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# Diagnosing Errors In Mathematics

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Remarks such as 'good work' when a pupil has obtained, say, 9 out of 12 sums correct and 'poor effort' when a pupil only obtained, say, 3 out of 12 sums correct may have their place in assessment but they definitely do not help the pupil who is really crying for help in those incorrect sums.

While teacher and pupil very often take pride in the percentage of correct responses, a second look at the scored exercise to see which exercises the pupil missed, what patterns of errors prevail and what can be done to correct the situation has to be the concern of all mathematics teachers.

Suppose, for example, that the teacher finds that a pupil is not doing well computationally. Where does the major breakdown occur – in the addition, subtraction, multiplication or division? Is he making one error twenty times, or twenty different errors? It is imperative that the teacher determines as accurately as possible the nature of the error. The treatment of one's difficulties, then, is dependent upon accurate diagnosis of errors made. Errors are marvellous vehicles, when put to a maximum use, for clarifying misunderstandings and diagnosing weaknesses.

Information for diagnosis can come from all the work pupils do. During regular class teaching the teacher may be diagnosing understanding continually from the puzzled look on the pupils' faces, from the number of hands raised to answer questions and so on. Questions and answers are a vital part of the method and continuous evaluation of teaching, but as the sole source of diagnosing understanding, class questioning can lead to an obvious misconception. Class questioning always leads to the correct answer being given by someone – if it does not, the teacher will supply the answer, repeat and recast the question and try again until a pupil supplies the correct response. As a collective, therefore, the class or group understands everything, but this is no guarantee that each

member of the class has grasped it all. It is only through the individual work of the pupils, considered separately and collectively, that full diagnosis can take place.

Diagnosis must be based to a large extent on individual responses, both oral and written, in classwork, homework, tests or whatever. In mathematics, in addition to the written record a pupil makes with pencil and paper, it is often helpful to know the *thinking associated* with that record.

The teacher, in diagnosing pupil performance, has at her disposal, basically, four ways to proceed:

(i) observe the pupil at work

(ii) interview the pupil and through discussion analyze his work

(iii) analyze his daily assignments or board work, or

(iv) administer a diagnostic test.

To make the best decision possible, it is necessary to collect evidence, or data, which is both adequate to the purpose at hand and accurate. Teachers should use the techniques that will best provide the data sought. Depending upon the objective measured, this may or may not involve individual tests. Once the information has been collected, accurate records of errors diagnosed must be maintained.

Very often identifying a pattern to explain a failing may not be easy. West (1971) gives as an example of an error which needed a great deal of ingenuity to spot. He refers to a pupil whose answers to simple divisions look like this:

$$\begin{array}{r} 701 \\ 7\,\overline{)\,5114} \end{array} \qquad \begin{array}{r} 701 \text{ r } 4 \\ 8\,\overline{)\,5765} \end{array} \qquad \begin{array}{r} 21 \\ 4\,\overline{)\,102} \end{array}$$

At first it seems that he does not know his division and the answers are haphazard expressions of this ignorance. But careful analysis shows that he does not know them, but is committing one systematic error. Instead of carrying the remainder as a number of 'tens' into the next column, he is adding it to the next digit so that 102 ÷ 4 becomes 10 ÷ 4 = 2 carry 2, 2 + 2 =4, 4 ÷ 4 = 1. He was failing to regroup (or rename) the remainders in the subdivisions. This is not easy to detect, but when successful, is a useful warning compared to making a hasty diagnosis from the written answers alone. Five minutes spent sitting with the pupil while he talked himself through a few divisions would show up the error of his ways. Hence the need to *work aloud.*

When at times even the closest examination of an incorrect exercise does not convey to the teacher just what the thought pattern is, a very good diagnostic procedure is to ask the student to work the exercise aloud to the teacher, who listens carefully. Often, the erroneous concept comes to the surface almost instantly. For example, a pupil of mine some years ago, when asked to multiply 3 times 38 did the following:

$$\begin{array}{r} {}^{2}38 \\ \times \quad 3 \\ \hline 154 \\ \hline \end{array}$$

When I asked him to work the sum aloud he said, "Three 8's are 24; put down the 4 and carry the 2; 2 plus 3 is 5, 3 times 5 is 15". He added the carry number, then multiplied. Every isolated step was correct, but his sequencing, based on a misconception, was incorrect. Through the work-aloud procedure, I was able to locate the difficulty with a single sum. Correction was a simple process.

Lankford (1974) offers in his article useful tips to the teacher who wishes to try some diagnostic interviews with his pupils.

Through individual oral interviews with several pupils in a class a teacher will soon become aware, very often to his surprise, of wide variations in computational strategies employed by his pupils. He can then plan his teaching either to encourage and reinforce those

variations that are acceptable or try to achieve more uniformity in computational practice for his pupils by getting them to adopt and use a single strategy approved by him.

It may seem desirable, but highly impractical, for a busy teacher to arrange individual oral pupil interviews for all the pupils in the class. It would be appropriate, however, that from time to time the failing pupils be interviewed. The interview may take only a few minutes in the corner of the room while other pupils are doing seat work. The interview may use the exercises assigned to the whole class.

In order to use the diagnostic method successfully, the teacher must have a genuine desire to help pupils identify their own problems. Unless this attitude is present, no procedure will work. A proper attitude on the part of the teacher manifests itself in many ways. One such way is when a teacher pays close attention to pupil output. The following illustration will clearly show why. A teacher gives his pupils some exercises to be worked in class. Out of 25 exercises, Mary correctly works 20. Although the teacher – and Mary – may take pride in her percentage of correct responses, it is really in the five she missed that Mary is crying for help. The diagnostically oriented teacher gives a second look at the scored paper to see which exercises Mary missed, what patterns of errors prevail and what can be done to correct the situation. While many teachers look only at "number right", the diagnostic teacher is more concerned with number wrong, which ones, and why.

I wish to conclude my article with an 'ingenious' error given by one of my pupils, fourteen years of age, in a question on standard form. She wrote.

$$10^{-8} = 0.0000000\frac{1}{8}$$

This is obviously an instructional error, because its origins lie clearly in what she has been taught. Ten to the minus something implies a string of zeros after the decimal place and she has put the right number of them. Ten to a negative power, also means one over

something, and so she included a one over something. The curious number which resulted comes about from an honest attempt to make sense of what she has heard. The major insight to be gained from diagnostic assessment in mathematics is that it cannot be taken for granted that what is heard and understood by the pupil is what was said and intended by the teacher. It is only by constantly checking and rechecking that the two can be made to correspond exactly.

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