Assessing conceptual learning from quantitative problem solving

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Abstract
In recent years more emphasis had been placed on conceptual learning and understanding in the sciences. Unfortunately, among advocates of such a position there were those who did not see any role for quantitative problem solving at the secondary/high school level or even the introductory university level. This may be due to the frequent use of typical 'plug and chug' problems in problem solving sessions. However a quantitative problem which requires students to illustrate their conceptual learning and understanding will reveal much to teachers and at the same time provide invaluable feedback. Such a problem was identified and presented to 118 secondary three students and 38 junior college year one students. Analyzing their performance on this quantitative problem, it was evident that these students do have problems related to conceptual learning. In this paper the author will discuss these conceptual learning problems based on the students' performance. At the same time, a number of suggestions on appropriate remedial experiences would be offered.
Introduction

In recent years more emphasis had been placed on conceptual learning and understanding in the sciences. Unfortunately, among advocates of such a position there were those who did not see any role for quantitative problem solving at the secondary/high school level or even the introductory university level. This may be due to the frequent use of typical 'plug and chug' problems in problem solving sessions. Moreover in advocating such a position, we may also be 'throwing away the baby with the bath water'.

Traditionally, problem solving has always been part and parcel of the teaching-learning process in science, especially the physical science.

Support from educators for problem solving is not lacking. Problem solving was recognized by Ausubel (1968) as a special case of meaningful learning. Mills & Deane (1960) described problem solving as a way of thinking that should be cultivated.

However, attempts had been made recently to stimulate a rethinking of the things we do so routinely. For example, the issue of defining problems and problem solving (example, Chiapetta & Russell, 1982; Garrett et al., 1990) had certainly been useful. Zoller (1987) used the term 'exercise solving instruction' to refer to one which required students to solve problems by merely applying a known procedure to obtain the teacher known, correct solution. Woods (1977) defined 'problem solving instruction' as one of finding the best answer for an unknown. Gagne (1965) described 'problem solving instruction as a strategy where students construct their own solution to a problem, not when a problem solution is stated by the teacher.'

Festinger's (1962) view of a problem is one which causes 'cognitive dissonance' in a learner. This could be appealing. It suggests a problem as one that can stimulate intellectual functioning of the mind, regardless of whether there is a solution or not. It is dependent on how a situation or context is posed. In an editorial on problem solving, Rigden (1987) pointed out that "there is a significant difference between rote computation and problem solving....Problem solving involves higher-order thinking skills."

For practical purposes, it appears useful to differentiate the different types of problems and problem solving. Qualitative problems are those where the use of investigative/inquiry/science process skills are involved. Quantitative or mathematical or computational problems are those where the use of mathematical models, equations or formulae are involved.

Whatever terms and definitions advocated, I certainly hope we can all agree that our main educational concern is that the use of worthwhile
higher order thinking skills are involved in problem solving. At the same time, a quantitative problem which requires students to illustrate their conceptual learning and understanding will reveal much to teachers and at the same time provide invaluable feedback.

The real issue is whether we can identify quantitative problems which can provoke higher order thinking skills on the part of our students and through which we can then recognize the lack of conceptual learning or understanding.

As part of our on-going proactive teacher training programme in content pedagogy, we attempt to motivate our pre-service teachers to identify and innovate quantitative problems that can not only provoke higher order thinking skills on the part of students but also provide information on the lack of conceptual learning and understanding. As an initial step, we reviewed typical problems given in textbooks and try to identify those problems that may be routine but have potential to be adapted and improved upon. One such problem would be used for discussion here. With this quantitative problem, we had conducted a study to look at students' conceptual understanding. This approach in content pedagogy is important in order to ensure that our future teachers will be 'thinking and innovative' teachers.

A Typical Quantitative Problem

I would illustrate in this presentation a fairly typical quantitative problem set at the 'O' level:

A plane mirror, 20 cm in height, is held vertically about one metre in front of a student whose height is 180 cm. The mirror is positioned so the top of the student's head appears at the top edge of the mirror. The student's eyes are 12 cm below the top of his/her head. (1) How much of his own image can be seen? (2) In order to see a full-length image of himself/herself, what is the minimum mirror height required?

It is possible for this problem to be solved very quickly and easily by 'applying' a very simple principle, that is, 'image size seen equal to twice mirror height'. In fact most students do use this quite routinely. This is an example of rote problem solving.

