Paper Title Knowing Mathematical Representations: Pedagogical Principles of Cultural Development

Author(s) SungWon Hwang, Nanyang Technological University, Singapore; Wolff-Michael Roth, University of Victoria; Mijung Kim, NSSE, National Institute of Education, Nanyang Technological University

Session Title Teachers and Students

Session Type Paper

Presentation Date 5/1/2010

Presentation Location Denver, Colorado, USA

Descriptors Science Education, Curriculum in Classroom, Literacy

Methodology Mixed Method

Unit SIG-Science Teaching and Learning

2010 AERA Annual Meeting

Knowing Mathematical Representations: Pedagogical Principles for Cultural Development[1]

SungWon Hwang, Wolff-Michael Roth*, & Mijung Kim

Nanyang Technological University, *University of Victoria

ABSTRACT

Mathematical representations constitute prevalent resources that mediate the understanding of science concepts. For example, in science lessons different types of mathematical representations (e.g., force diagram, chemical reaction equations, DNA models) assist science teachers in explaining science concepts. The inherently materials bodies (e.g., visual representations) are connected to other sense-making resources (e.g., science talk), and thereby come to stand for something (i.e., mathematical idea). Yet, students have difficulty reading mathematical representations that they encounter in science lessons and associated science talks. More so, this difficulty tends to be attributed to the matter of either students' individual capacities or the qualities of mathematical representations, which exists independent of the concrete practice of communication. In this study, we take a holistic approach to students' understanding of mathematical representations, which does not dichotomize sense-making from the sensuous

[1] All correspondence concerning this paper should be addressed to SungWon Hwang, Assistant Professor, Natural Sciences & Science Education, National Institute of Education, Nanyang Technological University, 1 Nanyang Walk, 637616, Singapore. Email: sungwon.hwang@nie.edu.sg.

experience of the world (objects) and therefore provides implications for the pedagogical problem of "representation." We thematize the dynamic experience of mathematical representations in communication, which we summarize into three claims. First, the body reproduces and transforms cultural resources for translating mathematical representations. Second, the increase of heterogeneous sense-making resources in communication increases possibilities for realizing a new way of talking. Third, knowing mathematical representations emerges from the different, irreducible modes of communication as an integrated whole. We support the three claims by analyzing a case example in which children talk about a (randomly generated) geometrical representation in a mathematics lesson. We conclude that knowing mathematical representations is equivalent to (bodily and embodied) reading between different sense-making resources that constitute a series of references to bring about scientific conceptions in its totality.

INTRODUCTION

Mathematical representations constitute pervasive sense-making resources in the pedagogical practice of science education (e.g., diagram, geometrical models, or graphs). Many science teachers draw on mathematical representations in their science talk (e.g., Roth, 2009). Despite the significance of mathematical representations and associated studies on learning, pedagogical theories related to the use of mathematical representation have been less informed by learning theories. Rather, prescriptive teaching models describe learning from a perspective that is constructed after representations are already known, and thereby make a gap with the students' position in which those representations may not mark any sense. Mathematical representations tend to be treated as if they possess meanings stably attached to them and should express meanings self-evidently. This assumption is exemplified when a (beginning) teacher attempts to present a number of mathematical representations in a short talk and does not provide sufficient amount of time to read. It is well known among experienced teachers that the simple introduction of nice representations does not necessarily promise students' successful understanding of scientific ideas. One teacher's effective use of a mathematical representation does not promise another teacher's success with it in another situation, either. This is so because mathematical representations constitute only one part of communication of which the whole stands for scientific/mathematical ideas. The reason for a successful or unsuccessful lesson may not be fully explained by the use or not-use of a specific representation but requires the consideration of sense-making resources simultaneously mobilized for the communication of scientific ideas. Therefore, the purpose of this article is to study the whole of different sense-making resources that dynamically appear in communication to constitute a process of talking mathematical representations. We aim to understand real people's experience of mathematical representations in real time and articulate pedagogical principles theorized from a perspective of students who do not yet know mathematical representations and therefore cannot intend to learn.

