Title	Positive feelings towards the learning of mathematics for low achievers
Author(s)	Tong, Cherng Luen, Leong, Yew Hoong and Quek, Khiok Seng
Source	*36th Annual Conference of the Mathematics Education Research Group of Australasia Incorporated (MERGA 2013) on "Mathematics Education: Yesterday, Today and Tomorrow", Melbourne, Australia, 7 – 11 July 2013*

Positive Feelings Towards the Learning of Mathematics for Low Achievers

Tong, Cherng Luen
Nanyang Technological University
<cherngluen.tong@nie.edu.sg>

Leong, Yew Hoong
Nanyang Technological University
<yewhoong.leong@nie.edu.sg>

Quek, Khiok Seng
Nanyang Technological University
<khiokseng.quek@nie.edu.sg>

A common area of difficulty and frustration for low achieving students is the operations involving negative numbers. The main objective of our innovations in this area with two classes of Year 7 low achieving students was to develop positive feelings towards mathematics, in particular of an appreciation for the 'reasonableness' and clear visualisation of the negative numbers 'in action'.

Mathematical Progress and Value for Everyone

The research reported in this paper is part of a project known as Mathematical Progress and Value for Everyone (MProVE). The overarching goal of MProVE is to explore the principles of a design that works for Singapore low-achieving students in the learning of lower secondary (Years 7 and 8) mathematics. A key feature of this project is that teachers in the project school play both the critical roles of actively designing the curriculum and consciously building the capacity needed to implement the curriculum successfully. We identified four domains to attend to: Mathematical content resources, problem solving disposition, study habits, and feelings towards the learning of mathematics. It is the last domain that is the focus of this paper.

Positive Feelings Towards the Learning of Mathematics

Fear of mathematics among many students, especially low achievers, is well-known. This negative feeling towards mathematics and poor mathematics achievement are closely linked (Ma & Kishor, 1997, McLeod, 1994). Thus, to help low achievers, teachers must have the skills to inculcate positive feelings towards the learning of mathematics (FLM). The general approach should be one of exposing students to repeated experience of positive emotions, as this seems to be the basis for fostering a stable positive FLM (Zan, Brown, Evans, Hannula, 2006). One concrete strategy to use, as advocated by Karsenty, Arcavi, and Hadas (2007), is to explore and legitimize the informal mathematical products by low achievers and helping them see their potential links to more conventional mathematical products—such as those required in standard examination scripts.

Through discussion with the teachers, we identified the topic of "operations with negative numbers" as a suitable place within the Year 7 syllabus to start the inquiry into FLM in curriculum design, for the following reasons: (1) The topic was scheduled to be taught early in the year. The teachers were of the view that FLM should be introduced early so that Year 7 students—their first secondary school year level—can have positive FLM right from the start of their secondary school experience; (2) the teachers viewed the topic as difficult to teach and confessed to using purely the teaching of arbitrary rules in their past approach. Thus, they welcomed the incorporation of FLM to make the learning of negative numbers more interesting to students; (3) in previous teacher development

In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA.

programmes with the project school, we developed manipulatives known as AlgeCards for the purpose of concretising algebra for the lower secondary students. For details on this innovation, the reader may refer to Leong et al. (2010) and Leong et al. (2011). The teachers saw the potential in extending the benefits of AlgeCards to the teaching of negative numbers, especially for the purpose of inculcating positive FLM.

The 'game' of adding and subtracting with AlgeCards is very similar to the use of different coloured counters that is well-known in the western-based literature: addition as "taking in" and subtraction as "taking away" counters. In the case of subtracting a negative number, it involves introducing 'zero pairs' of positive and negative counters and the removal of the negative counters. This two-stage process is equivalent to the one step of adding positive counters, thus illustrating, for example, that $3 - (-2) = 3 + 2$. Instead of using different-coloured counters to represent 1 and -1, the labels "1" and "-1" are imprinted on both sides of the AlgeCards. The approach was considered 'innovative' to the teachers as they were not aware of it prior to its introduction in the project. The teachers found the method 'concrete' and useful for sense-making, and thus amenable to the goal of helping students develop positive FLM.