However, this very same problem could be adapted to demand from students an indication of meaningful problem solving. At the same time, one could assess their conceptual understanding.

Assessing Conceptual Understanding
For the above problem to be solved meaningfully, students need to be asked to begin by sketching a reasonable ray diagram to indicate how the image is located (Appendix A). Much can be learnt by asking students to perform this additional task before they provide their quantitative solutions. In fact this approach to quantitative problem solving has become a feature adopted in our content pedagogy programmes for both pre-service and in-service teacher training. It is also a practice very much based on previous research studies.

As early as the seventies, researchers (example, Reif et al., 1976; Larkin, 1977a, 1977b; Simon & Simon, 1978; Larkin & Reif, 1979; Larkin, 1980; Finegold & Mass, 1985) adopting the novice-expert approach had observed that one main characteristic that distinguished experts from novices was the use of qualitative analyses before 'selecting' the appropriate equations or formulae. One concrete evidence or extension of such qualitative analyses is the use of appropriate diagrams/sketches. Yap, Toh and Lam (1995) had suggested that we could use this characteristic together with others to define a 'deep' approach to problem solving.

Content Analysis

Often when asked what physics principles are involved in this problem, teachers and students readily suggested the laws of reflection, especially that relating the incidence and reflection angles. However the author suggests that by asking students to draw a sketch a ray diagram, one could also assess their understanding of other image characteristics, such as image size and distance from the mirror, and their understanding of the principle of sight. We can also understand why a practical approach to solving the problem could also interfere with the solver's conceptual learning. It is very common for textbooks to treat discussions on principles of reflection, mirror images and properties, and principle of sight separately, without making any attempt to show how they are inter-related. Yet in everyday life, they are inter-related.

Students need to indicate a ray from the top of head incident upon top edge of mirror and reflected to the eye in accordance with law of reflection that the angle of incidence is equal to the angle of reflection. In order for this to hold, it is only possible if the mirror is midway between the top of the head and the eye.

To determine that lower part of the body that the student can see, it is easier to trace from the eye to that body part such that the same law is applicable. But this is contrary to how one should show the path of the ray.
Results of Study

While the adapted problem (Appendix A) was initially given to 118 secondary three and 38 JC1 students, the results in the discussion below would be based on only 96 secondary three students from 3 classes in two different schools. These students had been 'taught' the topics on plane mirror reflection and images.

A variety of responses were obtained from the students. Most of the students included the incident and reflected rays with the object and the mirror in place. However, as high as 35% of the students either did not show any image or showed an incomplete image in their sketches. Less than 15% of the students made use of their sketches to obtain or calculate their solutions. A very high proportion appeared to apply the relationship 'image size equal to twice mirror size' in a rote manner.

A large proportion of students (about 40%) indicated that the 'positions of top edge of mirror and the top of head are at same level'. In discussing with various groups of pre-service and in-service teachers, there had always been a suggestion that this might be the result of a language problem. They suggested that students interpreted the sentence "The mirror is positioned so that the student's head appears at the top of the mirror" to mean that the top edge of the mirror is at the same height as the top of the student's head. One such suggestion was to rephrase it to "The mirror is positioned so that the student will see the top of his/her head at the top edge of the mirror." However, initial tryout with a small group of students suggested that there would not be much changes in the responses. I would suggest that it is more a conceptual problem. If students were aware that the ray from the top of the head has to reflected on to the eye (principle of sight) and that in this process the angle of incidence must also be equal the angle of reflection, then they would at least correct their initial drawings.

Out of this proportion, some (about 10%) would draw in such a way as to clearly indicate that the angle of incidence could not be equal to the angle of reflection. This is represented by the sketch below.

Most of the rest would indicate the equality of the angles of incidence and reflection but did not show a ray incident at the top edge of the mirror and reflected to the eye. A representative sketch is shown as follows.
By asking for a sketch of a ray diagram before working out the numerical solution, a reasonable assessment of students' understanding of the principle of sight can be made. The directions of the light rays indicated in the sketch provide us a reasonable clue. A number of students (about 30%) indicated sketches that are similar to either of the diagrams shown below. It appears that such students may be holding the view that vision is possible by 'active' looking (in this case through the mirror) not by reflection of light rays to the eyes. This view is similar to that expressed by students in earlier studies on vision (e.g. Guesne, 1985)

Some students have the incident and reflected rays at the top edge of the mirror indicated in the correct directions but then not for those rays at the bottom edge of the mirror. A plausible explanation for this is tied up with the practical way of locating the lower portion of the body that can be seen. It is easier to draw(sketch from the eye to the bottom edge of the mirror first and then locate the lower portion of the body.