From a pedagogical perspective, knowing mathematical representations involves a dialectic problem. Students encounter new mathematical representations (e.g., force diagram) and depend on teacher's talk and other textual references to get some sense of them. However, teachers' talk with/about mathematical representations, for example, would make sense to students only when they become already familiar with aspects of the world that those representations refer to. This is the case when a student reads a difficult book and ultimately says he/she finds nothing but the reproduction of what he/she has been already familiar with or when a student listens to a science/mathematics talk and finds it sounding like a talk in a foreign language. From a linguistic perspective, understanding mathematical representations involves the problem of making a material body (i.e., a representation) stand for something, which is possible because of a series of connections that a representation makes in relation to (culturally) appropriate resources (i.e., *sign–interpretant* relation). Mathematical representations mark some sense to readers because of the web of cultural significations that they make available. Yet, the mutually presupposing relation between mathematical representations (as sign) and sense-making resources (as interpretant) indicates that this web of cultural significations consists of the infinite number of steps until it is stabilized at some stage of communication, which is known as the problem of *unlimited semiosis* (e.g., Eco, 1976). For example, a science teacher explains the concept of Newton's second law by presenting a mathematical equation followed by analyzing a constant-velocity motion graph in a zero-net-force situation, explaining the concepts of inertia and acceleration, showing some demonstrations, and so on. Still, the teacher has to make a decision whether he/she has to provide additional sense-making resources. The infinite number of sense-making resources required for setting up something as a representation raises a question about the very possibility of reading (understanding) mathematical representations: In what ways people overcome the infinite number of steps of cultural significations and come to read (see) mathematical representations as a reference to some scientific/mathematical idea? How students come to be able to read from a force diagram the (non-) equilibrium of multiples forces acting on an object and the resulting motion of the object? Since knowing mathematical representations (e.g., force diagram) means being able to enlist appropriate sense-making resources (e.g., configuration of real objects, verbal description) and therefore let a material body stand for something (i.e., making a sign–interpretant relation), the amount and extent of sense-making resources that students need turns out to be an empirical matter that depends on the concrete ways by which people experience the world and find their ways (i.e., culture). The role of pedagogical practice is to assist students in coming to know (read) mathematical representations and understand scientific (mathematical) ideas involved in them.

In this study, we approach the pedagogical problem of matheamtical representation by taking a holistic approach to communication (Vygotsky, 1986), which allows dealing with the problem by addressing cultural, linguistic, and bodily nature of learning science/mathematics rather than a matter of individual consciousness—that which we have no access and therefore only leads to the unsolvable problem. That is, we take a holistic perspective that does not theorize thinking as dependent of a specific form of representation (e.g., words) but considers different, irreducible modes of communication as an integrated whole. In what follows, we articulate a Vygotskian theoretical framework of knowing mathematical representations, which is centralized by the phenomenological concept of the living body as the source of sense-making in and through a cultural web of significations. We exemplify the significance of this Vygotskian, holistic approach by analyzing a process by which a randomly produced geometrical shape is first known to children in and through communication. For the analysis of the empirical data in the first-time-through method, we use analytic methods of *conversation analysis* and ethnomethodology and exemplify the development of representation-mediated communication. We thereby articulate pedagogical implications for assisting students' cultural development.

THE LIVING BODY AS CULTURAL SIGNIFICATION

In the real situation of teaching and learning, mathematical representations constitute one of communicative resources mobilized together with different forms of experience such as words (verbally spoken), gestures, body positions, and orientations. A speaker displays or hand-writes a specific form of representation (e.g., force diagram) and might simultaneously talk, point out some part of it, and changes a position/orientation that he/she takes up with respect to it.

