call for a shift in classroom pedagogy where the teacher should diminish their control of the class and allow the students take more ownership of their learning through a more cognitive and constructivist approach and for the teacher to take on the role of a cognitive coach. This is not to say that teacher-led instruction should be thrown out of the window as it still has a place in mathematics teaching and learning. Importantly, students should be provided with situations where they can transfer and apply their curricular mathematics to real world situations. Engaging students in mathematical modelling situation looks promising if we want our children to be involved in authentic and meaningful problem solving. This paper has provided some insights into mathematical modelling and the mathematical development of children through their construction of models that could serve to foster in children future-oriented problem-solving competencies, and that it has also provided greater consideration towards supporting and enacting the Singapore Mathematics Curriculum Framework.

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