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HOW MONTESSORI MATERIALS ENHANCE AUTISTIC CHILDREN MATHEMATICS LEARNING: CASE STUDY OF LEARNING DIVISION

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ABSTRACT
Dr. Maria Montessori educational pedagogy was originally developed by observing mentally retarded children and learning from special education pioneers, like Jean Itard and Edouard Seguin. Her success with special needs children was extended to normal children. It became so popular and effective for the past eighty years, with normal children in many countries with diverse cultures, that it is now forgotten as a viable pedagogy for special needs education. The purpose of the case study is to study how Montessori materials can help autistic children to learn, operate and apply the mathematical concept of division, which is generally not taught, as it is considered too challenging. It is the hope of this study to generate interest in considering using Montessori materials and educational pedagogy in special education.
Dr. Maria Montessori educational pedagogy was initially developed by intense observations (8 a.m. to 7 p.m.) of mentally retarded children for two years in lunatic asylum from 1899-1901, where she was the director of the State Orthophrenic School of Rome. Her methods proved extremely successful (Standing, 1957; Lillard, 2005), such that retarded children with her intervention, were not only able to read and write, but were also able to pass public examinations, taken together with normal children (Standing, 1957). She became convinced that the mental deficiency problem was a pedagogical problem rather than a medical one (Standing, 1957). Intuitively, she may have suspected the brain research finding (Wolfe & Brandt, 1998) that the brain changes physiologically as a result of experience from the environment. The environment affects how the genes work and genes determine how the environment is interpreted. The Treatment and Education of Autistic and related Communication handicapped Children (TEACCH) approach to Autism Spectrum Disorders (ASD), a validated program that is considered effective by the Committee on Educational Interventions for Children with Autism (2001), supports the need to create organization in the physical environment and to provide elements of structured teaching that will cater to the needs of ASD learners in terms of their thinking, learning and neurobehavioral patterns, also known as the culture of autism (Meisbov, Shea & Schopler, 2004). My literature search for peer-reviewed research literature for subjects on “Montessori and Autism” from databases like ERIC (Education Resources Information Center) via EbscoHOST, Psychinfo and ProQuest Education, failed to yield any results, other than articles that suggest how Montessori may be used for children with disabilities (Flowers, 1993; Pickering, 2004). Moreover, recently published peer-reviewed reports of internet survey of treatments used by parents of children with autism (Green, et al., 2006) and autism intervention updates (Francis, 2005) did not even list Montessori approach being used for autism. I found a book by Dr. Lillard (2005), “Montessori: The Science Behind the Genius”,...
whose evaluation, found Montessori theory and practice to be scientifically supported with more than five hundred published research papers. However, there is certainly a gap in research on how Montessori theory and practice can be used for autistic children.

Dr. Maria Montessori further developed her pedagogy by learning from special education pioneers, like Jean Itard and Edouard Seguin (Standing, 1957). She believes that “...the child constructs himself, that he has a teacher within himself and that this inner teacher also follows a programme and a technique of education, and that we adults by acknowledging this unknown teacher may enjoy the privilege and good fortune of becoming its assistants and faithful servants by helping it with our co-operation” (Montessori, 1997, p. 4). She then extended her pedagogy to normal children. It became so popular and effective, in many countries with diverse cultures for the past eighty years that it is now forgotten as a viable pedagogy for special needs education. Dr. Montessori does not believe in the didactic form of teaching of her days as she found them boring (Lillard, 2005) and endorses what we now called the play curriculum. Montessori, like Nutbrown (1994), believes play is a process rather than a subject. It is within subjects that one should look to play as a means of teaching and learning rather than as a separate entity which probably lead to development of specially designed “toys” for children’s incidental learning. These varied learning materials have been field tested and loved by children to work with. Those which the children failed to show interest or ignored, were discarded and replaced by those which are popular and pleasurable (Lillard, 2005). The existing set of materials in use today have stood the test of time to be developmentally appropriate and desired by children for their simplicity, as children do naturally seek them out in the environment (Gettman, 1987). As a parent with two ASD sons in the Structured Teaching for Exceptional Pupils (STEP) Programme in Margaret Drive School(MDS), my interest is on how these specially designed Montessori “toys”, can be used
for my ASD children familiar to TEACCH program. TEACCH multifaceted approach, encourages other compatible programs for autism to be integrated, such as “behavioral techniques (prompting, shaping, reinforcement and response cost procedures), neo-behavioral approaches (such as incidental teaching and functional behavioral analysis) and developmental appropriates practices (Mesibov, et al., 2004). Montessori pedagogy may thus be considered for integration into TEACCH especially where educational materials are concerned. The curriculum used in MDS for STEP program is mainly from the TEACCH Preschool Curriculum Guide, which does not include the teaching of division. It would be interesting to study how Montessori materials can help autistic children to learn, operate and apply the mathematical concept of division, which is generally not taught, as it is considered too challenging. My research questions and their corresponding study propositions (CSP) (Yin, 2003) are:

1) How can Montessori materials help Autistic children to learn mathematical concept of division?