Mathematical representations constitute a sense-making corpus (i.e., bodies) toward which teachers and students bring their attention and carry out physical interactions. This is so because representations occupy part of the space and could be co-present there for the interaction with human actors. Even in the case of not grasping the sense of mathematical representations yet (e.g., (un-) balanced multiple forces), students can still point out a part, juxtapose their talk with them (e.g., asking a question, propose a claim), or take up a position beside them. The human living body thereby is mixed with other (sense-making) bodies in the interaction and is located always with respect to those other bodies (e.g., representations). In this, the living body is caught in to a (infinite) network of significations (i.e., signifying–signified relation) and can be marked by the cultural sense that goes beyond individuals' intentions. The notion of the *living* body or the *flesh* in phenomenological philosophies indicates this radically passive capacity of the human body (Henry, 1975). The living body in the active participation in communication is already caught up to embody cultural possibilities and therefore is constitutive part of cultural significations from the beginning—otherwise, we cannot explain how students do things in their everyday life and learn science. Therefore, the living body constitutes the very possibility for participating in representations-related practice (i.e., communication) and being part of scientific thinking (i.e., cultural practice). The living body is not only an individual gesture of thinking but also (cultural) expression itself (Merleau-Ponty, 1962), which in communication makes sense not only to the other but also to the actor him/herself. The following four aspects summarize the dynamic role of the living body that mediates cultural significations in the representation-related communication. First, the child's body constitutes the mediating hub in experiencing the mathematical bodies (representations). Second, the child's body constitutes the mediating hub in producing/receiving communication with/about mathematical bodies. Third, communication in the presence of mathematical bodies is distributed over a communicative unit that is produced by the body (e.g., verbally spoken words, gestures, body movements with respect to representations). Fourth, gestures, body orientations/positions, body movements, which are involved in experiencing mathematical representations, also are involved in communicating them. To sum up, these four aspects highlight that the living body is a signifier of the sense marked by mathematical representations and therefore signified by the culture that it realizes. The living body is cultural signification that unites and reunites sensual experiences and sense-making in concrete situation of communication in the presence of mathematical representations.

In what follows, we exemplify this holistic approach to mathematical sense-making and the child's cultural development around mathematical representations. The purpose of this analysis is to exemplify the role of living body that is caught up in the representation-related praxis and therefore mediates children's cultural development. The empirical example analyzed below is randomly selected from a second-grade elementary mathematics class in which children learn different shapes of geometrical objects and associated mathematical concepts. The episode is selected from a group-work in which three boys are engaged in knowing a geometrical shape that one of them (Gavin) has randomly produced using a rubber band and a plastic board consisting of a matrix of vertical pins (see Figure 1). The other two boys (Martin, Jaden) work to re-produce the same shape on their individual boards. In the conversation below, Gavin and Martin talk to assist Jaden, who is still working on his board, following their teacher's suggestion to help him in ways other than doing his work instead. That is, the two boys participated in producing a pedagogical discourse of which the role is to produce sense-making resources with respect to the original geometrical shape. The effectiveness of this discourse would be evidenced by Jaden's trajectory of reproducing the geometrical shape successfully while he attends to the communication. That is, the questions of what will be appropriate sense-making resources and how they work belong to an empirical region, which depends on their role in helping to see and reproduce the *geometrical* structure of the original shape (e.g., the number of sides and corners in specific angles). Therefore, we analyze their conversation and exemplify some aspects of the process in which children concretely develop cultural signification (i.e., *semiosis*) in and through communication. We attend to the role of the living body that involves the capacity to be marked by mathematical sense in active engagement with mathematical representations (bodies). We summarize cultural dynamics of knowing mathematical representations into three claims that we articulate concurrently together with our case analysis. First, the body reproduces and transforms cultural resources and connect them to mathematical representations. Second, the increase of heterogeneous sense-making resources in communication increases possibilities for realizing a new way of talking (i.e., a network of signification). Third, knowing mathematical representations emerges from the different, irreducible modes of communication as the integrated whole.

Transcript

01 Martin remember? (look at?)

02 [INSide] [down the-] (?) (look at?)
[((*Martin stretches his rubber hooked onto a pin and moves it back and forth behind the pin*))] (Figure 2a)
[((*Jaden holds his rubber band with his left hand and changes its position from one side to the other of the pin. Gavin turns his face toward Jaden.*))] (Figure 2b)

03 Gavin IN [side]
[((*Jaden holds his rubber band with his left hand and grabs its right part with his right hand.*))] (Figure 2c)

04 Martin [Inside (inside?)↑]
[((*Martin moves his rubber band back and forth five times behind the pin. Jaden stretches the right part of his rubber band around the pin.*))] (Figure 2d)