We adopted *Lesson Study* as a way to plan, implement, and revise the lessons taught under this topic. Prior to the lessons, we met to discuss the overall approach as well as explore the instructional options at various junctures of the module. The resident teachers of the two selected Year 7 classes taught the lessons—Dexter in Cycle 1 and Alvin in Cycle 2. We observed the Research lessons in each cycle and had post-lesson meetings to discuss revisions.

The Study

The purpose of this exploratory study is to examine elements within the plan on "operations with negative numbers" that need to be tweaked in order to better inculcate positive FLM among the lower achieving students. Three further points need to be clarified at this juncture: (1) Although the focus is on improving instruction (esp in the area of FLM) for this particular topic of "negative numbers", the goals of MProVE go beyond the boundaries of this narrow scope. The learning about productive (or unproductive) ways to inculcate positive FLM in this module is ultimately meant for testing at other points along the lower secondary syllabus; (2) the "tweaking" is not meant to be a one-off act. In line with design experiment, which is the research methodology for MProVE, this iterative work of tweaking is an important aspect of improving the overall curricular design and theory revision; (3) the tweak is not just for instructional materials, but also extends to the prorgamme of capacity-building efforts for teachers.

To further narrow the scope of inquiry, the data used in this study were the video of the Research Lesson taught by Alvin on subtraction involving negative numbers. We first viewed the lesson cursorily and code for junctures where positive FLM among the students were conspicuously heightened. We made the judgments about heightening based on the overall level of participation of the students—evidence includes raising of hands to answer questions and actively working on tasks. The second step was to zoom-in for further analysis regions of the lesson where positive FLM (or the lack of it) was most conspicuous.

FLM in the Research Lesson

The length of the Research Lesson was 46 minutes. Alvin started the lesson by revising the 'rules' of addition involving negative numbers. He then highlighted—what he called

the "magic" of—"zero pairs" as being important when understanding how AlgeCards worked in number operations. At the 10-minute mark of the lesson, he began the formal introduction of "subtraction involving negative numbers". For the next 20 minutes, he proceeded along five sets of exercises: First set is of the type "5 – 3", followed by the types "3 – 5", "5 – (-3)", "(-5) – 3", and "(-5) – (-3)" respectively. He used AlgeCards to demonstrate for all the sets of questions. For the second and third sets, he gave some time for students' short practice following his demonstration. The remaining 16 minutes of class time was devoted to student practice and his response to their errors.

The student interest coding indicates that there was frequent positive response in the first 20 minutes of the lesson. As we scrutinised the video, we found that this segment of the lesson coincided with opportunities for students to answer the teacher's questions, his 'slips' when using the AlgeCards, his rather comical way of slapping the "zero pairs", as well as the participation of a student to present "4 – 6" using AlgeCards on the whiteboard. However, after the 20-minute mark, the difference was stark: there were no more indications of positive FLM responses from the students right up to the end of the lesson. This phenomenon is surprising to us, especially when the last 26 minutes of the lesson were heaviest on AlgeCards-use—the main tool we thought (in our planning phase) would stir positive FLM in students. As we relooked that portion of the lesson, the evidence seemed to suggest that students largely did not buy-in to the use of AlgeCards. This is first a reminder that, despite all the planning and discussion leading up to a lesson, we can still miss out on critical features for a successful enactment. In addition, the phenomenon prompts a deeper analysis into that segment of the lesson.

Zoom-In to Students' Resistance to AlgeCards

We were interested to investigate the reasons for the students' resistance to the use of AlgeCards even when they knew that they were getting the wrong answers. This was so despite Alvin's use of it and his emphasis of its importance throughout most of the lesson. As soon as he started the segment of the lesson on subtraction of numbers, he brought in the AlgeCards demonstration and distributed the sets—one for each student. He used the first set of exercises to highlight that "taking away" was the AlgeCards equivalence of subtraction. When he proceeded with the second set of exercises, he underlined the importance of AlgeCards in helping students make sense of this subtraction. For the subsequent sets of exercises, he used AlgeCards to demonstrate how he obtained the answers. He devoted the longest time for student practice after the fifth set of exercises. During that segment of the lesson, Alvin walked around to observe the students' work, and repeatedly urged individual students—but often loud enough so that the surrounding students could hear—to use the AlgeCards, especially when they could not obtain the correct answers without the tool:

32.54 If get stuck, use AlgeCards …
34.01 If don't use AlgeCards, make sure 100% correct.
34.23 Wrong already, can you try and use [AlgeCards] properly?
34.38 Wrong already, [then lifting up an AlgeCard tile to the class] this is not for fun. If you don't use this, and you are getting it wrong, it means you are not trying.
35.06 You also, can you do it properly? [Looking up to the whole class] Again, with these AlgeCards in your hand, you shouldn't get any question wrong, if you do it properly.

Despite Alvin's emphasis, generally very few students actually used the AlgeCards during the seatwork. The broad trajectory of use was as follows: After the second set of exercises, only about three students used it; immediately after the fifth set of exercise, none used it; after Alvin's repeated coaxing (as seen in the extracts above), about half the

students were seen lifting up the cards and placing them on the table; a few minutes after, the numbers seen actually using the AlgeCards dwindled to about five.

It seemed clear to us that the students in the lesson did not 'buy-in' to AlgeCards, nor derive enjoyment working with it, thus were short of meeting the FLM intent of introducing this approach. Based on t he data available, we conjecture a number of possibilities: (1) Alvin, although he used the AlgeCards and even encouraged its use, fell short of *insisting* its use right from the start. His was a qualified message, "use if you can't get the answer". Students might have interpreted this non-insistence as unessential encumbrance; (2) there was no clear juncture in the earlier sets of exercises where the motivation for using AlgeCards was compelling. After the "3 – 5" type of questions, we heard some students shouting out the correct answer "-2" even before Alvin used the AlgeCards to show the working. To them, the ability to obtain the 'correct answer' without AlgeCards would not have helped in providing the motivation to use it. That another student at about the same juncture asked, "'cher, exams no AlgeCards right?" might reflect the mood of the students then—"why learn it when I don't need it and can't use it for exams?" (3) In the earlier sets of exercises, the pauses for practice were too short for students to pick up sufficient familiarity with the AlgeCards to figure out the operations for themselves, so much so that even when they realised later that they could not get the answers right, they were not able to fallback productively on the AlgeCards despite promptings from Alvin; and (4) Just before the long practice after the fifth set of exercises, Alvin led the students to see the "general rule" that "(-5) – (-3) = (-5) + 3", and said to the class, "with this rule in mind, solve the questions …". In other words, since the students were already told the "rule", there was really no motivation—a further blow to the already low motivations up to that point in time—to use the AlgeCards.

Tweaks for the Future

We should have known better. In our earlier use of AlgeCards in an earlier project on quadratic factorisation with the same school (Leong et al., 2010), we found that careful 'buy-in' and subsequent 'wean-off' of the AlgeCards were important features of the lessons. In the planning of this module, we were perhaps not sure as yet how critical they were. This study has somewhat confirmed—as a kind of non-example—the twin features of 'buy-in' and 'wean-off' as central in the inculcation of FLM and in productive learning. We intend to build these features into subsequent instructional design both for the re-work of this module and for other parts of the lower secondary curriculum.

References

Leong, Y. H., Yap, S. F., Subramaniam, T., Mohd Zaini, I. K., Zhang, Q. T., Ong, E. Y., & Tham, Y. H. (2011). One worksheet, different learning trajectories. *Maths Buzz, 12*(1), 2-6.

Leong, Y. H., Yap, S.F., Teo, M. L., Thilagam, S., Karen, I., Quek, E. C., & Tan K. L. (2010). Concretising factorisation of quadratic expressions. *The Australian Mathematics Teacher, 66*(3), 19-25.

Karsenty, R, Arcavi, A., Hadas, N. (2007). Exploring informal mathematical products of low achievers at the secondary school level. *Journal of Mathematical Behavior, 26*, 156-177.

Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude towards mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education, 28*(1), 26-47.

McLeod, D. B. (1994). Research on affect and mathematics learning in the JRME: 1970 to the present. *Journal for Research in Mathematics Education, 25*(6), 637-647.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*, 113-121.