CSP1: Montessori materials provide a learning framework for the autistic child to build learning of division concept.

2) How can Montessori materials help the autistic children to operate the mathematical concepts of division?

CSP2: Montessori materials provide means for the autistic child to operate concretely on the concept of division.

3) How can Montessori materials help Autistic children to apply the mathematical concept of division?

CSP3: Montessori materials provide a learning framework for the autistic child to apply concretely the main concept of division.
METHOD

Case study research methodology is selected as the research requires the understanding of the complex processes (Yin, 2003) of how Montessori materials can help autistic children to learn, operate and apply the mathematical concept of division. The processes are considered complex as ASD children have differences in thinking, neurobehavioral patterns and thinking (Mesibov, et al., 2004) when compared to non-ASD children and also between individuals with ASD. Moreover there is no requirement to control behavioral events, but rather to focus on contemporary event of learning processes of division by ASD children, as opposed to historical events (Yin, 2003). Finally, the intention is to generalize to theoretical propositions (Yin, 2003) of compatibility with TEACCH approach and not to populations or universes.

The research methodology adopts Yin (2003) five components of research design, using a multiple-case replication design, where there are two units of analysis; a moderately ASD child, age 11, with intelligence quotient(IQ) of 55 and a mildly autistic child, age 10 with IQ of 72. Both have the prerequisites of being able to count from one to one hundred. There are altogether six sessions in this research, where there are three period lessons (Lillard, 2005) for learning vocabulary and worksheet exercises (Ex) that are used to build concepts gradually according to the goals below:

1. Understand concept of divisor by fixing numerator and changing the divisor (Ex1-6)
2. Understand concept of dividend/numerator by fixing the divisor and changing the numerator (Ex 7-10)
3. Understand the concept of quotient by including elements of people pictures receiving the quotient/Sharing (Ex. 11)
4. Reinforce concept of dividend/numerator by fixing the divisor and changing the numerator and concept of quotient (Ex. 12-17)

5. Transfer learning to real life pictures (sweets) (Ex. 18-20)

6. Symbolic division manipulation (Ex 21-26)

7. Learner to understand meaning of numerator and divisor via using real life objects and pictures (Ex. 27-29)

8. Learner to solve real world problem by using real objects for dividends and pictures for divisors. (Ex. 30-33)

The duration of each session is not fixed so as to allow time for child to develop the concepts, make mistakes, learn from mistakes and intervene where necessary to check for understanding or reinforce ideas. Essentially to follow-the-child (Lillard, 2005) and encourage the child’s learning processes. The six sessions are carried out according to the below:

Session 1: 3 Period Lesson (Skittle, bead, multiplication board) + Ex 1-3

Session 2: 3 Period Lesson (Skittle, bead, multiplication board) + Ex 4-6

Session 3: Ex 7-10

Session 4: Ex 11-17

Session 5: Ex 18 - 22

Session 6: Ex 22 - 33

Attempt to gather interrater reliability was made by showing video clips, where five educationalists responded.
Logic Linking Data to the Propositions and Criteria for Interpreting the Findings

**CSP1: Montessori materials provide a learning framework for the autistic child to build learning of division concept.**

Evidences in this study to support this proposition are from two perspectives. Firstly to analyse and make explicit whether there are learning framework evidences in the design and planned use of Montessori division board set and secondly from evidences of whether studied subjects actually make use of the learning framework to build the learning of division concept from worksheet exercises 1 to 17.

Squire and Bryant (2003) found in their research paper on children’s model of division that young children found it easier to solve concrete partitive division, that is, when the objects to be shared are grouped by divisor rather than by the quotient. The division board set is designed to provide such concrete experience of learning partitive division. Its organisation and layout are as in Fig.1.