05 ((*Jaden lets his rubber band loosened.*)) (Figure 2e)

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06 Gavin    [inside (??)]
            [((Jaden grabs his loosened rubber band with his hands and
            stretches it toward the pin.))] (Figure 2f)

07 Martin   (exactly?) ((Jaden stretches his rubber band over the pin*. Jaden
            loosens his rubber band to behind the pin**.)) *(Figure 2g)
            **(Figure 2h)

08 Gavin    [IN:

09 Martin   [not [outside] outside will be
                [((Martin released his rubber band. Jaden presses down his
                rubber band onto the pin with his right fingers and stretches
                it with his left hand.))] (Figure 2i)

10          [this (0.2) inside will be this]
            [((Martin grabs his loosened rubber band at the bottom and
            stretches it upward.))] (Figure 2j)

11 Gavin    ((Gavin turns his gaze toward Jaden.))(Figure 2k)

12          [ya:]
            [((Jaden stretches his rubber band and hooks it to a pin on the
            left))] (Figure 2l)
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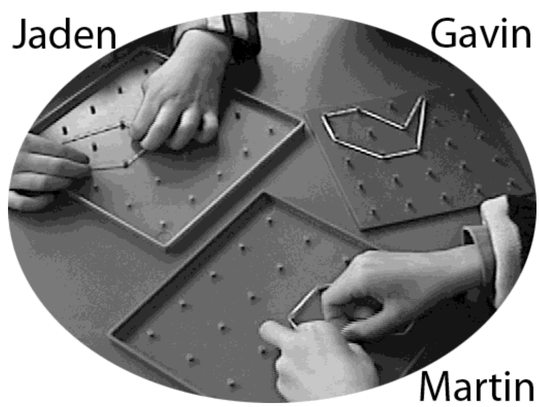

Figure 1. Three second-grade boys (Jaden, Gavin, and Martin) sit at a table in a mathematics classroom. On the table, there is a sample shape (a polygon with a reflex angle) that Gavin produced with a rubber band on a board. Jaden works to make a shape on his board. Martin and Gavin talk to help Jaden.