Another 'interesting' observation from the responses of students was that a high proportion (about 20%) of them sketched diagrams indicating a whole body image for a mirror not big enough. The diagram below is a typical representation (without any indication of the light rays). Apparently typical textbooks do not pay attention to such examples or exercises.

One student while indicating a whole body image also included rays to suggest that the image size was the same as the mirror size.

A few students (about 3%) were obviously 'attempting to draw' images based on a converging lens. The diagram below is such an example.

Discussion

From the results of the study, it is obvious that well selected or designed quantitative problems that do not follow the 'plug and chug'
routine have their role and place in the teaching-learning process. They should be part and parcel of physics teaching-learning. It will be for better physics teaching-learning if teachers are encouraged and given the time and motivation to identify and innovate similar quantitative problems.

A whole new perception and orientation towards problem solving sessions has to be adopted. We need to see physics problem sessions as not merely 'working out the solutions'. A variety of appropriate hands-on and minds-on activities has to be integrated in order to facilitate both conceptual understanding and problem solving. More proactive innovation may also be needed to try and extend problems in ways that are related with everyday events or experiences.

Extending the problem
In the given problem, the student stands 1 metre from the mirror. A good extension of this problem is to ask students to sketch a similar ray diagram when the distance is now 2 metres away from the mirror. Students are often surprised at the results. It would appear to contradict their daily experience, for example their observation with bathroom or dressing room mirrors. This provides another opportunity to extend this problem to deal with ray diagrams when the mirror is inclined at an angle relative to the observer. Typical textbooks and teaching do not often introduce problems that are based on everyday contexts. They do not deal with situations such as these. It is little wonder then that most of them could not handle such 'minds-on' variations of the problems.

Integrating Hands-on and Minds-on Activities
It is possible to integrate a number of 'concrete' activities related to this quantitative problem. Such activities are hands-on in nature but certainly useful and helpful to students acquiring a better conceptual understanding. Some of these activities could be:

Position plane mirror in front of student so that the student will see the top of his/her head at the top edge of the mirror. Another student could then determine the position of the top edge of the mirror relative to the top of the student's head and his/her eyes.

Compare (by measuring) the actual height of the image seen and the mirror height. The students will be able to determine that the actual height of the image seen will be double the mirror height.

Compare the amount of the image seen through the mirror for different distances away from the mirror. This will then help to reinforce their conceptual understanding that the amount of image seen is independent of the distance away from the mirror. This is true if the mirror is vertical relative to the observer.
Explore with an adjustable bathroom-type plane mirror to see how the angle of inclination affects the amount of image seen. This will also help to relate their conceptual knowledge with their daily experience.

Such hands-on and minds-on activities when integrated will help to remould our negative perception of the routine 'plug and chug' problem solving sessions.

Conclusion

There is no doubt that routine quantitative problem solving classes that involve 'plug and chug' type of problems are slowly but surely declining. As more and more educators become disenchanted with previous practices that are not educationally sound, we need to ensure the good are not thrown out as well. A number of innovative efforts have been introduced. Problems that can help to identify potential alternative conceptual views, problems that allow one to assess conceptual difficulties or understanding and problems that provide meaningful conceptual reinforcement are certainly useful. There is a need to build on such efforts. A more intimate and proactive partnership involving teacher educators and teachers is needed. Problems and problem solving need to be meaningful, challenging and related more to everyday contexts in order to ensure a truly complete learning experience for our students.

References


Appendix A

The Plane Mirror Problem

A plane mirror, 20 cm in height, is held vertically about one metre in front of a student whose height is 180 cm. The mirror is positioned so the top of the student's head appears at the top edge of the mirror. The student's eyes are 12 cm below the top of his head.

1) Sketch a ray diagram to indicate how the student's image would be located.

2) How much of his own image can be seen? Show your reasoning/calculation clearly.

3) In order to see a full-length image of himself/herself, what is the minimum mirror height required? Show your reasoning/calculation.
clearly.