**Fig 1. Division Board Set**
Visually, the division board organises clearly the locations for placement of divisors with green skittles (circular flat base) with corresponding sequentially labelled divisor values from one to nine and from left to right. Dividend quantity is represented by using green beads to be placed in small circular depressions and shared equally with the divisors/skittles, resulting in the quotient values read sequentially from top to bottom on the left of the board. For example, for the division problem of dividing 18(dividend value) with 9(divisor value), which gives the quotient value of 2 (see Fig. 2), the learner is to take out 9 skittles and place them sequentially from left to right, providing a concrete experience of handling and seeing divisors and counting them as they are placed in position. Secondly, the learner is to count 18 beads from the pool of beads and place them in the nearby empty container, which not only makes the dividend value concrete, but also allows them to physically manipulate and count meaningfully. Thirdly, the counted 18 beads are to be distributed equally with all the skittles, allowing physical and concrete experience of the division process. Finally, the quotient value is to be read from the left, revealing the value of 2, which is another visually concrete

**Fig 2. Sample Problem : 18 ÷ 9 = 2**

1. Scaffolding/Structure For Dividend placement
2. Concrete visual materials and quantity for dividend
3. Control of error if bead number unequal

*Quotient value can be read* sequentially from left to right, providing a concrete experience of handling and seeing
experience of deriving value of the quotient. The division board set is designed with control of error for the correct number of skittles to be placed, as the placement of skittles is visually numbered with any missing skittle clearly observed. It is also designed for control of error for number of beads for division problems without remainders, where incorrect counting of beads will result in some skittle not having the same number of beads as the rest. The design of the board with its depressions, beads and skittles also allows visual and physical counting to check for the correct quantities and results. Therefore, Montessori division board set is pedagogically designed for learning division in that it operationalizes the abstract concept of division by making the processes concrete, thereby providing appropriately for ASD children, who are concrete thinkers rather than abstract thinkers (Mesibov, et al., 2004). Moreover, the visually structured activities with visual organisation and visual clarity, helps ASD children with difficulty in organisation and sequencing to overcome their weakness. Moreover, the design of the activity also allows them to overcome difficulty to combine or integrate the ideas (Mesibov, et al., 2004) in division by visual clarity of the relationships (divisor, dividend and quotient). Finally, the division board set activity also allows ASD children to overcome their cognitive difficulty in generating meaning or make sense of the division activity event (Mesibov et al., 2004) and also removes anxiety, as the process of doing division, within the learning framework of the division board is sequentially predictable and structured (Mesibov, et al., 2004). As explained, the division board set has sound pedagogy embedded in the design for teaching division. Moreover, the design caters to TEACCH understanding of ASD learners in their thinking, neurobehavioral patterns and learning. The division board set is certainly a learning framework suitably designed for ASD learners. The evidences to support CSP1 from second perspective as well as for CSP2 and CSP3 are described in the results.
Validity of Design

**Construct Validity**

The construct validity of the research design is by establishing chain of evidence with documenting progress of learning through checking for the listed evidences of the corresponding study propositions. Backward design (Wiggins & McTighe, 2005) is used for constructing the curriculum, instructional plan and worksheets through process of engineering learning based on goals listed earlier.

**Internal Validity**

The internal validity is build using explanation-building of current observed learning processes with TEACCH approach, on how it caters to ASD children differences in thinking, neurobehavioral patterns and learning (Mesibov, et al., 2004) and use of backward design (Wiggins & McTighe, 2005) approach to establish goals of activities.

**External Validity**

The external validity is build using replication (multiple case) of mild and moderate ASD child.

**Reliability of Design**

The reliability of the research is established by using case study protocol. This involves using the three period lessons for learning vocabulary and worksheet exercises that build concepts gradually and adopting structured teaching methodology (Mesibov, et al., 2004). Moreover, attempt was made for simple interrater reliability of the evidences by showing video clips to five educationalists on whether the evidences are present. The value was determined by percentage of agreements of presence of evidence.
RESULTS

**CSP1: Montessori materials provide a learning framework for the autistic child to build learning of division concept.**

Yes, as explained earlier, the analysis showed that the division board set has a learning framework in the design and planned use as explained earlier. The findings from observations made did show that both the moderately and mildly autistic child actually make use of the learning framework to build the learning of division concept from worksheet exercises from 1 to 17 with details described below. There is more than eighty percent agreement from the five educationalists that listened to the presentation and viewed the short video clip that there are evidences of the learning framework and its actual usage in both autistic children.