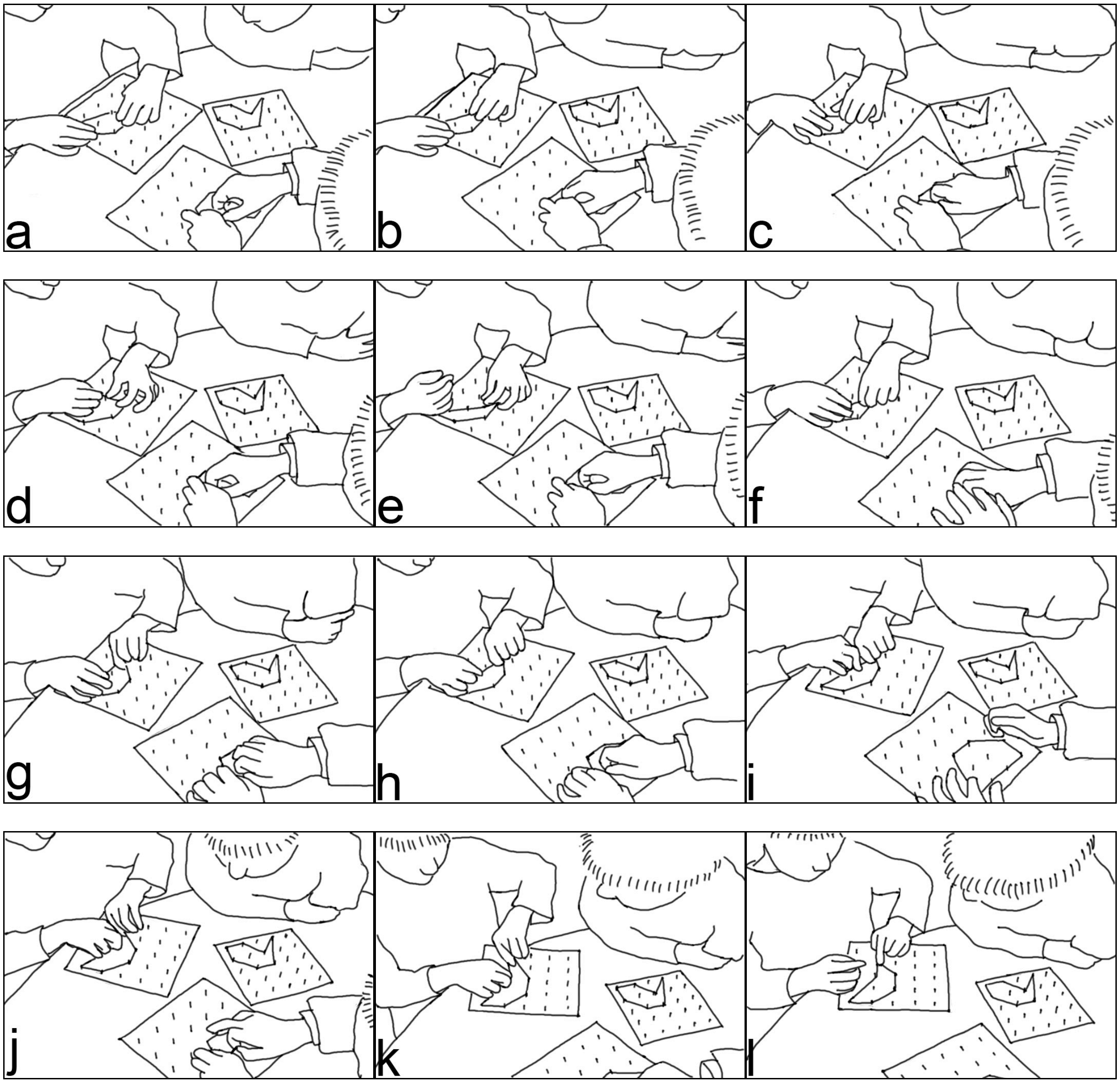

Figure 2. (a) Martin stretches his rubber hooked onto a pin and moves it back and forth behind the pin. (b) Jaden holds his rubber band with his left hand and changes its position from one side to the other of the pin. Gavin turns his face toward Jaden. (c) Jaden holds his rubber band with his left hand and grabs its right part with his right hand. (d) Martin moves his rubber band back and forth five times behind the pin. Jaden stretches the right part of his rubber band around the pin. (e) Jaden lets his rubber band loosened. (f) Jaden grabs his loosened rubber band with his hands and stretches it toward the pin. (g) Jaden stretches his rubber band over the pin. (h) Jaden

loosens his rubber band to behind the pin. (i) Martin released his rubber band. Jaden presses down his rubber band onto the pin with his right fingers and begins to stretch it with his left hand. (j) Martin grabs his loosened rubber band at the bottom and stretches it upward. (k) Gavin changes his gaze toward Jaden. (l) Jaden stretches his rubber band and hooks it to a pin on the left.

Description

The three boys sit at a table in a mathematics classroom (see Figure 5.1). On the table, there is a sample shape (a polygon) that Gavin produced (Figure 5.2a). Jaden makes a shape with a rubber band on his board. Martin holds a part of his rubber band on his board and says, “remember, look at” (turn 01, Figure 5.2a). He utters “inside” in increased speech intensity and simultaneously stretches his rubber band back and forth behind a pin (turn 02, Figure 5.2a). Martin continues uttering “down the” and Jaden simultaneously changes the position of his rubber band from one side to the other side of a pin (turn 02, Figure 5.2b). Gavin repeats the same word following Martin, “inside,” in increased speech intensity (turn 03). Jaden keeps holding the rubber band with his left hand and grabs the right part with his right hand (turn 03, Figure 5.2c). Martin repeats “inside” two more times and moves his rubber band back and forth five times behind the pin (turn 04, Figure 5.2d). Jaden stretches the right part of his rubber band around the pin (turn 04, Figure 5.2d) and lets it loosened (turn 05, Figure 5.2e). Gavin repeats “inside” and Jaden grabs the loosened rubber band (turn 06, Figure 5.2f). He stretches it toward the pin below (turn 06, Figure 5.2f). Martin says “exactly” (turn 07). Jaden stretches his rubber band over the pin and returns it back to a position behind the pin (turn 07, Figures 5.2g & 5.2h). Gavin speaks “in” in louder speech intensity (turn 08). Simultaneously Martin says “not outside” (turn 09). Martin releases the rubber band, and thereby lets it unhooked away from the pin (turn 09, Figure 5.2i).

Jaden presses down the rubber band onto the pin with his right fingers and stretches its left part with his left hand (turn 09, Figure 5.2i). Martin continues saying that “outside will be this” (turn 09). Martin grabs the loosened rubber band and begins stretching (turn 10, Figure 5.2j). Martin says, “inside will be this” and hooks his rubber band on the upper pin (turn 10). Gavin gazes at Jaden (turns 11) and utters “ya” (turn 12). Jaden hooks the left part of his rubber band onto a pin beside (turn 12).

Analysis

In this situation, Martin holds the rubber band with his hands and speaks “remember, look at” (turn 01). Martin’s utterances and his body orientation constitute a conversational turn that asks the other two boys’ attention toward him. Martin continues saying “inside down the,” which overlaps with his back-and-forth hand movement of a rubber band behind the pin (turn 02). Simultaneously, Jaden changes the position of the rubber band with his left hand from one to the other side of the pin behind (turn 02, Figures 5.2a–5.2b). Therefore, Martin’s actions constitute communicative resources for concretely articulating structural conditions for making a corner that is bent toward the inside of a polygon on his board (i.e., the same shape with the sample). The current position of the rubber band in Jaden’s board (Figure 5.2a) does not allow Jaden making the same shape of the corner in the sample. Once the left part of the rubber band is stretched down, the stretch of the rubber band around the pin would slip down together. Martin concretely shows this condition by verbally saying “inside” in the increased speech intensity and by moving his rubber band back and forth on one side of the pin—those bodily actions thereby make available where his rubber band is located around the pin. Martin’s actions constitute sense-making resources for distinguishing the sample shape on Gavin’s board from Jaden’s shape on his board. A way of talking geometrical representations emerges in the presence of them, which therefore shows the knowledgeability of the speaker with respect to the two representations. Therefore, the example shows that the knowledgeability is articulated through the bodily communicative resources including verbally spoken words, speech intensity and pitch, and hand movements. The living body constitutes cultural resources for signifying a physical structure of the geometrical representation that is materially present at hand but not salient (invisible) for Jaden.

The increase of sense-making resources in communication increases possibilities for the development of a network of cultural significations. Martin continues talking in the next (turn 02), which overlaps Gavin's utterance "inside" in the next (turn 03). Martin utters "inside" two more times and moves his hands holding the part of the rubber band five more times (turn 04, Figure 5.2d). The repetition of the literally same word (in different prosodies) together with wider hands-movements than before constitute sense-making resources for presenting the position of the rubber band even more saliently. In the next he provides another set of sense-making (embodied) resources. Martin releases the rubber band and lets it loosened at the bottom of his board (see Figure 5.2i). He utters, "not outside, outside will be this," which constitutes a contrast to his next utterance ("inside will be this") and hand-movement that hooks the rubber band up to the pin again (turn 09). The speaker's body produces additional sense-making resources (the word "outside," hands movements) that further clarify the spatial configuration of polygon and the position of the rubber band. The body increases sense-making resources available for communication participants by utterances and hand movements. This increase of heterogeneity (words, prosodies, bodily actions) develops a form of talking about the two mathematical representations on the table, in which each component marks sense with respect to one another. For example, the word "outside" stands together with "inside" and the pair of terms stands for the position of the pin and the spaces separated by the rubber band.