1. **Scaffolding/Elements of Structure**

Generally, both children are observed to be able to correctly read the questions and then to make use of the scaffolding, by firstly identifying the divisor value, counting the number of skittles and placing it accordingly in the respective positions. Thereafter both look for the value of the dividends, counted the number of beads, and then placed them by distributing equally with the skittles orderly. Both then read the quotient value on the left of the division board and wrote the value into the worksheet. Both have thus made use of the scaffolding and elements of structure of the division board to work out the solution to the problems in the worksheet. All five educationalists agreed that there is evidence from watching the short video clip.
2. **Acceptable Visual Learning/Concrete Visual Materials that Develop Solution**

As both are observed to make use of the division board set appropriately to develop solutions to problems in the worksheet, the concrete visual learning materials are acceptable to them. All five educationalists agreed that there is evidence from watching the short video clip.

3. **Consistent Work System/Reusable Consistent Material to Build Learning**

As both reuse the division board set materials to solve the problems from worksheet 1 to 17 successfully, the division board set, supports a consistent work system to solve division problems and also provides a reusable consistent material to build learning of division. Four of the five educationalists agreed that there is evidence from watching a short video clip.

4. **Attachment to Routines/Stable Learning Framework**

As both can understand the use of the division board set and does routinely use it to solve the division problems at different difficulty level, the division board set provides a stable learning framework for the learning of division. All five educationalists agreed that there is evidence from watching the short video clip.

5. **Appropriate organisation and sequencing of learning processes**

As both follows the order of making use of skittles for divisors and then beads for dividend distribution on the division board, the division board provides appropriate organisation and sequencing for learning process of division. Four of the five educationalists agreed that there is evidence from watching the short video clip.
CSP2: Montessori materials provide means for the autistic child to operate concretely on the concept of division.

Yes, the findings of observations showed that the division board provide means for the mildly and moderate autistic child to operate concretely on the concept of division from worksheet exercises from 18 to 22. All the five educationalists that viewed the short video clip all agreed on presence of all the evidences in both autistic children. There are three interesting and surprising findings. Firstly, both autistic children have pride and do not like to be wrong. There is a need to develop sensitive and tactful ways not to hurt child’s ego, such as requesting them to check without revealing what is wrong, finger pointing to the problem but not saying anything other than check. Secondly, handling the beads is observed to be difficult for both of them as both sought ways to reduce using them. Thirdly, given freedom, autistic children can develop their own algorithms for handling beads. The moderately autistic child resorts to using the lowest dividend value problem and then builds up to problems with higher dividend value by adding the difference value of beads with rearranging the existing beads to the new divisor value. The mildly autistic child resorts to using finger to count on the bead positions which to him is equivalent to putting the beads. Moreover, instead of using all the skittles, he only use two to mark the first and last position of the skittles for his visual organisation to distribute the beads. Details of the evidences are as below:

1. Identify divisor from worksheet.

Both are observed to correctly identify divisor from the worksheet as they are looking at the worksheet to determine the correct number of skittles and then putting accordingly. With time, an interesting development is observed for the mildly autistic child who decided to use the shortcut of putting only the first and last skittle to demarcate the positions needed to help him distribute correctly the beads.
2. *Take correct number of skittles.*

Both are observed to take the correct number of skittles. The moderately autistic child faithfully takes the correct number of skittles all the time, whereas the mildly autistic child with time, decided to use only two skittles to demarcate the first and last positions of the number of skittles needed to help him distribute correctly the beads. In any case, the meaning of reflecting the correct number of skittles is demonstrated.

3. *Put skittles in divisor positions from left to right.*

Both are observed to correctly reflect the correct number of skittles from left to right. The moderately autistic child faithfully takes puts the skittles from left to right, whereas the mildly autistic child with time, decided to put only the leftmost skittle and the rightmost skittle to demarcate the first and last positions of the number of skittles needed to help him distribute the beads. In any case, the goal of reflecting physically the numbers of skittle positions are observed.

4. *Identify dividend from worksheet.*

Both are observed to correctly identify the dividend from the worksheet as they reflect that they know the number of beads needed to solve the problem. With time, an interesting development is observed with the moderately autistic child in that he will chose the problem with the smallest dividend first to work with. This could have been the result of frustrations faced due to weak fine motor skills for handling large number of small beads. His algorithm is to use all the counted beads in the next problem with larger dividend value by adding the balance of beads required after rearranging all the beads based on the new divisor.

5. *Take correct number of beads*

Both are observed to take the correct number of beads. With time, both develop mechanisms to overcome the “chore” of this task in different ways because of difficulty in handling the small beads which can roll away. The moderately autistic child would work from problems
with the smallest dividend first and then work up to the largest number so as to only take the balance of beads needed to top-up to the new larger dividend number. On the other hand, the mildly autistic child would avoid taking the beads altogether and use his finger to count on the bead positions on the division board distributing equally from left to right based on the divisor skittle positions until he hits the dividend value. Occasionally, he lost count and needs to restart the counting again.

6. **Distribute beads equally to all the skittles with counting.**

Both showed evidences of understanding the concept, with the moderately autistic child faithfully putting all the beads while the mildly autistic child distributes virtually via his finger counting of the bead positions according to number of skittles.

7. **Identify quotient as total number of beads for each skittle.**

Both are observed to be able to identify the quotient value by correctly observing the value on the division board and then transferring the value to the worksheet or pointing to the value on the board and then writing them in the worksheet.

8. **Write the quotient in the worksheet.**

Both are observed to write the quotient value correctly into the worksheet.
CSP3: Montessori materials provide a learning framework for the autistic child to apply concretely the main concept of division

Yes, the findings from observations showed that the division board set provide means for the mildly and moderate autistic child to apply concretely on the concept of division from worksheet exercises 22 to 33. There is more than eighty percent agreement from the five educationalists that viewed the short video clip that there are evidences of both autistic children applying concretely the main concept of division. An interesting accidental discovery of transfer of division know-how is observed when my mildly autistic son observed my preparation of the worksheet and desires to show that he can do it. I told him to wait until work time but he went over to take a blank piece of paper and starting working out the problem of $21 \div 3$, by drawing out the equivalents of skittles and beads (see Fig. 3) needed and arrived at the answer. The finding implies that with time and practice, the division board know-how is transferable.

![Fig. 3 Joel transfer of division board learning to paper](image)

The following evidences are observed to support CSP3:

1. **Count number of sweets correctly**

Both showed difficulty in counting the number of sweets when the number is greater than twelve. This may be explained by their difficulty in organisation and sequencing (Mesibov, et al., 2004) especially for laying out the sweets to be counted properly.
2. **Write the total number of sweets in the worksheet.**
Both are observed to be able to write the total number of sweets correctly in the worksheet.

3. **Count the number of people to share sweets with.**
Both are observed to be able to count the number of people to share sweets with.

4. **Write the total number of people to share sweets with in the worksheet.**
Both are observed to be able to write the total number of people to share sweets with.

5. **Represent correct number of skittle**
Both are observed to be able to represent the correct number of skittle. The mildly autistic child will only use two skittles to represent the total number of skittle positions to consider when sharing.

6. **Take correct number of beads and distribute among the skittles**
Both are observed to understand the concept of the correct number of beads. The mildly autistic child will substitute the actual beads by counting with his fingers on the bead positions from left to right.

7. **Identify quotient as total number of beads for each skittle and write the quotient in the worksheet.**
Both are observed to be able to identify conceptually the total number of beads for each skittle by looking at the quotient value on the left of the division board.

**DISCUSSION**

Findings of the case study research, shows that both moderate and mildly autistic children can learn, operate and apply division concept with appropriate educational pedagogy. This is in contrast to the general belief that division concept is considered too abstract and thus not taught in the curriculum for autistic children. The findings certainly support Dr. Maria
Montessori insight that the mental deficiency problem in asylums of her time was a pedagogical problem rather than a medical one (Standing, 1957), as division can be taught with appropriate pedagogy. Moreover, as was described, the division board set has embedded pedagogy consistent with TEACCH approach to ASD (Mesibov, et al., 2004) in that it is a visually structured activity with visual organisation and visual clarity for visual and concrete learners that have difficulty to combine or integrate ideas, allowing the solutions to be worked out with organisation and sequence in a predictable learning framework. The division board set is thus suitable to be used and integrated as one of the activity within the TEACCH work system. This also highlights the potential that Montessori materials and its curriculum, which are currently used for normal children with effectiveness (Lillard, 2005), can also be used for ASD children, opening up opportunity for ASD children to access normal children education curriculum. This may lead to a more inclusive society as ASD children may learn together with normal children using a differentiated curriculum with help of appropriate Montessori materials and TEACCH approach to support their learning. Perhaps Montessori educational pedagogy and materials may be considered and used in Universal Design for Learning (UDL)(Mcguire, Scott & Shaw, 2006; Hitchcock, Meyer, Rose, & Jackson,2002) by conversion of its materials and intent, to suitable equivalent digital versions, allowing electronic manipulation and customisation for different learner preferences and needs.

LIMITATIONS OF STUDY

As researcher is new and inexperience in qualitative research, the quality of data captured can be improved. Richer data may also be captured if the researcher is not directly participating in the intervention. Researcher may be bias in interpreting observation.
REFERENCES