The development of cultural signification increases possibilities for knowing the conditions that Jaden deals with. Jaden changes the position of the rubber band from one side to the other when Martin talks (turn 02). In the next, he presses down the rubber band to the pin and stretches the right part to make a sharp corner (turn 03–04, Figures 5.2c–5.2d). Yet, this action turns out to be unsuccessful in making the shape as it leads him to turn the rubber band around the pin almost to the other side (180 degrees). Jaden releases the rubber band and moves his hands away from it (turn 05, Figure 5.2e). Jaden grabs the loosened rubber band with his hands and stretches it again down to the pin (turn 06, Figure 5.2f). Gavin's utterance, "inside" (turn 06), overlaps it. Jaden continues stretching the rubber band, which actually passes over the pin (turn 07, Figure 5.2g). Jaden a bit loosens the rubber band and makes it placed at the other side of the pin (Figure 5.2h). Jaden presses the stretched rubber band down to the pin with his right hand (turn 09, Figure 5.2i). This allows him stretching the left part of the rubber band with his left hand and not having to change the angle of stretching. The series of coordinated action show increased competency in dealing with the structural conditions, which allows him to hook the left part of the rubber band to a pin on the left (turns 10–12, Figures 5.2j–5.2l). Geometrically knowledgeable actions appear as Jaden participates in communication (listening, seeing) while he actively engage in the deployment of situational resources. The totality of all different communicative resources affects this organization of action. Jaden's organized action appears together with the development of Martin's talk. The two are concurrent rather than in the temporally linear cause-effect relationship. Knowing mathematical representations emerges from the different, irreducible modes of communication as an integrated whole.

Discussion

The three children's communication in this simple re-production task exemplifies the real-time development of semiosis in which a randomly produced shape is juxtaposed with the cultural/linguistic resources and with respect to them it comes to stand for something—i.e. geometrical representation that can be reproducible and sharable among the three students or with anyone else. A child may reproduce a shape on a piece of paper by using a pencil, by using a computer, or by using materials such as wooden sticks. The case example shows that the pedagogical practice for assisting someone in reproducing the shape is built upon a series of cultural signification and that this work is done by the living body, for example, that makes a distinction (conceptualization) between inside and outside around a pin. Martin's word "inside" marks sense by being juxtaposed with his body holding a part of rubber band on one side of a pin and talking while gazing at the position of Jaden's a rubber band. Martin's assisting talk might have increased the amount of sense-making work that Jaden has to conduct to re-produce the shape because now he has to attend to both the original sample and Martin's talk over his board and connect the two simultaneously. The increase of sense-making resources would involve the problem of unlimited semiosis until the connections among them become clear to Jaden. Following Vygotskian framework the theoretically endless process of significations would stop only when communication and thinking, which are two lines of development, encounters—i.e., the development of sense-making resources in communication (by Martin and Gavin) encounters Jaden's thinking with his rubber band and board while hearing and gazing at the other talking. At this point of encounter, the sense of the word "inside" and "outside" is coordinated with the material conditions of the board. Therefore, the role of the pedagogical discourse is to let this encounter between communication and thinking happen, which pertains to the child's cultural development. The encounter is made possible in and through the living body that can actively engage in knowing the objects (e.g., seeing the other's hand movement), and thereby can host and be marked (coordinated) by the whole different sense-making resources. The following three claims summarize the three aspects of the sense-making process. First, the body reproduces and transforms cultural resources and connects them to mathematical representations. Second, the increase of heterogeneous sense-making resources in communication increases possibilities for the development of a web of cultural signification. Third, knowing mathematical representations emerges from the different, irreducible modes of communication as the integrated whole.

CONCLUSIONS

Knowing mathematical representations consists of social practice in which heterogeneous communicative resources mediate children's experience of mathematical representations and its transformation. In this study, we articulate cultural dynamics of the sense-making process from a Vygotskian, holistic perspective. We exemplify that the living body constitutes a mediating hub that makes sense-making resources available in communication and develops a web of significations. The body mediates experiencing mathematical representations and associated science talk about/with them, and thereby constitutes a hub that hosts the dynamic relation between the development of communication and the experience of mathematical representations. The three theoretical claims presented in this paper propose that this encounter appears from the totality of communication performances. Mathematical representations come to stand for something not by itself but in relation to a web of (bodily) sense-making resources. Therefore, knowing mathematical representations is equivalent to (bodily and embodied) reading between different sense-making resources that constitute a series of references to bring about scientific/mathematical conceptions in its totality. This reading is performed by the living body that hosts the totality of communication in representation-related communication, which pertains to the child's cultural development.

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